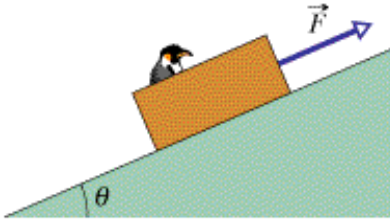


Problem 1

A loaded penguin sled weighing 65 N rests on a plane inclined at angle $\theta = 23^\circ$ to the horizontal (see the figure). Between the sled and the plane, the coefficient of static friction is 0.26, and the coefficient of kinetic friction is 0.17.



A) Draw a FBD on the sled. Assume that the force is sufficient to accelerate the box up the incline.

B) Assume that the sled is initially at rest, what is the minimum magnitude of the force, \vec{F} , that will accelerate the box up the incline.

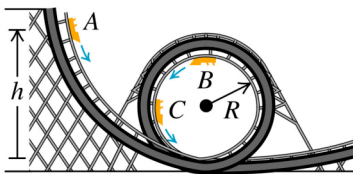
ANSWER: $F_N = 59.83N, f_{s,max} = 15.56N, F = 41N$

C) For the force calculated in part b, what is the acceleration of the box?

ANSWER: $a = 0.81 \frac{m}{s^2}$

Problem 2

A **50.0 kg** roller coaster has a speed of $v_A = 12 \frac{m}{s}$ at point A, it travels through a loop of radius $R = 15.0$ m passing through points B and C. In the diagram, Point A is at height $h = 40.0$ m above ground.



A) Find the speed of the coaster at point B. Treat the car as a particle, and assume that there is no loss of energy due to friction.

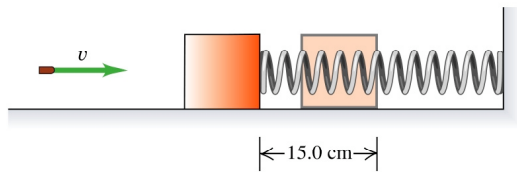
Answer: $v_B = 18.4 \frac{m}{s}$

B) Draw a free body diagram of all the forces acting on the coaster at point B. Hence, use Newton's law to determine the normal force of the track on the coaster at point B.

ANSWER: 640N

Problem 3

Three particles A bullet with a mass of $8.00 \times 10^{-3} \text{ kg}$ strikes and embeds itself in a block with mass 1.25 kg that rests on a **frictionless surface** and is attached to a coil spring with a force constant $k = 315 \frac{\text{N}}{\text{m}}$. If the speed of the bullet before it hits the block is $v = 373 \frac{\text{m}}{\text{s}}$.



A) Find the **speed** of the block + bullet **just after the impact**.

Answer: $V_F = 2.38 \frac{\text{m}}{\text{s}}$

B) Find the maximum compression of the block.

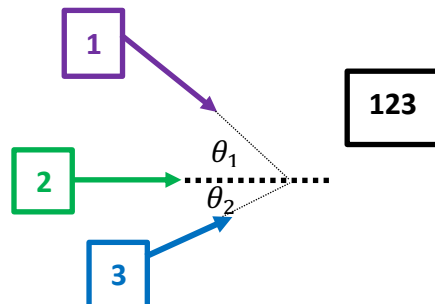
ANSWER: 15 cm

C) Calculate the **impulse (magnitude and direction)** on the **block** due to its collision with the bullet. What is the impulse on the bullet?

ANSWER: Impulse $\vec{J} = \Delta\vec{P}$ change in momentum, and take +x as right. $J_x = P_2 - P_1 = 2.975 \frac{\text{kg}\cdot\text{m}}{\text{s}}$, to the right.

Problem 4

Three particles $m_1 = 2\text{kg}$, $m_2 = 3\text{kg}$ and $m_3 = 3\text{kg}$ move at speed $v_1 = 3 \frac{\text{m}}{\text{s}}$, $v_2 = 2 \frac{\text{m}}{\text{s}}$ and $v_3 = 4 \frac{\text{m}}{\text{s}}$, respectively. The direction is indicated by the angle $\theta_1 = 53.1^\circ$ and $\theta_2 = 36.9^\circ$. They collide and stick together.



A) Find the final velocity of the composite mass.

$$\vec{v}_{final} = \left(2.4 \frac{m}{s}\right)\hat{i} + \left(0.3 \frac{m}{s}\right)\hat{j}; v_{final} = \sqrt{\left(2.4 \frac{m}{s}\right)^2 + \left(0.3 \frac{m}{s}\right)^2} = 2.42 \frac{m}{s}$$

B) Calculate the change in kinetic energy. Is this an elastic collision?

ANSWER: $\Delta K = -15.6J$

C) Calculate the impulse on particle 1 due to the collision

ANSWER: $\vec{J}_1 = \left(2.6 \frac{kg \cdot m}{s}\right)\hat{i} + \left(5.84 \frac{kg \cdot m}{s}\right)\hat{j}$

Problem 5

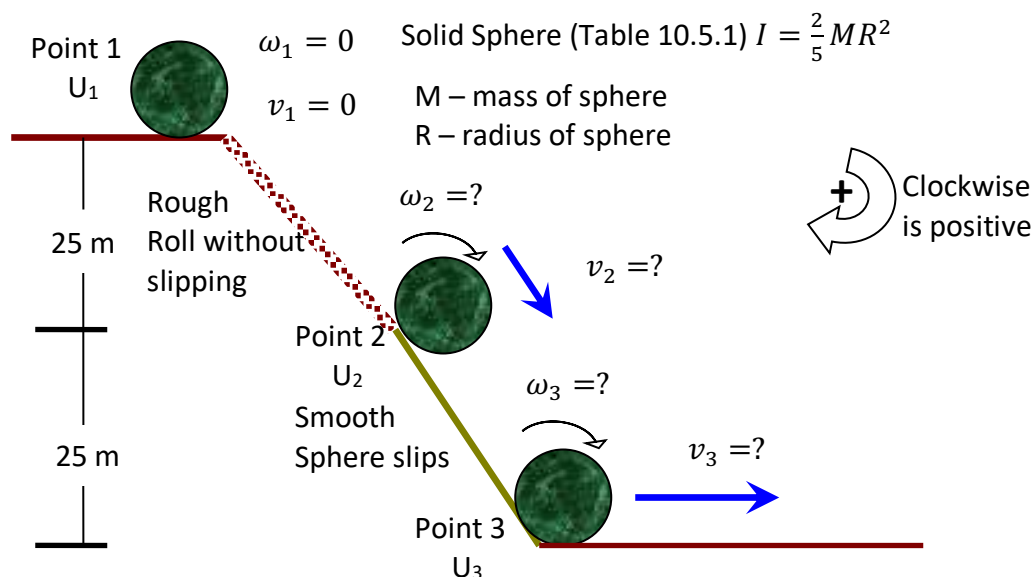
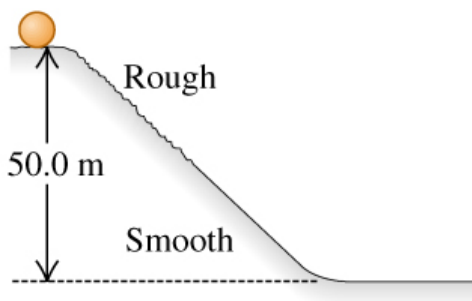
In the diagram below, a uniform spherical ($R = 0.1 \text{ m}$) boulder starts from rest and rolls down a 50.0 m high hill. The top half of the hill is rough so that the boulder rolls without slipping, but the bottom half is covered with ice, and so is frictionless.

A) What is the angular speed of the sphere at the middle of the hill, and at the bottom of the hill?

$$\omega_2 = 187 \frac{rad}{s}$$

B) What is the translational speed of the boulder when it reaches the bottom of the hill?

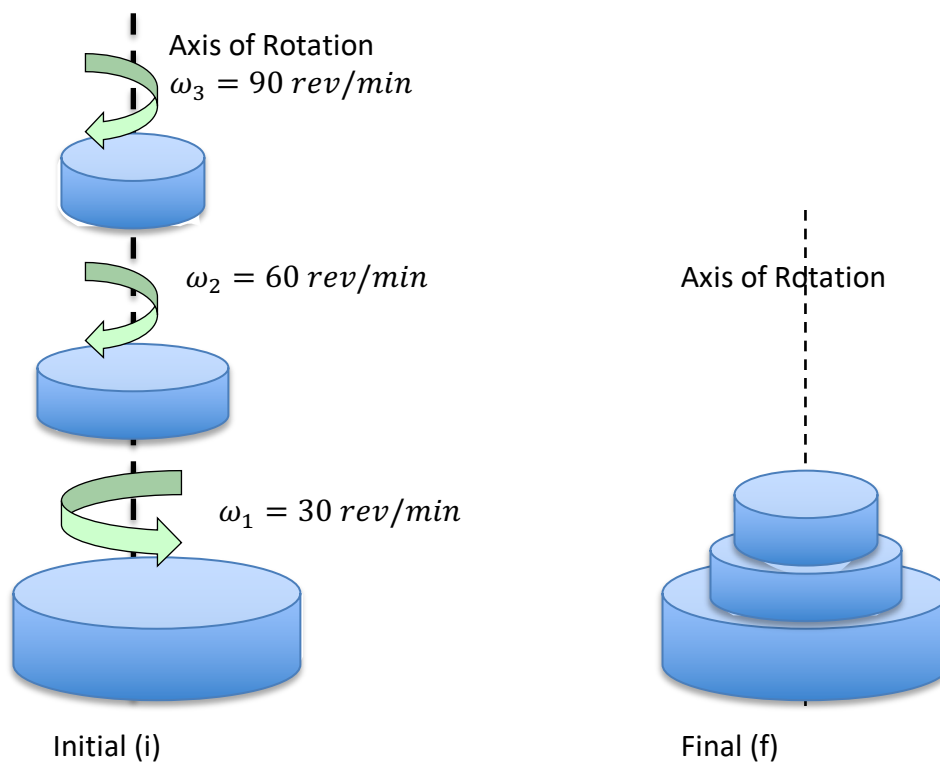
$$v_2 = 18.7 \frac{m}{s}$$



Problem 6

In the diagram below, disk 1 is hollow with $M_1 = 3\text{ kg}$ and radius $R_1 = 0.5\text{ m}$, Disk 3 is solid with $M_3 = 0.8\text{ kg}$ and radius $R_3 = 0.15\text{ m}$. There is no data on disk 2. Initially (left), the disks are rotation as shown. They undergo a collision to the final state (right), where the disks stick together, with an angular velocity of $\omega_f = 2.453\text{ rad} \cdot \text{s}^{-1}$, in the same direction of ω_1 .

- A)** Calculate the **initial** (left) angular momentum of each of the disks 1 and 2, and indicate the directions beside each disk. Hence find the moment of inertia and angular momentum of disk 2.
B) Calculate the final angular momentum of all three disks on the right. Indicate the direction of the final angular momentum, \vec{L}_f . **C)** Find the final angular velocity (magnitude and direction) if the I_2 you calculated is **tripled**.



ANSWER

A) BEFORE COLLISION

Disk 1: $L_1 = I_1\omega_1 = 3.927 \frac{\text{kg}\cdot\text{m}^2}{\text{s}}$, with the direction of \vec{L}_1 determined by the **right-hand rule (rhr)** to be pointing **up**, as shown in the figure.

Disk 3: $L_3 = -I_3\omega_3 = -0.084823 \frac{\text{kg}\cdot\text{m}^2}{\text{s}}$, with the direction of \vec{L}_3 determined by the **right-hand rule (rhr)** to be pointing **down**, as shown in the figure.

Disk 2: $I_2 = 0.0863\text{ kg} \cdot \text{m}^2$; $L_2 = -I_2\omega_2 = 0.54 \frac{\text{kg}\cdot\text{m}^2}{\text{s}}$

$$L_i = L_1 + L_2 + L_3 = 3.3 \frac{\text{kg} \cdot \text{m}^2}{\text{s}}$$

B) Final Moment of inertia

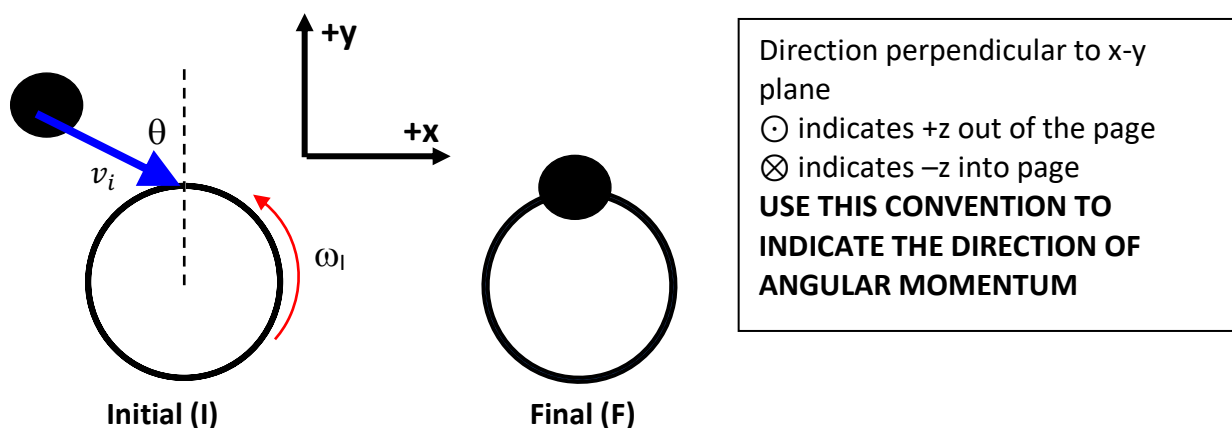
$$L_f = 3.3 \frac{kg \cdot m^2}{s}, \text{ up}$$

C) Initial $L_i = 2.215kg \cdot m^2 \cdot s^{-1} = I_{total}\omega_f \rightarrow \omega_f = 1.403rad \cdot s^{-1}$

Final angular momentum is $L_f = 2.215kg \cdot m^2 \cdot s^{-1}$, up (CCW).

Problem 7

In figure below, a carousel has a radius of 3.0 m and a moment of inertia of $I_C = 2000kg \cdot m^2$, for rotation about axis perpendicular to the its center. The carousel is rotating unpowered and without friction with an angular velocity of 1.1 rad/s. A 120-kg super-athlete runs with a velocity of $v_i = 10 \frac{m}{s}$, in a direction at $\theta = 30^\circ$ with the vertical, at the rim of the carousel, as shown below. He grabs hold of a pole on the rim.



A) Before the collision, what is the magnitude of the angular momentum of the rotating carousel, \vec{L}_C , with respect to the center of the carousel? What is the **direction of \vec{L} ? Directions (+x, +y, +z, -x, -y, -z) are as indicated in the above figure.**

ANSWER: $L_C = 2200kg \cdot m^2 \cdot s^{-1}$, with direction \odot , +z, or $\vec{L}_C = (2200kg \cdot m^2 \cdot s^{-1}) \hat{k}$

B) Before the collision, what is the magnitude of the angular moment of the running 80-kg man, \vec{L}_M , with respect to the center of the carousel? What is the **direction of \vec{L}_M ?**

ANSWER: $L_M = 1800kg \cdot m^2 \cdot s^{-1}$, with direction \otimes , -z, or $\vec{L}_M = -(1800kg \cdot m^2 \cdot s^{-1}) \hat{k}$

C) After the collision when the man is on the carousel, what is the magnitude of the final angular velocity of the carousel (with the man on it), ω_F ? What is the **direction of the final angular velocity $\vec{\omega}_F$? **Note:** $I_{total} = I_C + mR^2$**

ANSWER: $\omega_F = 0.136rad \cdot s^{-1}$, with direction \odot , +z, or $\vec{\omega}_F = (0.136rad \cdot s^{-1}) \hat{k}$.