Section 4.61: Permeability of Artificial Membranes

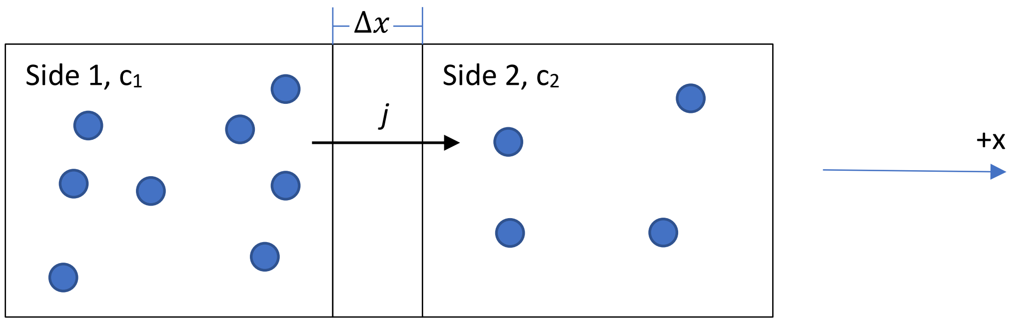
Fick's Law (4.19) $j = -D\frac{dc}{dx}$

Students are advised to read section 4.4.2 to understand the above equation.

In the figure below, a system of two sides (as shown below), say side 1 and side 2, separated by a partition of thickness Δx, Fick's law becomes:

$$j = -D\frac{\Delta c}{\Delta x},$$

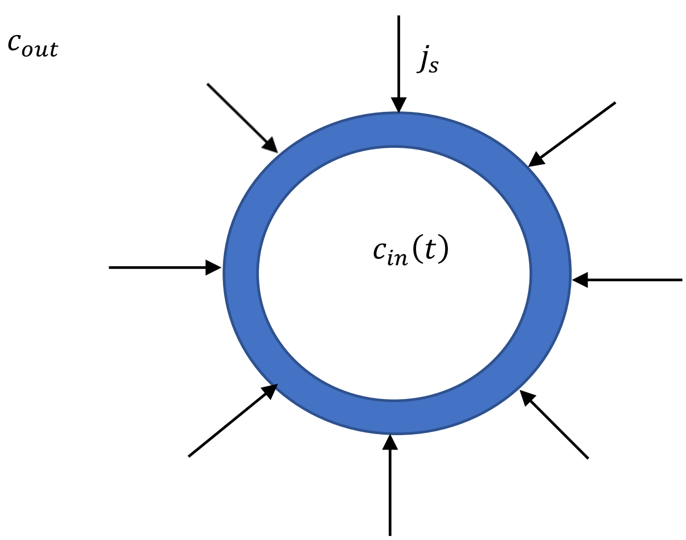
where $\Delta c = c_2 - c_1$. +x is taken to be to the right, and since it is clear that $c_2 - c_1 < 0, j > 0$, and the flux of particles is to the right, which makes sense since particles must flow from high to low concentration.



You should now read section 4.6.1, where Fick's law is applied to a biological cell (say a bacteria), where a lipid membrane separated the inside (in) of the cell from the outside (out) environment. Let's assume that the cell is anaerobic, in that it consumes oxygen (O_2) to create energy. The concentration of oxygen in the outside environment is usually assumed to be constant, c_{out} (a typical value is $c_{out} = 0.2 mole \cdot m^{-3}$). Assume that oxygen concentration inside $c_{in}(t)$ is less than outside, $c_{in}(t) < c_{out}$. The flux of oxygen is

$$j_s = -\wp_s \Delta c, \quad \Delta c = c_{out} - c_{in}(t),$$

Where $\wp_s$ is the permeability of the membrane to a solute, which depends on the type of solute, and the type and thickness of the membrane. Typical number is $\wp_s \sim 3 \times 10^{-6} m \cdot s^{-1}$. Note that the unit of j_s is $m \cdot s^{-1} \times m^{-3} = m^{-2} \cdot s^{-1}$. Since $\Delta c > 0 \to j_s < 0$, so that the flux is into the cell.



It is clear that as oxygen flows into the cell the oxygen concentration $c_{in}(t)$ inside will increase. Detailed balance gives:

Rate of increase of oxygen inside **EQUALS** rate of flow of oxygen from outside to inside

$$\frac{d(c_{in} \times V)}{dt} = A \times |j_s|,$$

Where $V = \frac{4}{3}\pi r^3$ is the volume of the spherical cell (radius r), and $A = 4\pi r^2$ is the surface area of the lipid membrane that envelops the cell. Using $j_s = -\wp_s \Delta c$, and assuming that V and A are constants, and that $\Delta c = c_{out} - c_{in}(t) > 0$:

$$\frac{d(c_{in})}{dt} = \frac{A\wp_s}{V}\Delta c,$$

But if we note that c_{out} is constant, $\frac{d(c_{out})}{dt} = 0$, we have $\frac{d(c_{in})}{dt} = -\frac{d(c_{out} - c_{in}(t))}{dt} = -\frac{d(\Delta c)}{dt}$, and the above equation becomes

$$-\frac{d(\Delta c)}{dt} = \frac{A\wp_s}{V}\Delta c \quad [1]$$

which is equation 4.22 of the textbook. This can be solved by assuming a solution

$$\Delta c = c_{out} - c_{in}(t) = Be^{-\frac{t}{\tau}}, \qquad \tau = \frac{V}{A\wp_s}$$

where B is a constant. This is easily verified by direct substitution into [1]

$$-\frac{d(\Delta c)}{dt} = -\frac{d\left(Ae^{-\frac{t}{\tau}}\right)}{dt} = -\left(-\frac{B}{\tau}e^{-\frac{t}{\tau}}\right) = \frac{A\wp_s}{V}Be^{-\frac{t}{\tau}} = \frac{A\wp_s}{V}\Delta c$$

We can find B by assuming that a t = 0, the inside concentration is $c_{in}(0)$, hence at t = 0

$$\Delta c = c_{out} - c_{in}(0) = Be^{-\frac{0}{\tau}} \rightarrow c_{out} - c_{in}(0) = B$$

Finally the solution is

$$c_{out} - c_{in}(t) = \left(c_{out} - c_{in}(0)\right)e^{-\frac{t}{\tau}},$$

You should now be able to do part a of problem 4.7.