Kinematics in 2D and 3D

Chapter 4
Position Vector in Three Dimensions (3D)

• $\hat{i} = (1,0,0)$ along x-axis
• $\hat{j} = (0,1,0)$ along y-axis
• $\hat{k} = (0,0,1)$ along z-axis
• Position Vector
• $\vec{r}(t) = (x(t), y(t), z(t))$
\[ \overrightarrow{r} = x\hat{i} + y\hat{j} + z\hat{k}. \] (4.1.1)

The vector components are \(x\hat{i}\), \(y\hat{j}\), and \(z\hat{k}\), and the coefficients \(x\), \(y\), and \(z\) in front of the unit vectors are the scalar components.

In the following figure, a particle is located at coordinates \((-3 \text{ m}, 2 \text{ m}, 5 \text{ m})\).
Displacement in Three Dimensions (3D)

- **Position Vectors**
  - Initial time $t_1$
    - $\vec{r}_1(t_1) = (x_1(t_1), y_1(t_1), z_1(t_1))$
  - Final time $t_2$
    - $\vec{r}_2(t_2) = (x_2(t_2), y_2(t_2), z_2(t_2))$

- **Displacement**
  - $\Delta \vec{r} = \vec{r}_2 - \vec{r}_1$
  - $\Delta \vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$
Average Velocity in Three Dimensions (3D)

- **Position Vectors**
  - Initial time $t_1$
    - $\vec{r}_1(t_1) = (x_1(t_1), y_1(t_1), z_1(t_1))$
  - Final time $t_2$
    - $\vec{r}_2(t_2) = (x_2(t_2), y_2(t_2), z_2(t_2))$

- **Average Velocity**
  - $\vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t} = \frac{(x_2-x_1)}{(t_2-t_1)} \hat{i} + \frac{(y_2-y_1)}{(t_2-t_1)} \hat{j} + \frac{(z_2-z_1)}{(t_2-t_1)} \hat{k}$
  - $\vec{v}_{avg} = \frac{\Delta x}{\Delta t} \hat{i} + \frac{\Delta y}{\Delta t} \hat{j} + \frac{\Delta z}{\Delta t} \hat{k}$
  - $\vec{v}_{avg} = \vec{v}_{avg,x} \hat{i} + \vec{v}_{avg,y} \hat{j} + \vec{v}_{avg,z} \hat{k}$

PATH of object

$\Delta \vec{r} = \vec{r}_2 - \vec{r}_1$

$\vec{v}_{avg}$
Instantaneous Velocity (velocity) in Three Dimensions (3D)

**Average Velocity**

- \( \vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t} \)
- \( \vec{v}_{avg} = \frac{\Delta x}{\Delta t} \hat{i} + \frac{\Delta y}{\Delta t} \hat{j} + \frac{\Delta z}{\Delta t} \hat{k} \)
- \( = v_{avg,x} \hat{i} + v_{avg,y} \hat{j} + v_{avg,z} \hat{k} \)

**Velocity**

In the calculus limit, \( \Delta t \to 0 \)

- \( \vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{dx}{dt} \hat{i} + \frac{dy}{dt} \hat{j} + \frac{dz}{dt} \hat{k} \)
- \( = v_x \hat{i} + v_y \hat{j} + v_z \hat{k} \)
- \( \vec{v}_1 = \vec{v}_1(t_1) \) and \( \vec{v}_2 = \vec{v}_2(t_2) \) is the velocity at time, \( t_1 \) and \( t_2 \), respectively.
- In general, the velocity is a function of time, \( \vec{v} = \vec{v}(t) \)
Average Acceleration, $\vec{a}_{avg}$, Acceleration, $\vec{a}$, in Three Dimensions (3D)

### Average Acceleration

- $\vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\Delta v_x}{\Delta t} \hat{i} + \frac{\Delta v_y}{\Delta t} \hat{j} + \frac{\Delta v_z}{\Delta t} \hat{k}$
- $\vec{a}_{avg} = \frac{v_{2x} - v_{1x}}{t_2 - t_1} \hat{i} + \frac{v_{2y} - v_{1y}}{t_2 - t_1} \hat{j} + \frac{v_{2z} - v_{1z}}{t_2 - t_1} \hat{k}$
- $\vec{a}_{avg} = a_{avg,x} \hat{i} + a_{avg,y} \hat{j} + a_{avg,z} \hat{k}$

### Acceleration

In the calculus limit, $\Delta t \to 0$

- $\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{dv_x}{dt} \hat{i} + \frac{dv_y}{dt} \hat{j} + \frac{dv_z}{dt} \hat{k}$
- $\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$
- $\vec{a}_1 = \vec{a}_1(t_1)$ and $\vec{a}_2 = \vec{a}_2(t_2)$ is the velocity at time, $t_1$ and $t_2$, respectively.
- In general, the velocity is a function of time, $\vec{a} = \vec{a}(t)$. 

PATH of Object
Example of Kinematics in 2D

An object follows a path \( \mathbf{r} = \left( \frac{1}{s^2} \right) t^2 \mathbf{i} + \left( \frac{1}{s^4} \right) t^4 \mathbf{j} + (2m) \mathbf{k} \)

A) Draw the path of the object from \( t > 0 \).
B) Find average velocity from \( t = 1 \text{ s} \) to \( 2 \text{ s} \).
C) Find average acceleration from \( t = 1 \text{ s} \) to \( 2 \text{ s} \). Plot.
D) Find acceleration at \( t = 1 \text{ s} \) and at \( 2 \text{ s} \).

Solution of part A

It’s clear, \( x = \left( \frac{1}{s^2} \right) t^2 \), \( y = \left( \frac{1}{s^4} \right) t^4 \), \( z = 2m \)

\[ y = \left( \frac{1}{s^4} \right) t^4 = \left[ \left( \frac{1}{s^2} \right) t^2 \right]^2 \to y = x^2 \]

Path is a parabola in the plane defined by \( z = 2m \)

The relation \( y = x^2 \) omits unit. If units were included \( \to y = (1m^{-1})x^2 \)
Example of Kinematics in 2D

An object follows a path \( \vec{r} = \left( \frac{1}{s^2} \right) t^2 \hat{i} + \left( \frac{1}{s^4} \right) t^4 \hat{j} + (2m)\hat{k} \)

- B) Find the average velocity from \( t = 1s \) to \( 2 \) s.

**Solution of part B**

At \( t = 1s \), \( x(1s) = \left( \frac{1}{s^2} \right) (1s)^2 = 1m \), \( y(1s) = \left( \frac{1}{s^4} \right) (1s)^4 = 1m \), \( z = 2m \)

\( \vec{r}(1s) = (1m)\hat{i} + (1m)\hat{j} + (2m)\hat{k} \)

At \( t = 2s \), \( x(2s) = \left( \frac{1}{s^2} \right) (2s)^2 = 4m \), \( y(2s) = \left( \frac{1}{s^4} \right) (2s)^4 = 16m \), \( z = 2m \)

\( \vec{r}(2s) = (4m)\hat{i} + (16m)\hat{j} + (2m)\hat{k} \)

\[ \vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \left( \frac{x_2 - x_1}{t_2 - t_1} \right) \hat{i} + \left( \frac{y_2 - y_1}{t_2 - t_1} \right) \hat{j} + \left( \frac{z_2 - z_1}{t_2 - t_1} \right) \hat{k} \]

\[ \vec{v}_{avg} = \left( \frac{3}{s} \right) \hat{i} + \left( 15 \frac{m}{s} \right) \hat{j} \]
Example of Kinematics in 3D

An object follows a path \( \vec{r} = \left( \frac{1}{s^2} \right) t^2 \hat{i} + \left( \frac{1}{s^4} \right) t^4 \hat{j} + (2m) \hat{k} \)

• C) Find average acceleration from \( t = 1s \) to \( 2s \). Plot

**Solution of part C**

First find velocity as a function of time \( t \) by differentiation wrt time, \( t \).

- \( \vec{v} = v_x \hat{i} + v_y \hat{j} + v_z \hat{k} = \frac{dx}{dt} \hat{i} + \frac{dy}{dt} \hat{j} + \frac{dz}{dt} \hat{k} \)

- \( v_x = \frac{dx}{dt} = \frac{d}{dt} \left( \frac{1}{s^2} \right) t^2 = 2 \times \left( \frac{1}{s^2} \right) t^{2-1} = \left( \frac{2}{s^2} \right) t \)

- \( v_y = \frac{dy}{dt} = \frac{d}{dt} \left( \frac{1}{s^4} \right) t^4 = 4 \times \left( \frac{1}{s^4} \right) t^{4-1} = \left( \frac{4}{s^4} \right) t^3 \)

- \( v_z = \frac{dz}{dt} = \frac{d}{dt} (2m) = 0 \rightarrow \vec{v} = \left( \frac{2}{s^2} \right) t \hat{i} + \left( \frac{4}{s^4} \right) t^3 \hat{j} \)

At \( t = 1s, \vec{v}(1s) = \left( \frac{2}{s^2} \right) (1s) \hat{i} + \left( \frac{4}{s^4} \right) (1s)^3 \hat{j} = \left( \frac{2}{s^2} \right) \hat{i} + \left( \frac{4}{s^2} \right) \hat{j} \)

At \( t = 2s, \vec{v}(2s) = \left( \frac{2}{s^2} \right) (2s) \hat{i} + \left( \frac{4}{s^4} \right) (2s)^3 \hat{j} = \left( \frac{4}{s^2} \right) \hat{i} + \left( 32 \frac{m}{s^2} \right) \hat{j} \)

D) Find acceleration at \( t = 1s \) and at \( 2s \).

\( \vec{a} = a_x \hat{i} + a_y \hat{j} + \frac{dv_x}{dt} \hat{i} + \frac{dv_y}{dt} \hat{j} \)

\( \vec{a} = \frac{d}{dt} \left( \frac{2}{s^2} \right) t \hat{i} + \frac{d}{dt} \left( \frac{4}{s^4} \right) t^3 \hat{j} \)

\( \vec{a}(1s) = \left( \frac{2}{s^2} \right) \hat{i} + \left( \frac{12}{s^2} \right) \hat{j} \)

\( \vec{a}(2s) = \left( \frac{2}{s^2} \right) \hat{i} + \left( \frac{192}{s^2} \right) \hat{j} \)
Projectile Motion

• An object launched near the earth’s surface with an initial velocity, $\vec{v}_0$
• The Object then it follows a path (i.e. a projectile) that dictated by the force of gravity

$a_y = -g\hat{j}$
Projectile Motion: Motion diagram illustrates that the x- and y- components are independent

- At $t_0 = 0$, position $x_0 = 0$ and $y_0 = 0$, ball with initial velocity $\vec{v}_0$
- Initial velocity components:
  - $x$-comp, $v_{0x} = v_0 \cos \theta_0$
  - $y$-comp, $v_{0y} = v_0 \sin \theta_0$

$x$ and $y$ components are independent
Projectile Motion: Kinematics Equations

- At $t_0 = 0$, position $x_0 = 0$ and $y_0 = 0$, ball with initial velocity $\vec{v}_0$
- Initial velocity components:
  - $X$-comp, $v_{0x} = v_0 \cos \theta_0$
  - $Y$-comp, $v_{0y} = v_0 \sin \theta_0$

Equations to find **position**, $x(t)$, $y(t)$ and **velocity**, $\vec{v}(t) = v_x(t)\hat{i} + v_y(t)\hat{j}$ at time $t$.

**Horizontal (x)**
- $x = x_0 + v_{0x}t$, $E1$

**Vertical (y)**
- $y = y_0 + v_{0y}t - \frac{1}{2}gt^2$, $E2$
- $v_y^2 = v_{0y}^2 - 2g(y - y_0)$, $E3$, $v_y = v_{0y} - gt$, $E4$
Simple Projectile Problem

• The airplane shown is in level flight at an altitude of 0.50 km and a speed of 150 km/h. At what distance d should it release a heavy bomb to hit the target X? Take \( g = 10 \text{ m/s}^2 \).

• A) 150 m; B) 295 m; C) 417 m; D) 1500 m; E) 15000 m.

\[
\begin{align*}
\text{Horizontal (x)} \\
x &= x_0 + v_{0x}t, \quad \text{E1} \\
\text{Vertical (y)} \\
y &= y_0 + v_{0y}t - \frac{1}{2}gt^2, \quad \text{E2} \\
\left( v_y \right)^2 &= v_{0y}^2 - 2g(y - y_0), \quad \text{E3} \\
v_y &= v_{0y} - gt, \quad \text{E4}
\end{align*}
\]
Simple Projectile Problem: Solution

- The airplane shown is in level flight at an altitude of 0.50 km and a speed of 150 km/h. At what distance \(d\) should it release a heavy bomb to hit the target \(X\)? Take \(g = 10 \text{ m/s}^2\).
- **A)** 150 m; **B)** 295 m; **C)** 417 m; **D)** 1500 m; **E)** 15000 m.

**Solution:**
Consider **Vertical** and use \(E2\), to find time bomb falls 500m (0.5 km):

\[
y = y_0 + v_{0y} t - \frac{1}{2} g t^2
\]

\[-500 \text{ m} = 0 + 0 \times t - 5 \frac{m}{s^2} t^2 \rightarrow t = 10 \text{ s}\]

**Horizontal Range**
Initial Horizontal velocity \(v_{ox} = 41.7 \text{ m/s}\)
Horizontal Range: **E1** \(x = v_{ox} t = 417 \text{ m}\)
Multiple Choice

• The figure below (top of next page) shows trajectories of four artillery shells. Each fired with the same initial speed. Which trajectory remains in the air for the longest time? Circle the right answer. **Hint**: ask yourself how to throw a ball so that it remains in the air for the longest period.

Answer: A
• (10 points) A child is standing 10m from the edge of a cliff. He kicks a soccer ball is kicked on the ground with an initial speed of 17.7 m/s at an upward angle of 42.7° above the horizontal. The cliff is 12.25 m high.

A) Draw the path of trajectory (or a motion diagram) of the soccer the ball, which shows the direction of the velocity and acceleration at the following points: i) the instant it leaves the ground; ii) its maximum height; iii) the instant when it hits the ground. Calculate the x and y component of the initial velocity

Solution of A)
Initial Velocity, \( v_0 = 17.7 \, \text{m/s} \) at \( \theta = 42.7^\circ \)
x-com, \( v_{0x} = v_0 \cos \theta = 13 \, \text{m/s} \) (1 point)
y-com, \( v_{0y} = v_0 \sin \theta = 12 \, \text{m/s} \) (1 point)

Initial Velocity. \( v_x = 17.7 \, \text{m} \) at an upward angle of \( \theta = 42.7^\circ \)
Previous Test Questions: Solution part B

- **(10 points)** A child is standing **10m** from the edge of a cliff. He kicks a soccer ball is kicked on the ground with an initial speed of 17.7 m/s at an upward angle of 42.7° above the horizontal. The cliff is 12.25 m high.

B) Calculate the **time** it takes the ball to **hit** the **ground**. Does it land on the bottom of the cliff? **HINT:** The answer is yes, but you must show this using projectile equations. **Partial Answer:** \( t = 3.23 \) s

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**Solution of B)**

First, we must show that the ball does **not** land on the cliff. Since it must travel **10m** **horizontally** to the cliff edge, the time it takes is: \( 10 m = v_{0x} t \rightarrow t = 0.77 s \). We used **E1**.

Using **E2**, the vertical position is: \( y = v_{0y} t - \frac{1}{2} g t^2 = 12 \frac{m}{s} \times 0.77 s - \frac{1}{2} \left( 9.8 \frac{m}{s^2} \right) (0.77 s)^2 = 6.33 m \).

Hence at the cliff edge it is **above** top of the cliff. **(2 points)**

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Horizontal \( (x) \)  \( x = x_0 + v_{0x} t, \text{ E1} \)

Vertical \( (y) \)
\( y = y_0 + v_{0y} t - \frac{1}{2} g t^2, \text{ E2} \); \( v_y^2 = v_{0y}^2 - 2g(y - y_0), \text{ E3} \)

\( v_y = v_{0y} - gt, \text{ E4} \)

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**Diagram (2 points)**

- **Range** is \( C) \)
- **A)** the horizontal.
- **kicked** \( \)10 \)
- **ground**

\( \text{Physical Solution is } t = 3.23 s \)

To find the time to the bottom use **6** equations.

**Draw the path of trajectory (or a motion diagram) of the soccer the ball, which shows**

- ii) its **height**; iii) the instant when it hits the ground.

**Partial Answer:** \( S \)

\( \text{HINT:} \) The cliff is \( 12.25 \) m high. \( \)
Previous Test Questions: Solution part B, Continued

• (10 points) A child is standing 10m from the edge of a cliff. He kicks a soccer ball is kicked on the ground with an initial speed of 17.7 m/s at an upward angle of 42.7° above the horizontal. The cliff is 12.25 m high.

B) Calculate the time it takes the ball to hit the ground. Does it land on the bottom of the cliff? HINT: The answer is yes, but you must show this using projectile equations. Partial Answer: t = 3.23 s

Solution of B) Continued

We have shown that the ball will clear the cliff’s edge and fall to the ground

To find time to the ground use \( E2 \)

\[
y = y_0 + v_{0y}t - \frac{1}{2}gt^2
\]

\[-12.25m = 12 \frac{m}{s} t - 4.9 \frac{m}{s^2} t^2 = 0 \rightarrow t^2 - 2.45t - 2.5 = 0 \]

Solution, \( t = \frac{2.45 \pm \sqrt{2.45^2 - 4 \times 2.5}}{2} = \frac{2.45 \pm 4}{2} \rightarrow t = 3.23s \text{ and } -0.78s. \]

Physical Solution is \( t = 3.23 \text{ s} \) (3 points)

Diagram (2 points)

Horizontal (x) \( x = x_0 + v_{0x}t, \ E1 \)

Vertical (y) \( y = y_0 + v_{0y}t - \frac{1}{2}gt^2, \ E2 \); \( v_y^2 = v_{0y}^2 - 2g(y - y_0), \ E3 \)

\( v_y = v_{0y} - gt, \ E4 \)
Previous Test Questions: Solution part C

• (10 points) A child is standing 10m from the edge of a cliff. He kicks a soccer ball is kicked on the ground with an initial speed of 17.7 m/s at an upward angle of 42.7° above the horizontal. The cliff is 12.25 m high.

C) Hence calculate the horizontal distance by the ball traveled just before it hits the ground at the bottom of the cliff, i.e. calculate the range.

Solution of C)

To find range to the ground use $E1$

Also from part A) $v_{0x} = v_0 \cos \theta = 13\text{ m/s}$, $v_{0y} = v_0 \sin \theta = 12\text{ m/s}$

From part B, $t = 3.23$ s

Range is $x = v_{0x}t = 13 \times 3.23s = 42m$. (1 point)
Projectile Motion: Equation of Motion revisited

- At $t_0 = 0$, position $x_0 = 0$ and $y_0 = 0$, ball with initial velocity $\vec{v}_0$
- Initial velocity components:
  - X-comp, $v_{0x} = v_0 \cos \theta_0$ \(E13\)
  - Y-comp, $v_{0y} = v_0 \sin \theta_0$ \(E15\)

Equations to find position, $x(t)$, $y(t)$ and velocity, $\vec{v}(t) = v_x(t)\hat{i} + v_y(t)\hat{j}$ at time $t$.

**Horizontal (x)**
- $x = x_0 + v_{0x} t$, \(E14\)

**Vertical (y)**
- $v_y = v_{0y} - gt$, \(E16\)
- $y = y_0 + v_{0y} t - \frac{1}{2} gt^2$, \(E17\)
- $v_y^2 = v_{0y}^2 - 2g(y - y_0)$, \(E18\),
Equations for Midterm 1

• 1D Kinematics:

\[ \begin{align*}
\text{E1: } & \quad \frac{\Delta x}{\Delta t} = \frac{x_2-x_1}{t_2-t_1} \\
\text{E2: } & \quad v = \frac{dx}{dt} \\
\text{E3: } & \quad s_{avg} = \frac{\text{total distance}}{\text{total time}} \\
\text{E4: } & \quad \frac{\Delta v}{\Delta t} = \frac{v_2-v_1}{t_2-t_1} \\
\text{E5: } & \quad a = \frac{dv}{dt} \\
\text{E6: } & \quad v = v_0 + at \\
\text{E7: } & \quad x = x_0 + v_0 t + \frac{1}{2} at^2 \\
\text{E8: } & \quad v^2 = v_0^2 + 2a(x-x_0) \\
\end{align*} \]

• 2D Kinematics:

\[ \begin{align*}
\text{E9: } & \quad \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}_2-\vec{r}_1}{t_2-t_1} = \frac{x_2-x_1}{t_2-t_1} \hat{i} + \frac{y_2-y_1}{t_2-t_1} \hat{j} + \frac{z_2-z_1}{t_2-t_1} \hat{k} = v_{avg,x} \hat{i} + v_{avg,y} \hat{j} + v_{avg,z} \hat{k} \\
\text{E10: } & \quad \frac{\vec{v}}{dt} = \frac{dx}{dt} \hat{i} + \frac{dy}{dt} \hat{j} + \frac{dz}{dt} \hat{k} = v_x \hat{i} + v_y \hat{j} + v_z \hat{k} \\
\text{E11: } & \quad \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_2-\vec{v}_1}{t_2-t_1} = \frac{v_{2x}-v_{1x}}{t_2-t_1} \hat{i} + \frac{v_{2y}-v_{1y}}{t_2-t_1} \hat{j} + \frac{v_{2z}-v_{1z}}{t_2-t_1} \hat{k} = a_{avg,x} \hat{i} + a_{avg,y} \hat{j} + a_{avg,z} \hat{k} \\
\text{E12: } & \quad \frac{d\vec{a}}{dt} = \frac{dv_x}{dt} \hat{i} + \frac{dv_y}{dt} \hat{j} + \frac{dv_z}{dt} \hat{k} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k} \\
\end{align*} \]

• Projectile Motion

• Horizontal: \( v_{0x} = v_0 \cos \theta _0 \) \text{ E13; } x = x_0 + v_{0x} t \text{ E14 }

• Vertical: \( v_{0y} = v_0 \sin \theta _0 \) \text{ E15; } \( v_y = v_{0y} - gt \) \text{ E16; } y = y_0 + v_{0y} t - \frac{1}{2} gt^2 \text{ E17; }

\[ \begin{align*}
\text{E18: } & \quad v_y^2 = v_{0y}^2 - 2g(y-y_0) \\
\end{align*} \]
Tossed ball from rising Balloon

A balloon is rising at a constant speed of $6.7 \, \text{m/s}$. At 88.2 m above the ground, a ball is launch from the balloon at $10 \, \text{m/s}$ (with respect to himself) at $53.1^\circ$ above the horizontal.

A) Find the maximum height of the ball.
B) At the maximum height, what is the speed and acceleration of the ball?
C) Find the range (horizontal distance from the launch point) of the ball.
D) Find the velocity and speed of the ball when it hits the ground.

Solution of Part A
Initial Velocity component:
Use $\textbf{E13} \quad v_{0x} = 10 \times \cos 53.1^\circ = 6 \, \text{m/s}$
Use $\textbf{E15} \quad v_{0y} = 10 \times \sin 53.1^\circ = 8 \, \text{m/s}$
But this neglect the balloon’s vertical velocity
\[ v_{0y} = 10 \times \sin 53.1^\circ + 6.7 \, \frac{m}{s} = 14.7 \, \frac{m}{s} \]
Tossed ball from rising Balloon: Part A

A balloon is rising at a **constant** speed of \(6.7 \frac{m}{s}\). At 88.2 m above the ground, a ball is launched from the balloon at \(10 \frac{m}{s}\) (with respect to himself) at 53.1° above the horizontal.

A) Find the maximum height of the ball.

**Solution of Part A (continued)**

Initial Velocity component:

\[ v_{0x} = 6 \frac{m}{s}; \quad v_{0y} = 14.7 \frac{m}{s} \]

At the Maximum height....??

Use E18, \( v_y^2 = 0 = v_{0y}^2 - 2g(y - y_0) \)

\[
0 = v_{0y}^2 - 2gy_{\text{max}}
\]

\[
y_{\text{max}} = \frac{(14.7 \frac{m}{s})^2}{2 \times 9.8 \frac{m}{s^2}} = 11.025m = 11m
\]

Maximum height is 11m + 88.2 m = 99.2 m above the ground.
Tossed ball from rising Balloon: Part B

A balloon is rising at a constant speed of \(6.7 \text{ m/s}\). At 88.2 m above the ground, a ball is launched from the balloon at \(10 \text{ m/s}\) (with respect to himself) at 53.1° above the horizontal.

A) Find the maximum height of the ball.

B) At the maximum height, what is the speed and acceleration of the ball?

Alternate Solution of Part A, Use E16 to find the time to \(y_{max}\), then use E17 to find \(y_{max}\)

Solution of Part B

Acceleration is \(g = 9.8 \text{ m/s}^2\) down towards the ground.

Speed is simply the x-comp, \(v_{0x} = 6 \text{ m/s}\)

\[ y = -88.2 \text{ m} \]
Tossed ball from rising Balloon: Part C

A balloon is rising at a constant speed of $6.7 \frac{m}{s}$. At 88.2 m above the ground, a ball is launch from the balloon at $10 \frac{m}{s}$ (with respect to himself) at 53.1° above the horizontal.

C) Find the range (horizontal distance from the launch point) of the ball.

**Solution of C**

From A, $v_{0x} = 6 \frac{m}{s}$; $v_{0y} = 14.7 \frac{m}{s}$

Use E17, $y = y_0 + v_{0y}t - \frac{1}{2}gt^2$

$-88.2m = 0 + 14.7 \frac{m}{s} t - 4.9 \frac{m}{s^2} t^2$

Divide by 4.9, rearrange and omit unit:

$t^2 - 3t - 18 = 0 \rightarrow (t - 6s)(t + 3s) = 0$

Physical Solution: $t = 6s$

Use E14, $x = x_0 + v_{0x}t$, range = $6 \frac{m}{s} \times 6s = 36m$

Other Solution? $t = -3s$
Tossed ball from rising Balloon: Part D

A balloon is rising at a **constant** speed of \(6.7 \frac{m}{s}\). At 88.2 m above the ground, a ball is launch from the balloon at \(10 \frac{m}{s}\) (with respect to himself) at 53.1° above the horizontal.

D) Find the velocity and speed of the ball when it hits the ground.

**Solution of D**

From A, \(v_{0x} = 6 \frac{m}{s} \); \(v_{0y} = 14.7 \frac{m}{s}\)

Use E16, \(v_y = v_{0y} - gt\), and \(t = 6s\)

\[v_y = v_{0y} - gt = -44.1 \frac{m}{s} \rightarrow \vec{v} = 6 \frac{m}{s} \hat{i} - 44.1 \frac{m}{s} \hat{j}\]

**Speed:** \(v = |\vec{v}| = \sqrt{\left(6 \frac{m}{s}\right)^2 + \left(-44.1 \frac{m}{s}\right)^2} = 44.5 \frac{m}{s}\)

**Worthwhile exercise:**

1. Show that for final velocity, at \(t = 6s\), \(\theta = 278°\)
2. Find velocity for other solution, \(t = -3s\)
Tangential, \( \vec{a}_t \), and Radial, \( \vec{a}_{rad} \) component of Acceleration.

- **Tangential Component**, \( \vec{a}_t \), is **parallel** to the velocity \( \vec{v} \)
- **Radial Component**, \( \vec{a}_{rad} \), is **perpendicular** to the velocity \( \vec{v} \)
- How does the velocity change?
  - In time \( \Delta t \), the velocity will change by \( \Delta \vec{v} = \vec{a} \Delta t \)
  - Final velocity is \( \vec{v}_f = \vec{v} + \vec{a} \Delta t \)
- System on right is **speeding up** and **turning right**!

\[ \Delta \vec{v} = \vec{a} \Delta t \]

Example of **Slowing down** and **turning left**
A Multiple Choice

• Shown Below are the velocity and acceleration vectors for an object in several different types of motion. In which case is the object slowing down and turning left?

ANSWER: E
Circular Motion Section 4.5

• An object that follows a circular path of radius $r$, at constant speed, $v$, is said to be in a **uniform circular motion** as shown in the Figure below:

  ![Diagram of uniform circular motion](image)

  The **acceleration**, $\ddot{a}$ of an **object** in **uniform circular motion** is perpendicular to its velocity, $\dot{v}$.

  The **acceleration**, $\ddot{a}$ points to the center of the circle and has a **magnitude** of:

  $$a = \frac{v^2}{r}$$

  The figure on the **left** is an **example** of a **circular motion** at **variable speed**.
Another Multiple choice

- An object is moving around in a circle at constant speed in the **clockwise** direction (shown below), from P₁ to P₂ to P₃ to P₄ and back to P₁. At which point does the object have a **negative x-velocity** and a **negative y-velocity**? Circle the correct answer.
- a) P₁  b) P₂  c) P₃  d) P₄  e) none of these answers

**ANSWER: B**