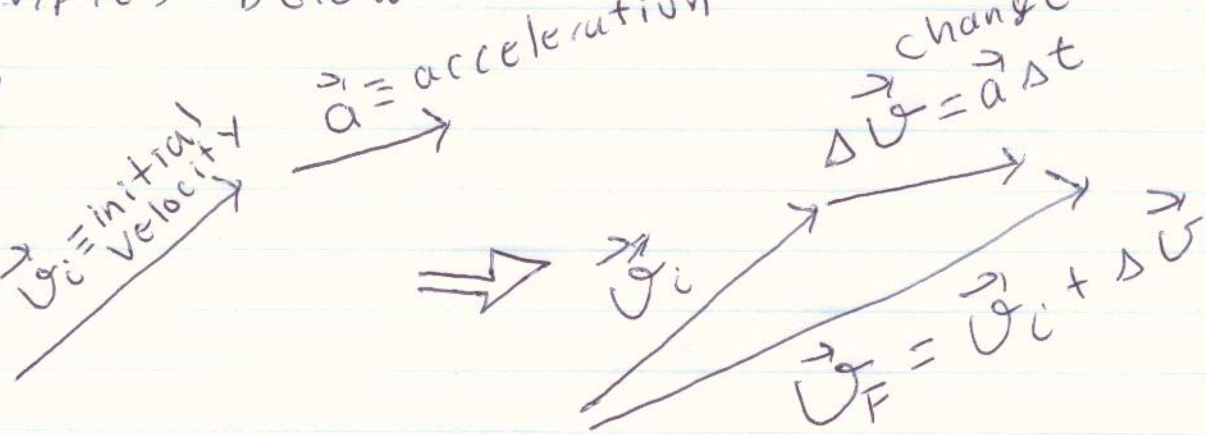
# Speeding up/Slowing down & turning Left/Right

acceleration $\vec{a}$ is the rate of change of velocity $\Delta\vec{v}/\Delta t$. Consider an Object moving at a velocity $\vec{v}$ and acceleration $\vec{a}$. To determine the effect of $\vec{a}$ on $\vec{v}$, assume $\vec{a}$ lasts for a period $\Delta t$ to create a change in velocity $\Delta\vec{v} = \vec{a}\,t$. Consider the examples below

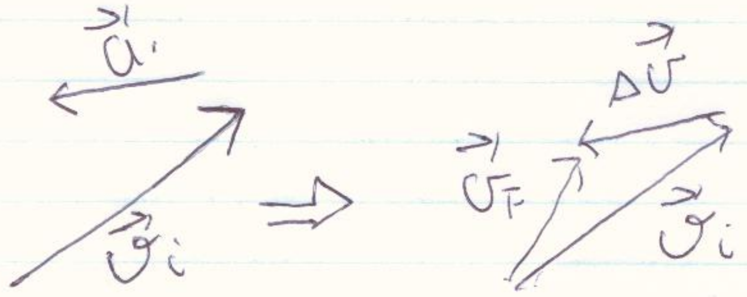
$\boxed{1}$




$v_F > v_i$ speeding up

turning right w.r.t $\vec{v}_i$

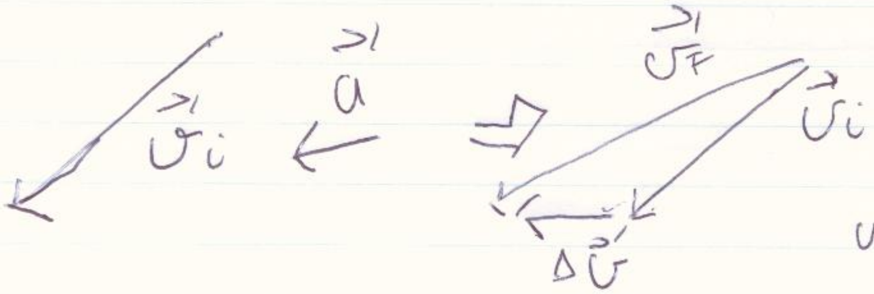
$\boxed{2}$




$v_F < v_i$ slowing down

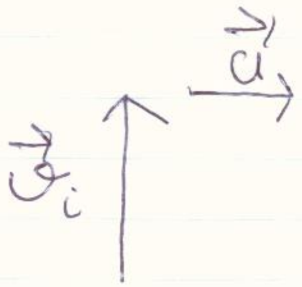
turning left

3




speeding up
turning right
w.r.t $\vec{v}_i$

4



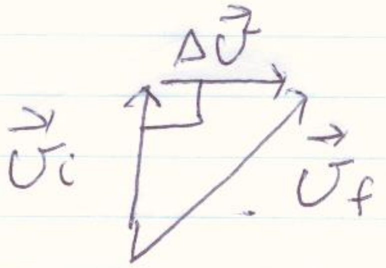

$\vec{a} \parallel \Delta\vec{v} \perp \vec{v}_i$

$\vec{a}\,\Delta t = \Delta\vec{v}$


constant speed
turning right

BUT IS THERE A PROBLEM ?

Consider again 4



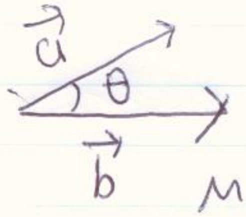

Using Pythagoras theorem $v_f > v_i$

So the speed shouldn't be constant

<u>REsolution</u> The reason for this apparent contradiction is due to the fact that when $\vec{v} \perp \vec{a}$ it is no longer appropriate to use $\Delta t > 0$, but instead we must use the limit $\Delta t \to 0$.

# Math Preamble

scalar product section 3.8 pp 47-49




$\vec{a}\cdot\vec{b} = ab\cos\theta \Rightarrow$ if $\vec{a}\perp\vec{b}$ $\theta = 90^\circ$

$\vec{a}\cdot\vec{b} = ab\cos 90^\circ = 0$

Magnitude of $\vec{a}$: $|\vec{a}| = a = \sqrt{\vec{a}\cdot\vec{a}}$

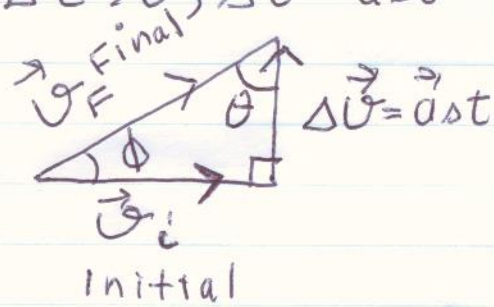
or $a^2 = \vec{a}\cdot\vec{a}$

Consider Again an object moving with $\vec{v}$ & acceleration $\vec{a}\perp\vec{v}$

$\vec{v} \longrightarrow \quad \uparrow \vec{a}$

i) Finite time

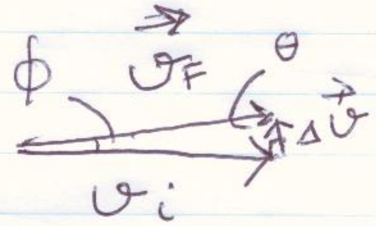
$\Delta t > 0$, $\Delta\vec{v} = \vec{a}\Delta t$




$\phi > 0$, $\theta < 90^\circ$

ii) Small time

$\Delta t <<< 1$




$\phi$ is small

$\theta$ is close to $90^\circ$

iii) Calculus limit

$\Delta t \to 0$

$\phi \to 0$

$\theta \to 90^\circ$

Hence as $\Delta t \to 0$ $\Delta\vec{v}$ is $\perp$ to $\vec{v}_F$ & $\vec{v}_i$

$\Delta\vec{v}\cdot\vec{v}_i = 0$ & $\Delta\vec{v}\cdot\vec{v}_F = 0$

Consider square Final speed

$$v_F^2 = \vec{v}_F\cdot\vec{v}_F = \vec{v}_F\cdot(\vec{v}_i + \Delta\vec{v})$$

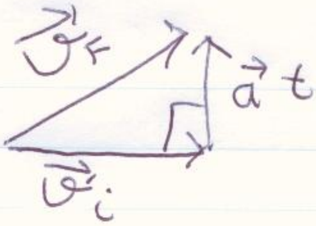
$$v_F^2 = \vec{v}_F\cdot\vec{v}_i + \underbrace{\cancel{\vec{v}_F\cdot\Delta\vec{v}}}_{\text{zero}}$$

$$= (\vec{v}_i + \Delta\vec{v})\cdot\vec{v}_i$$

$$v_F^2 = \vec{v}_i\cdot\vec{v}_i + \cancel{\Delta\vec{v}\cdot\vec{v}_i} \text{ zero}$$

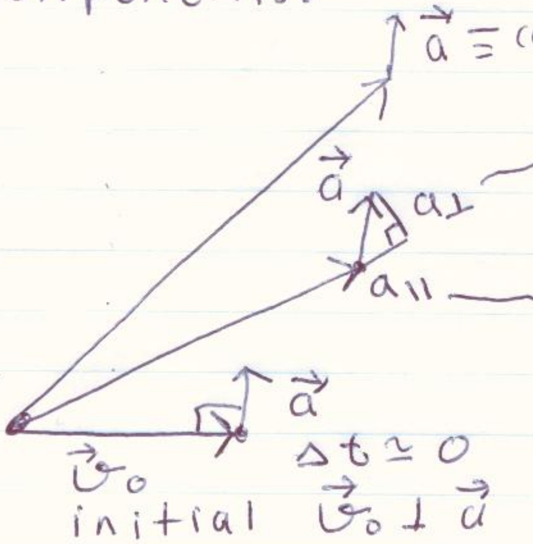
$$v_F^2 = v_i^2 \qquad \text{QED}$$

$$\boxed{\text{Final speed } v_F = v_i \text{ initial speed}}$$

Why for finite $\Delta t > 0$  gives $v_F > v_i$ ?



ANSWER Initially $\Delta t \cong 0$ & acceleration $\vec{a}$ is perpendicular $\perp$ to velocity $\vec{v}$. But as $\Delta t > 0$ increases the velocity $\vec{v}_F = \vec{v}_i + \vec{a}\cdot\Delta t$ will no longer be $\perp$ to $\vec{a}$, instead $\vec{a}$ will have parallel $a_{\parallel}$ & perpendicular $a_{\perp}$ components.

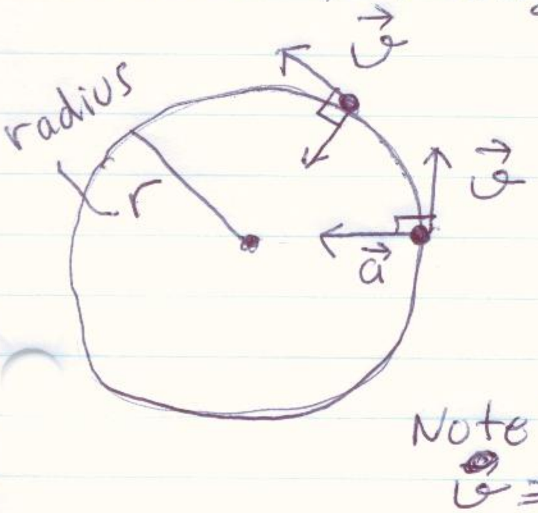




component of $\vec{a}$ $\perp$ $\vec{v}$, which changes Direction of $\vec{v}$

component of $\vec{a}$ // to $\vec{v}$, which changes the speed of $\vec{v}$ (i.e. makes $\vec{v}$ longer)

For UNIFORM CIRCULAR MOTION the acceleration $\vec{a}$ is constantly changing direction so that $\vec{a}$ is always $\perp$ $\vec{v}$, and the direction of $\vec{v}$ is constantly changing while ~~with~~ its speed $v$ remains constant.





$\vec{v}$ is always // to the circle
" " " $\perp$ to $\vec{a}$
$\vec{a}$ always points to center
magnitude $a = v^2/r$

Note: For uniform circular motion $v$ = constant. IF $v$ is not constant $\vec{a}$ can have both $a_{\perp}$ & $a_{\parallel}$.