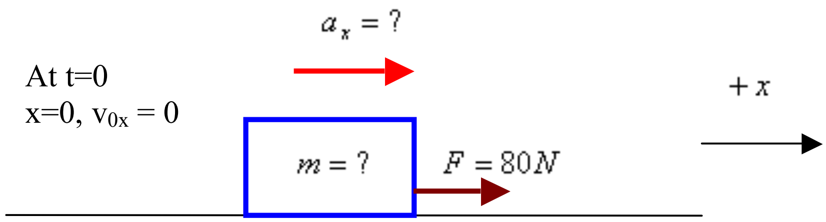
Newton's Law

Problem 1

A dockworker applies a constant horizontal force of 80N to a block of ice on a smooth horizontal floor. The frictional force is negligible. The block starts from rest and moves a distance 11.0m in a time 5.00s.

(a) What is the mass of the block of ice?



Use the kinematic equation $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 = \frac{1}{2}a_x t^2$

or $a_x = \frac{2x}{t^2} = \frac{2(11m)}{(5s)^2} = 0.88\frac{m}{s^2}$

Using Newton's 2nd law $F_x = F = ma_x$, and $m = \frac{F}{a_x} = \frac{80N}{0.88\frac{m}{s^2}} = 90.9kb$

(b) If the worker stops pushing at the end of 5.00s, how far does the block move in the next 5.00s?

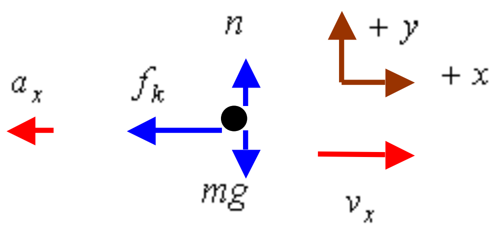
At the end of 5.00s the speed of the ice will be $v_x = a_x t = \left(0.88\frac{m}{s^2}\right)(5.00s) = 4.4\frac{m}{s}$

After 5.00s, the ice travels at a constant velocity (force is zero) of $v_x = 4.4\frac{m}{s}$, so in the next 5.00s it will move $v_x t = (4.4m/s)(5.00s) = 22m$.

Problem 2

Stopping Distance.

(a) If the coefficient of kinetic friction between tires and dry pavement is $\mu_k = 0.80$, what is the shortest distance in which an automobile can be stopped by locking the brakes when traveling at $v_{0x} = 28.7m/s$? v_x



y-component

Use **Newton's first law**

$n = mg$

x-component

friction force is $f_k = \mu_k n = \mu_k mg$

Use **Newton's second law**

$-f_k = -\mu_k mg = -ma_x \qquad a_x = \mu_k g = (0.80)(9.8m/s^2) = 7.84m/s^2$

Apply the kinematic equation

$v_x^2 = (0)^2 = v_{0x}^2 + 2a_x x = (28.7m/s)^2 + 2(-7.84m/s^2)x \;, x = \dfrac{(28.7m/s)^2}{2(7.84m/s^2)} = 53m$

(b) On wet pavement the coefficient of kinetic friction may be only $\mu_{wet} = 0.25$. How fast should you drive on wet pavement in order to be able to stop in the same distance as in part (a)? (*Note:* Locking the brakes is *not* the safest way to stop.)

In this case the friction force is

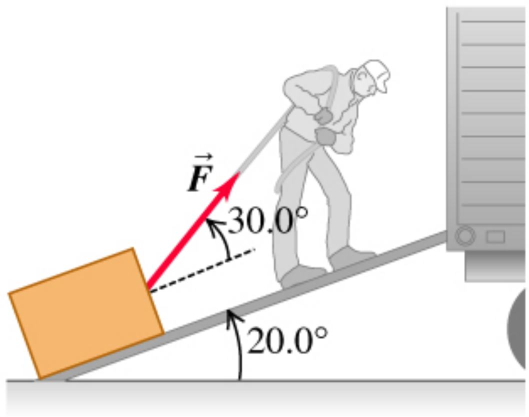
$-f_k = -\mu_{wet} mg = -ma_x \rightarrow a_x = \mu_{wet} g = (0.25)(9.8m/s^2) = 2.45m/s^2$

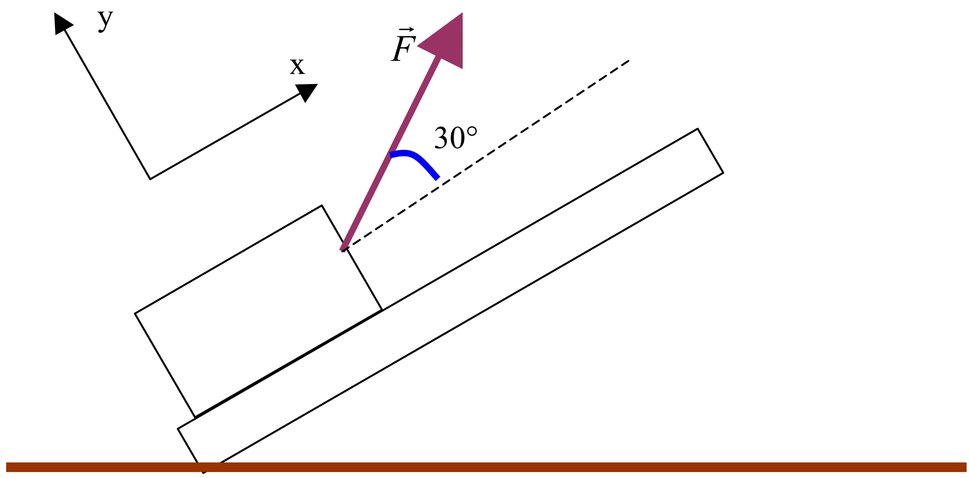
$v_x^2 = (0)^2 = v_{0x}^2 + 2a_x x = v_{0x}^2 + 2(-2.45m/s^2)(53m)$

$v_{0x} = \sqrt{2(2.45m/s)(53m)} = 16m/s$

Problem 3

In the diagram below a) What is the magnitude of $\vec{F}$, so that the component parallel to the ramp is $F_x = 60N$? b) How large will the component perpendicular to the ramp, F_y be in this case?



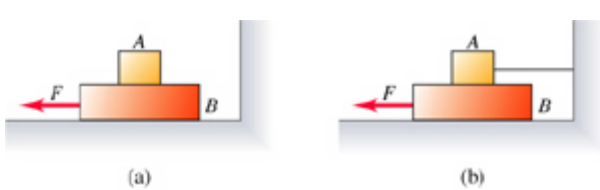


It is most convenient to use the x-y coordinate as shown. The x axis is parallel to the ramp, while the y axis is perpendicular to the ramp.

a) x-component If $F_x = FCos30^\circ = 60N \Rightarrow F = \dfrac{60N}{Cos30^\circ} = 69.3N$

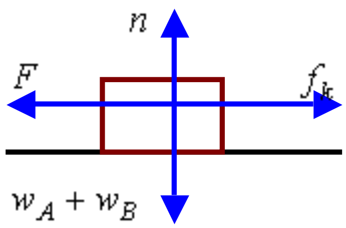
b) y-component If $F_y = FSin30^\circ = 69.3N(0.5) = 34.7N$

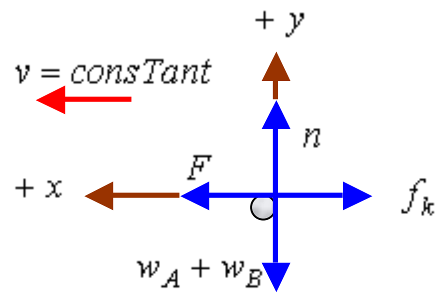
Problem 4



Block *A* in the figure weighs w_A =1.20 N and block *B* weighs w_B =3.60N. The coefficient of kinetic friction between all surfaces is μ_k =0.300.

A. Find the magnitude of the horizontal force $\vec{F}$ necessary to drag block B to the left at constant speed if A rests on B and moves with it (figure (a)).
In this case, the situation is the same as a combined block that weights $w_A + w_B$ as depicted below





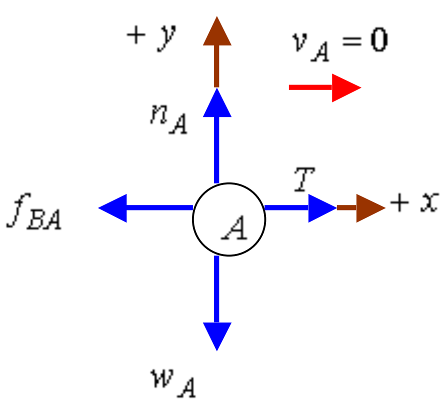
<u>y-component</u> Using **First law** $n = w_A + w_B$

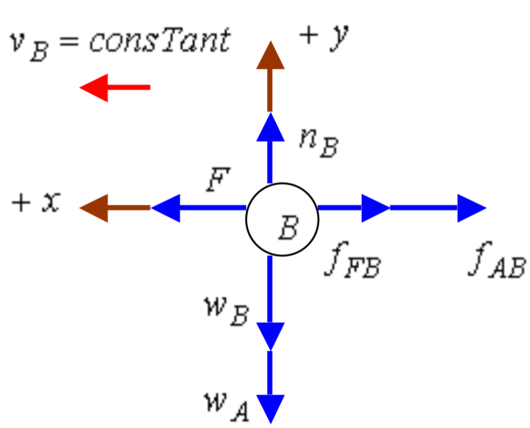
<u>x-component</u> $f_k \equiv$ friction force between floor and combined crate

$f_k = \mu_k n = \mu_k (w_A + w_B) = 0.300(1.20N + 3.60N) = 1.44N$

Using **First law** $F - f_k = 0$ $\qquad F = f_k = 1.44N$

B. Find the magnitude of the horizontal force $\vec{F}$ necessary to drag block B to the left at constant speed if A is held at rest (figure (b)).





BLOCK A

<u>y-component</u> $n_A = w_A = 1.20N$

x-component $v_A = 0$, so $T = f_{BA}$, the friction force of block B on A is
$f_{BA} = \mu_k n_A = 0.3(1.20N) = 0.360N$

BLOCK B

y-component $n_B = w_A + w_B = 1.20N + 3.60N = 4.80N$

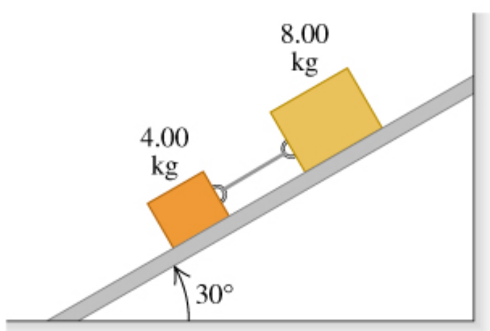
x-component There are two friction forces

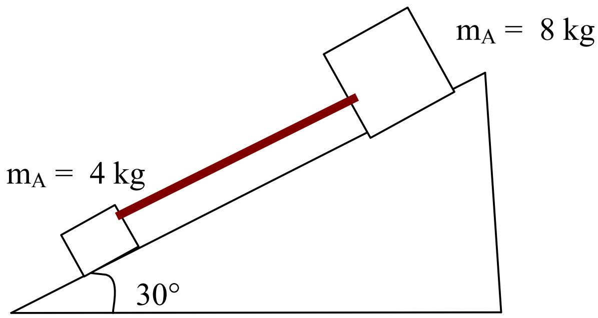
$f_{FB} \equiv$ **friction** force of floor on block B, $f_{FB} = \mu_k n_B = 0.3(4.80N) = 1.44N$

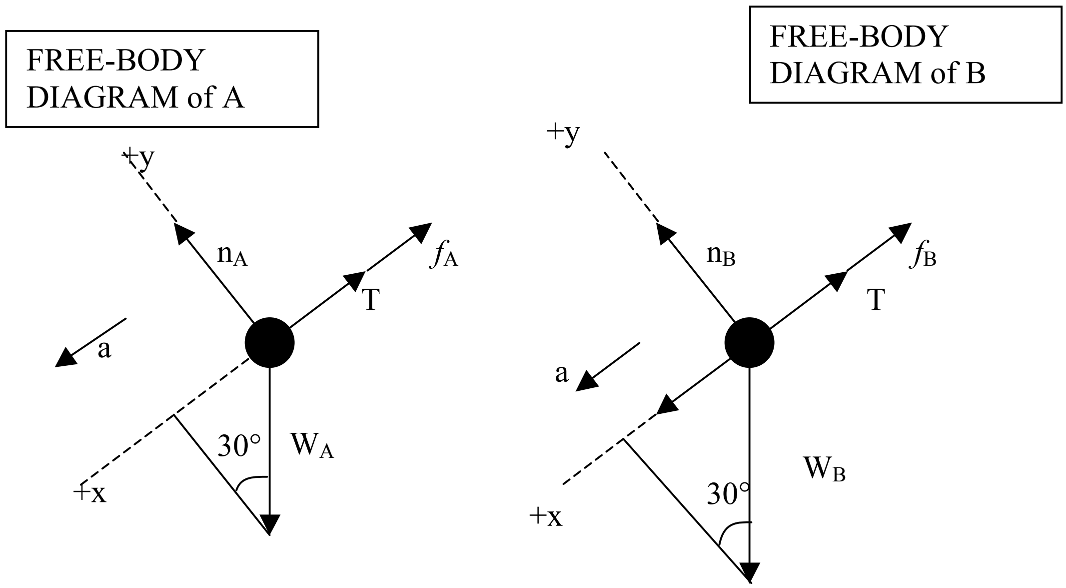
$f_{AB} \equiv$ **friction** force of block A on B. f_{AB} and f_{BA} are **action-reaction** pair (**third law**) $f_{AB} = f_{BA} = 0.360$

$v_B = consTant$,so using **first law** $F = f_{FB} + f_{AB} = 0.360N + 1.44N$

Problem 5
In the figure below two blocks connected by a rope slides down an incline, where the coefficient of kinetic friction between the 4kg box and the incline is 0.25 and between the 8kg box and the incline is 0.35. What is the acceleration of the boxes and the tension in the rope? What happens if the positions of the boxes are exchanged?







(a) First note that the friction between block A and the incline ($\mu_{kA} = 0.25$) is less than between block B and the incline ($\mu_{kB} = 0.35$). In the absence of the rope, block A will accelerate down the incline at a greater rate than block B. So their acceleration will not be the same. But since the blocks are attached by a rope, the rope will become taut, and, as shown in the diagram, the tension T will pull block A up decreasing the acceleration rate, and pull block B down increasing its acceleration rate. In the end both blocks will have the same acceleration.

For block A:

y-component, $F_y^{net} = n_A - W_{Ax} = 0 \rightarrow n_A = m_A g \cos 30^\circ = 4kg\left(9.8m/s^2\right)0.87 = 34N$

$f_A = \mu_{kA} n_A = (0.25)(34N) = 8.49N$

x-component, $F_x^{net} = W_{Ax} - f_A - T = m_A a \rightarrow m_A g \sin 30^\circ - 8.5N - T = 4a$

$(4.0kg)\left(9.8m/s^2\right)0.5 - 8.5N - T = 4a \rightarrow 4a + T = 11.1$, (1)

where units are omitted for simplification.

For block B:

y-component, $F_y^{net} = n_B - W_{Ax} = 0 \rightarrow n_B = m_B g \cos 30^\circ = 8kg\left(9.8m/s^2\right)0.87 = 68N$

$f_B = \mu_{kB} n_B = (0.35)(68N) = 23.8N$

x-component, $F_x^{net} = W_{Bx} - f_B + T = m_B a \rightarrow m_B g \sin 30^\circ - 23.8N + T = 8a$

$(8.0kg)(9.8m/s^2)0.5 - 23.8N + T = 8a \rightarrow 8a + T = 15.4$, (2)
where units are omitted for simplification.
Add (1) and (2) $12a = 26.5 \rightarrow a = 2.21m/s^2$

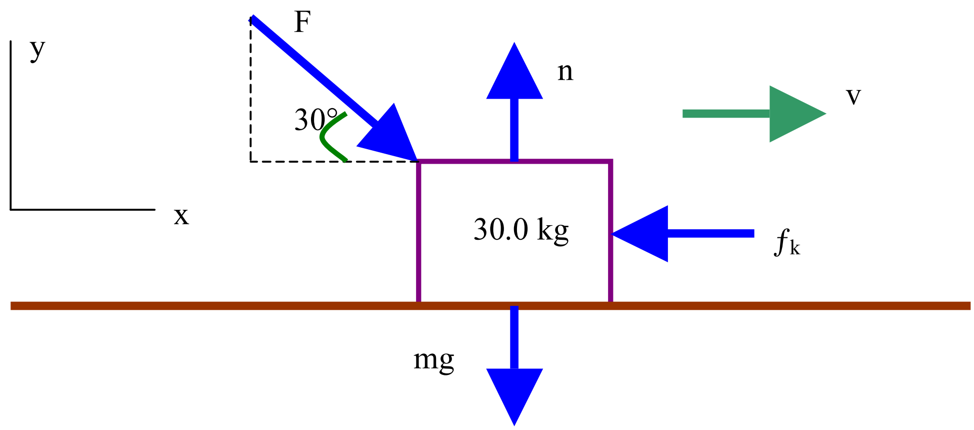
(b)
Using (1) $4a + T = 11.1 \rightarrow T = 11.1N - (4kg)(2.21m/s^2) = 2.26N$
Using (2) $8a - T = 15.4 \rightarrow T = (8kg)(2.21m/s^2) - 15.4N = 2.26N$

(c) If the positions of the blocks were reversed, block B will be at lower position will accelerate at a lower rate than block A, and the rope will be slack.

Work-Energy

Problem 6

A worker pushes a 30.0 kg crate a distance of 4.5m by pushing with a downward force at an angle of 30° below the horizontal. The coefficient of friction between the crate and the floor is 0.25. What magnitude of force must be applied to keep the crate moving at constant velocity? How much work is done by this force after the crate has been pushed 4.5m? How much work is done by friction? What is the total work done by the crate?



(A) <u>y-component</u> $F_y^{NET} = n - F\sin 30^\circ - mg = 0 \Rightarrow n = \frac{F}{2} + (30kg)(9.8m/s^2) = \frac{F}{2} + 294N$

<u>Friction</u> $f_k = \mu_k n = \mu_k\left(\frac{F}{2} + 294N\right) = 0.25\left(\frac{F}{2} + 294N\right) = \frac{F}{8} + 73.5N$

<u>x-component</u> $F_x^{NET} = F\cos 30^\circ - f_k = 0 \Rightarrow F\cos 30^\circ - \frac{F}{8} - 73.5N = 0 \Rightarrow 0.741F = 73.5N$

F = 99.2 N.

Using the above $f_k = \frac{F}{8} + 73.5N = \frac{99.2N}{8} + 73.5N = 85.9N$.

(B) $W_A = F_x(4.5m) = F\cos 30^\circ(4.5m) = (99.2N)(0.866)(4.5m) = 387J$. $W_A > 0$ since the x-component of the applied force is in the direction of displacement.

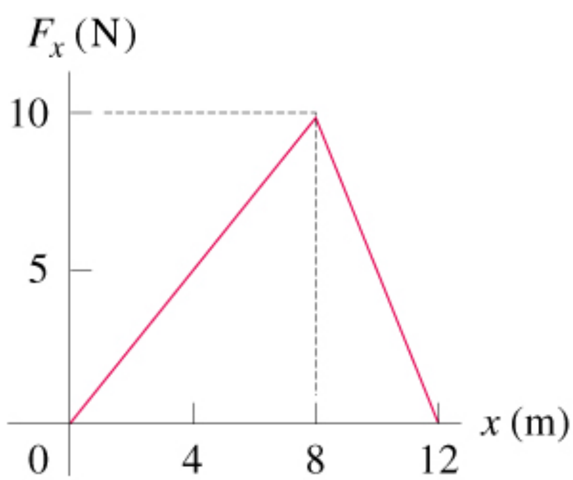
(C) $W_f = -f_k(4.5m) = -(85.9N)(4.5m) = -387J$. $W_f < 0$ since the friction force is in the opposite direction of displacement.

(D) The normal force and gravity is perpendicular to the direction of motion, so the work done by these forces are zero.

(E) The total work done is $W_T = W_A + W_f = 387J - 387J = 0$. Note here that by the work-energy theorem is $W = \Delta K = 0$, and there is no change in kinetic energy of the box (the speed of the box remains constant).

Problem 7

In the diagram below a force is applied on a 10.0kg crate moving in a straight line (the x axis). Calculate the work done from a) x = 0 to x = 8.0 m; b) x = 8 m to x = 12.0 m; c) x = 0 to x = 12.0 m.



As discussed in class for 1D motion, given an F_x vs. x graph the work done is the area under the curve.

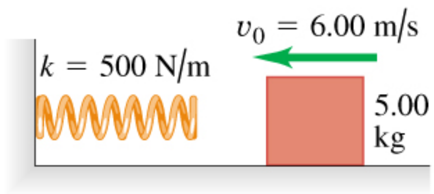
(A) For the interval from x = 0 m to x = 8.0 m, the work done is the area of triangle A, $W = \frac{1}{2}(height)(width) = \frac{1}{2}(10N)(8.0m - 0m) = 40J$.

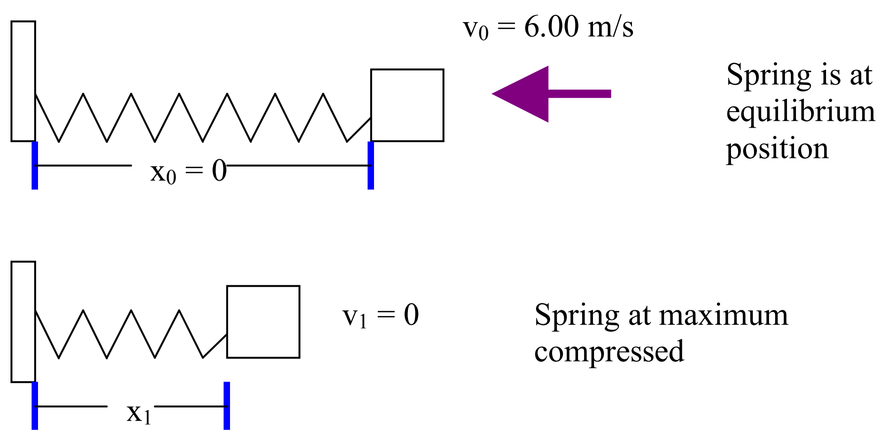
(B) For the interval from x = 8.0 m to x = 12.0 m, the work done is the area of triangle B, $W = \frac{1}{2}(height)(width) = \frac{1}{2}(10N)(12.0m - 8.0m) = 20J$.

(C) For the interval from x = 0.0 m to x = 12.0 m, the work done is the area of triangle A + B, $W = \frac{1}{2}(height)(width) = \frac{1}{2}(10N)(12.0m - 0.0m) = 60J$.

Problem 8

In the diagram below, a 5kg box with initial speed of $v_0 = 6m/s$ collides and compresses a mass-less spring with k = 500N/m. There is no friction. Calculate the maximum distance the spring is compressed. If the spring is compressed by no more than 0.150 m what is the initial speed $v_0 = ?$





(A) Use the work-energy theorem, $W = \Delta K$, where W is the work done on the box (by the spring) and ΔK is the change in the kinetic energy:

$$W = \Delta K \Rightarrow -\left(\frac{1}{2}kx_1^2 - \frac{1}{2}kx_0^2\right) = \frac{1}{2}mv_1^2 - \frac{1}{2}mv_0^2 \Rightarrow -\frac{1}{2}kx_1^2 = -\frac{1}{2}mv_0^2$$

$$x_1 = \sqrt{\frac{mv_0^2}{k}} = \sqrt{\frac{(5.00kg)(6.00m/s)^2}{500N/m}} = 0.60m$$

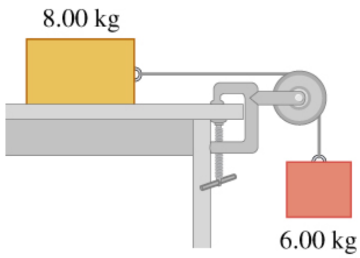
(B) If the spring is compressed by 0.150 m (x_1 = 0.150 m), find v_0 =?. Again

$$W = \Delta K \Rightarrow -\left(\frac{1}{2}kx_1^2 - \frac{1}{2}kx_0^2\right) = \frac{1}{2}mv_1^2 - \frac{1}{2}mv_0^2 \Rightarrow -\frac{1}{2}kx_1^2 = -\frac{1}{2}mv_0^2$$

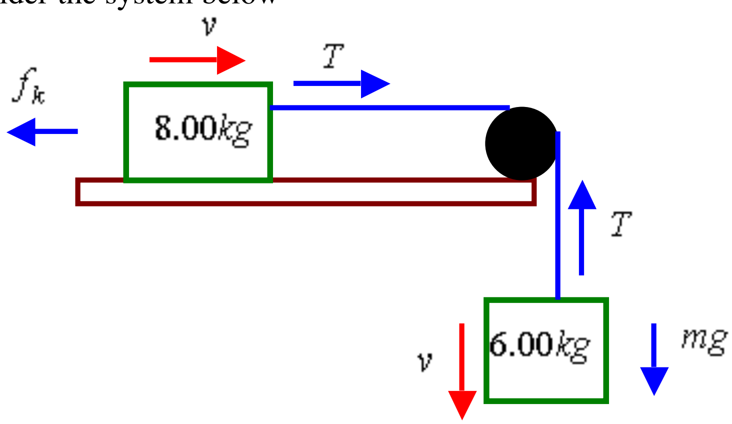
$$v_0 = \sqrt{\frac{kx_1^2}{m}} = \sqrt{\frac{(500N/m)(0.150m)^2}{5.00kg}} = 1.5m/s$$

Problem 9

In the diagram below the coefficient of kinetic friction between the table and the 8kg crate is 0.250. Use the work energy theorem to calculate the speed of the 6kg block after it has fallen 1.50 m.



Consider the system below



If the blocks is released from rest find the speed of the blocks after the 6.00 kg block has descended 1.50 m. $\mu_k = 0.250$ .This problem can be solved by combining **Newton's law** and **kinematic equations**, but it is easiest to use the **work-energy** method:

- Note that the work done by the tension T on the 8.00 kg block is positive and is equal in magnitude to work done by the tension T on the 6.00 kg block, which is negative. So the **work done** by the **tension** T in the rope is **zero**. **Make sure you understand this**.
- Hence we need only consider the work done by the **friction** f_k and **gravitational force**, mg.

Work done by friction on 8 kg block is

$$-f_k s = -mg\mu_k s = -(8.00kg)(9.8m/s^2)(0.250)(1.50m) = -29.4J$$

Work done by gravitational force on 6.00 kg block is

$$mgs = (6.00kg)(9.8m/s^2)(1.50m) = 88.2J$$

The total work done on the system is

$$W = mgs - f_s s = 88.2J - 29.4J = 58.8J$$

Using the **work-energy theorem** and the fact that the initial speeds are zero

$$W = \Delta K = \frac{1}{2}mv_F^2 \qquad v_F = \sqrt{\frac{2W}{m}}$$

Since the 8.00 kg and 6.00 kg blocks move together with the same speed, m is the combined mass of the 6.00 kg and 8.00 kg block. Hence m = 14.00kg. Again **make sure you understand this**. So we obtain

$$v_F = \sqrt{\frac{2(58.8J)}{14.00kg}} = 2.89m/s$$

Problem 10

Power of the Human Heart. The human heart is a powerful and extremely reliable pump. Each day it takes in and discharges about 7500 L of blood. Assume that the work done by the heart is equal to the work required to lift this amount of blood a height equal to that of the average American female (1.63 m). The density (mass per unit volume) of blood is 1.05×10^3 kg/m^3. A) How much work is done by the heart in a day? B) Calculate the power in watts.

A) We know that $1L = 10^{-3}m^3$. In term of mass, this is equal to

$$m = (7500L)\left(10^{-3}\frac{m^3}{L}\right)\left(1.05\times10^3\frac{kg}{m^3}\right) = 7.88\times10^3 kg$$ of blood.

To transport this amount of blood 1.63 m upward, the heart must perform

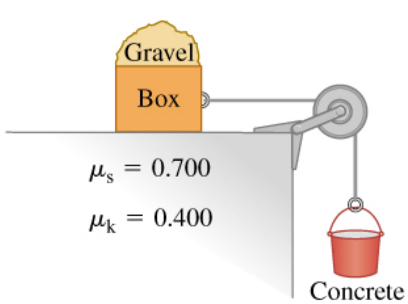
$$W = mgh = (7.88\times10^3 kg)(9.8m/s^2)(1.63m) = 1.26\times10^5 J$$

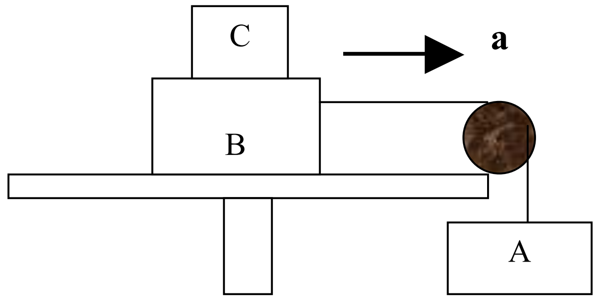
B) In one day there is $1day\times24\frac{hr}{day}\times60\frac{\min}{hr}\times60\frac{s}{\min} = 86400s$

So the average power output of the heart is $P_{av} = \frac{1.26\times10^5 J}{86400s} = 1.46watt$.

Problem 11

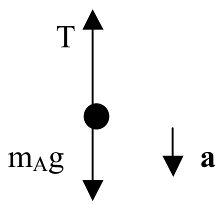
In diagram below a 65 kg bucket of concrete hangs from a cable connected to a 80 kg box with 50 kg of gravel on top. The friction between the box and the floor is as shown. A) If the system is at rest find the friction force on the box and the tension in the rope. B) If the gravel were removed the bucket will start to move down. Find the speed of the bucket after it has descended 2 m.





The rope is massless and there is no friction in the pulley

(A) Free Body diagram on object A (bucket)



System at equilibrium (at rest) so net force is zero

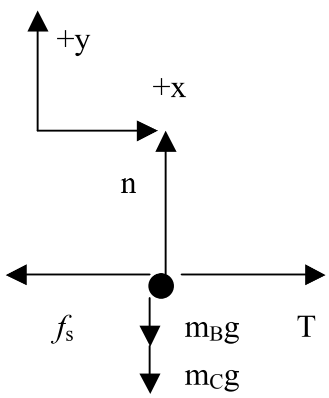
$F_y^{NET} = T - m_A g = 0$

$T = m_A g + m_A a$

$= (65.0\text{ kg})(9.8\text{ m/s}^2)$

$T = 637\text{ N}$

Free-Body diagram on composite B+C object (box + bag of gravel)



<u>y-component</u>

Using **Newton's first law**

$n = (m_B + m_C)g$

<u>x-component</u>

$f_s < n\mu_s = (m_B + m_C)g\mu_s = (80\text{kg}+50\text{kg})(9.8\text{m/s}^2)(0.7) = 892\text{ N}$

System at equilibrium so net force must be zero

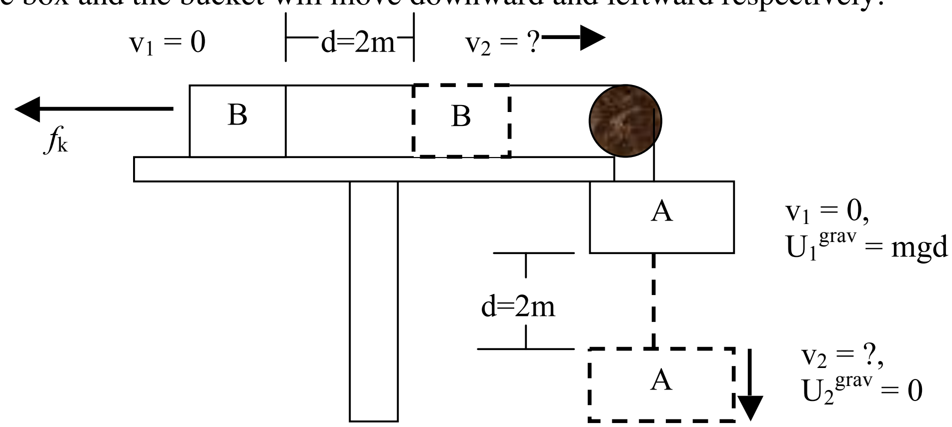
$F_x^{NET} = T - f_s = 0$

Since the tension is less than the maximum value of static friction, $T = 637\text{ N} < 892\text{ N} = n\mu_s$, friction equals the tension

$f_s = T = 637\text{ N}$

Since the bag of gravel rest on the box without moving the friction force, the only force that can act on it, on it is **zero**.

(B) If the gravel were removed, there would not be enough friction to keep the box stationary. The box and the bucket will move downward and leftward respectively.

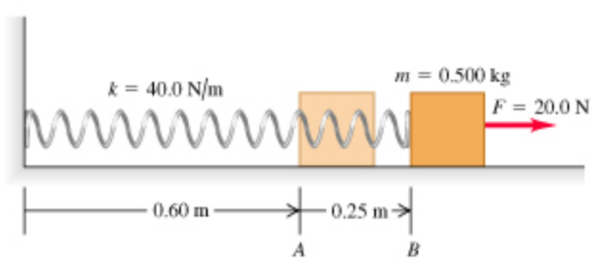


Use **conservation of total energy** with position 1 and 2 the initial and final positions. $U_1^{grav} + K_1 + W_{other} = U_2^{grav} + K_2$, where $W_{other} = -f_k d$ is the negative work done by the **kinetic** (the box is now moving) **force of friction**, $f_k = n\mu_k$, but now the gravel is no longer on top of the box so n = $m_B g$ and $f_k = m_B g\,\mu_k$ = (80kg)(9.8m/s^2)(0.4) = 313.6 N.
Hence $W_{other} = -f_k d = -(313.6N)(2.0m) = -627.2J$. Using above diagram, this gives

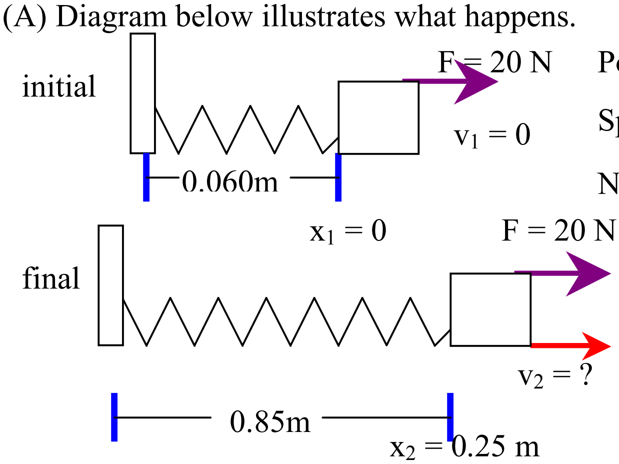
$$U_1^{grav} + K_1 + W_{other} = U_2^{grav} + K_2 \rightarrow m_A g d + 0 - 627.2J = 0 + \frac{1}{2}(m_A + m_B)v_2^2$$

$$(65kg)(9.8m/s^2)(2m) - 627.2J = \frac{1}{2}(65kg + 80kg)v_2^2 \rightarrow v_2 = \sqrt{\frac{646.8J}{72.5kg}} = 2.99m/s$$

<u>Problem 12</u>
In diagram below a 0.5kg block is connected to a spring (k = 40 N/m) at equilibrium 0.6 m from a wall. A 20N force pulls the block as shown. There is no friction. A) When the block is 0.25m form equilibrium, what is its speed? B) The block is then let go. How close to the wall does the block get?



(A) Diagram below illustrates what happens.



Position 1

Spring at equilibrium $U_1^e = \frac{1}{2}kx_1^2 = 0$

No Kinetic Energy $K_1 = 0$

Position 2

Spring PE $U_2^e = \frac{1}{2}kx_2^2$

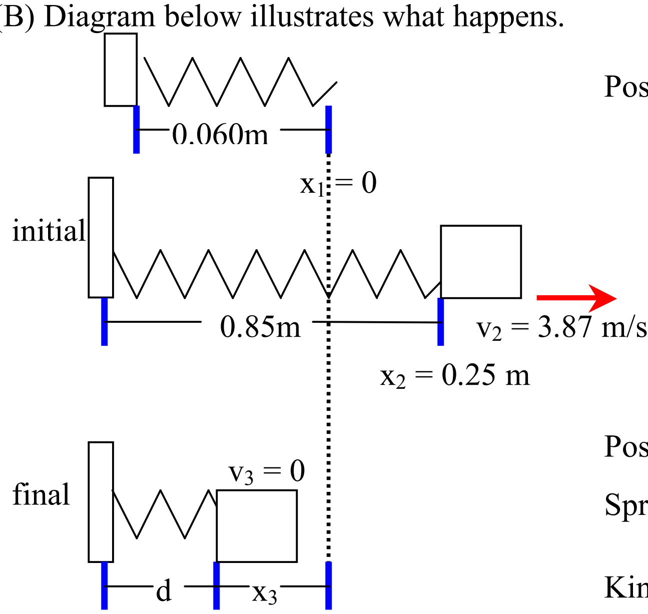
Kinetic Energy $K_2 = \frac{1}{2}mv_2^2$

Use conservation of energy:
$U_1^e + K_1 + W_{other} = U_2^e + K_2$, with $W_{other} = F(x_2 - x_1)$ being the work done by the 20N force in moving the box 0.25 m to the right: $W_{other} = 20N(0.25m) = 5J$.

$$U_1^e + K_1 + W_{other} = U_2^e + K_2 \rightarrow 0 + 0 + 5J = \frac{1}{2}kx_2^2 + \frac{1}{2}mv_2^2$$

$$v_2 = \sqrt{\frac{2(5J)}{m} - \frac{k}{m}x_2^2} = v_2 = \sqrt{\frac{2(5J)}{0.5kg} - \frac{40N/m}{0.5kg}(0.25m)^2} = 3.87\frac{m}{s}$$

(B) Diagram below illustrates what happens.



Position 1: spring at equilibrium
$x_1 = 0$

Position 2

Spring PE $U_2^e = \frac{1}{2}kx_2^2$

Kinetic Energy $K_2 = \frac{1}{2}mv_2^2$

Position 3

Spring PE $U_3^e = \frac{1}{2}kx_3^2$

Kinetic Energy $K_3 = \frac{1}{2}mv_3^2 = 0$

x_3 = ? is actually to the left of x_1 = ? Hence, the spring will be compressed

Use conservation of energy between positions 2 and 3 (there is no W_{other})

$$U_2^e + K_2 + = U_3^e + K_3 \rightarrow \frac{1}{2}kx_2^2 + \frac{1}{2}mv_2^2 = \frac{1}{2}kx_3^2 + 0$$

$$x_3 = \sqrt{x_2^2 + \frac{m}{k}v_2^2} = v_2 = \sqrt{(0.25m)^2 + \frac{0.5kg}{40N/m}(3.87m/s)^2} = 0.5m$$

From the diagrams, $d = 0.6m - x_3 = 0.1m$. The box reaches 0.1m from the wall.