CMAS EXAM, PHYS 1101 (INTRODUCTORY PHYSICS), 15 December 2009

NAME: **STUDENT ID:**

This exam book has 8 pages including an equation sheet on the last page

PART I: MULTIPLE CHOICE QUESTIONS (question 1 to 8)

For each question **circle** the correct answer (a,b,c,d or e).

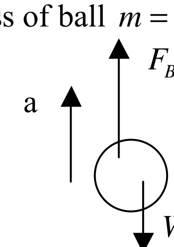
1. (**1 point**) A cat runs in a straight line (the x-axis) from point A to point B to point C, as shown below. The distance between point A and C is 5.00m, between point B and C is 10.0m, and the positive direction of the x-axis points to the right. The time to run from A to B is 20.0s and from B to C is 8.00s. The **average speed** for the whole trip is closest to
   a) -0.179 m/s b) 0.536 m/s c) 0.179 m/s d) 0.893 m/s e) -0.893 m/s

   Definition of average speed $v_{av} = \frac{\text{total distance}}{\text{total time}} = \frac{\text{AB + BC}}{t_{AB} + t_{BC}} = \frac{15.0m + 10.0m}{20.0s + 8.0s} = 0.893\frac{m}{s}$.

   Answer d) Note that average speed must be positive!!

2. (**1 point**) A golf ball is hit so that it leaves the ground at 60° above the horizontal and feels no air resistance as it travels. Which of the following statement about the subsequent motion of the ball while it is in the air is true? Only one correct answer!
   a) Its speed is zero at its highest point.
   b) Its velocity is zero at its highest point.
   c) Its acceleration is always 9.8 m/s^2 downward.
   d) Its forward acceleration is always 9.8 m/s^2.
   e) Its acceleration is zero at its highest point.
   Answer c)

3. (**1 point**) A plastic ball in liquid is acted upon by its weight and by a buoyant force. The weight of the ball is 2.5N. The buoyant force has a magnitude of 4.4N and acts vertically upward. At a given instant, the ball is released from rest. The acceleration of the ball at that instant, including direction, is closest to:
   a) zero b) 7.4 m/s^2,up c) 7.4 m/s^2,down d) 3.7 m/s^2,up e) 3.7 m/s^2 down
   mass of ball $m = W / g = 2.5N / 9.8m/s^2 = 0.255kg$

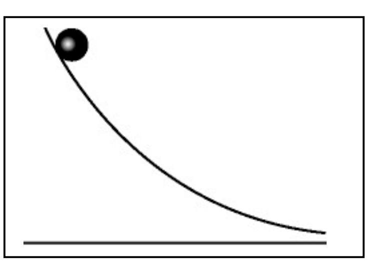


   $F_B = 4.4N$, Buoyant Force

   second law $F_y^{net} = F_B - W = ma \rightarrow a = \frac{4.4N - 2.5N}{0.255kg} = 7.4\frac{m}{s^2}$

   $W = 2.5N$ **ANSWER** b)

4. (**1 point**) In the figure below a ball rolls down a hill. Which of the following statements is the correct one? **ANSWER** e)



   a) both the speed and acceleration remain constant.
   b) its speed decreases and its acceleration increases.
   c) both its speed and its acceleration increase.
   d) both speed and acceleration decrease.
   e) its speed increases and its acceleration decreases

5. (**1 point**) A 13.5-kg box slides over a rough patch 1.75 m long on a horizontal floor. Just before entering a rough patch the speed of the box was 2.25 m/s, and just before leaving it its speed was 1.20 m/s. The magnitude of the average force of friction on the rough patch exerted on the box is closest to:
a) 14.0 N b) 13.7N c) 19.5 N d) 5.55 N
e) it is impossible to calculate since the coefficient of friction is not given
Using work-energy theorem, the work done by friction is

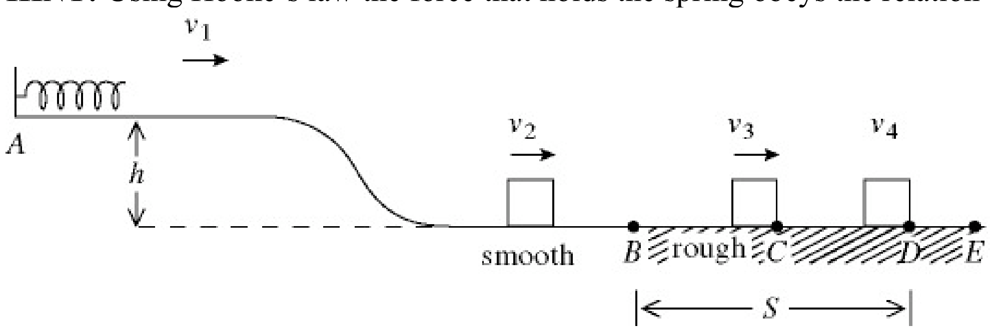
$$W_f = \Delta K = \frac{1}{2}(13.5kg)\left(1.20\frac{m}{s}\right)^2 - \frac{1}{2}(13.5kg)\left(2.25\frac{m}{s}\right)^2 = -24.45J$$

$$W_s = -f_k d = -24.45J \rightarrow f_k = \frac{24.45J}{1.75m} = 13.97N = 14.0N$$

ANSWER a)

6. (**1 point**) A 0.54-kg block is held in place against the spring by a 65-N horizontal external force. The external force is removed, and the block is projected with a velocity v_1 = 1.2 m/s upon separation from the spring. The block descends a ramp and has a velocity v_2 = 1.4 m/s at the bottom. The track is frictionless between points A and B. The block enters a rough section at B, extending to E. The coefficient of kinetic friction is 0.36. The velocity of the block is v_3 = 1.4 m/s at C. The block moves on to D, where it stops. In the figure below the initial compression of the spring, in cm, is closest to:
a) 0.64 b) 0.43 c) 1.6 d) 1.2 e) 2.4

HINT: Using Hooke's law the force that holds the spring obeys the relation $kx = 65N$!



To answer the question we only need to consider all data in the vicinity where the box is in contact with the spring. Using the fact that the box start from rest and has a "final" speed of v_1 = 1.2 m/s, and using conservation of energy (no friction) we obtain $\frac{1}{2}mv_1^2 = \frac{1}{2}kx^2$, where x is the maximum (initial) compression of the spring. Rearranging we obtain $x = \frac{mv_1^2}{kx}$. Also we can use the hint $kx = 65N$,

$$x = \frac{0.54kg \times (1.2m/s)^2}{65N} = 0.01196m = 1.2cm$$ **ANSWER** d)

7. (**2 points**) A block of mass m = 4.2 kg, moving on a frictionless surface with a speed v_i = 2.9 m/s, makes a perfectly **elastic collision** with a block of mass M at rest. After the collision, the 4.2 kg block recoils with a speed of $|v_f|$= 0.9m/s. In figure below, the mass M after the collision is closest to:

V

a) 4.2 kg b) 5.5 kg c) 18 kg d) 8.0 kg e) 3.8 kg

Hint: 1)You must first find V. For an elastic collision the **initial relative velocity** between the colliding objects has the same **magnitude as the final relative velocity** (v_f – V), but the two velocities are in opposite directions. 2) Once V is found ask yourself the meaning of an elastic collision.

Using the hint $v_i = -(v_f - V) = V - v_f$. Using $|v_i| = 0.9$ and the fact that m is traveling left after the collision (see diagram) gives $v_f = -0.9m/s$. Combine with $v_i = 2.9m/s$ we have $V = v_i + v_f = 2.9m/s - 0.9m/s = 2.0m/s$. Now use conservation of momentum (or if you like, conservation of energy, but this is harder):

$$mv_i = mv_f + MV \rightarrow M = \frac{m(v_i - v_f)}{V} = \frac{4.2kg(2.9m/s + 0.9m/s)}{2m/s} = 7.98kg$$ **ANSWER** d)

8. (**2 points**) At time t = 0, a wheel has angular displacement of zero radians and an angular velocity of +15 rad/s. The wheel has a constant acceleration of –0.48 rad/s^2. The time at which the angular displacement is +78, and decreasing, is closest to:
a) 6 s b) 57 s c) 31 s d) 5 s e) 67 s

Use $\theta = \omega_{0z}t + \frac{1}{2}\alpha_z t^2$ with $\theta = 78rad$, $\omega_{0z} = 15\frac{rad}{s}$ and $\alpha_z = -0.48\frac{rad}{s^2}$. Since initially $\omega_z = \omega_{0z} > 0$, but $\alpha_z < 0$, θ will increase until at time t_1 it will be $\theta = 78rad$. Assuming that at time t_1, $\omega_z > 0$ θ will keep increasing until $\omega_z \leq 0$, when θ will decrease at time t_2 it is once again $\theta = 78rad$. To find time t_1 and t_2 we use the above info to get

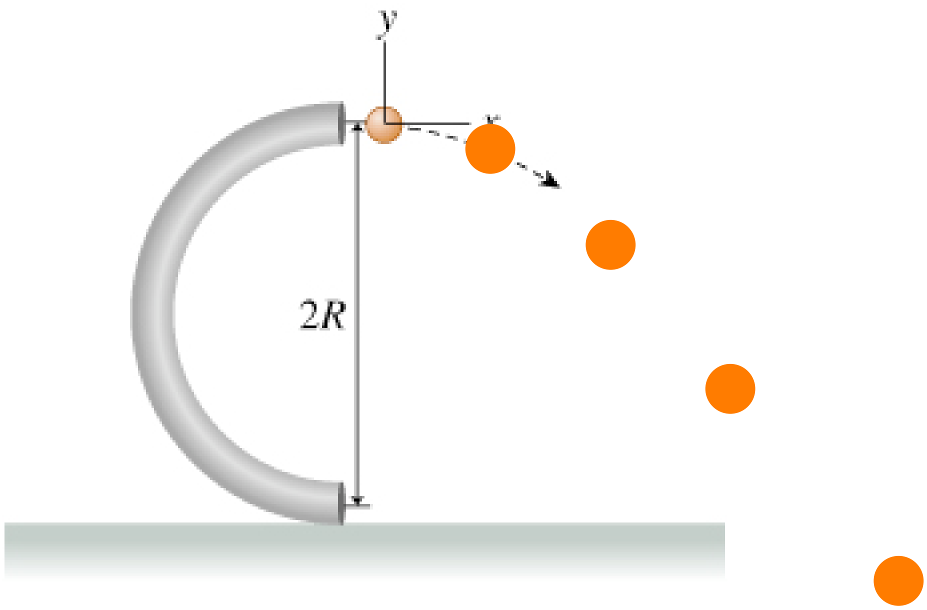
$\theta = \omega_{0z}t + \frac{1}{2}\alpha_z t^2 \rightarrow 78 = 15t - 0.24t^2 \rightarrow t^2 - 62.5t + 325 = 0$ with solution

$t = \frac{62.5 \pm \sqrt{62.5^2 - 4 \times 325}}{2} = 5.73s$ and 57s with $t_1 = 5.7s$ and $t_2 = 57s$. Of course $t_2 = 57s$ is the time when the angular displacement is +78, and decreasing. **ANSWER** b)

PART II: FULL ANSWER QUESTIONS (question 9 to 12)

Do all four questions on the provided exam booklets. Show all works.

9. (**10 points)**) A ball is launched up a semicircular chute of radius R = 1.0 m. At the top of the chute, just before it goes into free fall, the ball has a **centripetal acceleration** of 19.6 m/s^2.



a) Draw a motion diagram of the trajectory from when the ball emerges from the top of the chute to when it hits the ground.

The motion diagram is as shown above. To receive full credit, the dots spacing must increase and the trajectory must curve downward (e.g. it is not a straight line) to indicate the increase in the y-component of the velocity.(**2 points**)

b) Determine the speed of the ball just as it emerges from the top of the chute. **HINT:** the **centripetal acceleration** of an object in **circular motion** of radius R and speed v is $a_{rad} = \frac{v^2}{R}$

From $a_{rad} = \frac{v^2}{R} \rightarrow v = \sqrt{Ra_{rad}}$ (**1 point**), R = 1 m and a_{rad} = 19.6m/s^2 (**1 point**), $v = \sqrt{1m\left(19.6m/s^2\right)} = 4.43m/s$ (**1 point**).

c) Determine the vertical and horizontal component of the velocity just as the ball emerges from the top of the chute.

$v_{0x} = 4.43m/s$ (**1 point**), $v_{0y} = 0m/s$ (**1 point**),

d) Determine the horizontal distance from the bottom of the chute to where the ball lands

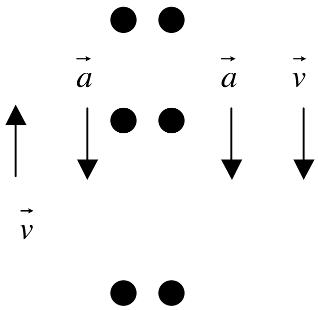
Time to the ground is found with $y = y_0 + v_{0y}t - \frac{1}{2}gt^2$ with $y_0 = 0$ and y = -2.0 m giving

$t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2(2m)}{9.8m/s^2}} = 0.64s$. (**1.5 points**)

The range is simply range= $v_{0x}t = 4.43 \times 0.64s = 2.83m$ (**1.5 points**)

10. (**10 points**)) Jack throws an egg straight up with a speed of 8.4 m/s. The egg is released at the same height as his head. It rises and then falls down and hits his head.

    (a) Draw a **motion diagram** of the path of the egg from when it was released to the time it lands on his head. In the diagram, indicate the direction of the velocity and acceleration of the egg.



In the diagram the dots represent the positions of the egg at equal time interval. To obtain full marks the spacing between dot must decreases on the way up and increases on the down, in order to illustrate that the velocity is changing. Further the direction of the velocity and acceleration on the way up/down must be clearly indicated as shown. (**2 points**)

(b) Find the velocities of the egg at the height of 2.0 m above Jack's head. Note that the egg will be 2.0 m above Jack's head twice, once on the way up, the other on the way down.

Easiest method

$v_y^2 = v_{0y}^2 - 2g(y - y_0) \Rightarrow v_y = \pm\sqrt{v_{0y}^2 - 2g(y - y_0)}$ (**1 point**)

v_{0y} = 8.4 m/s, y_0 = 0, y = 2.0m, v_y = velocity at 2.0 m (**1 point**)

$v_y = \pm\sqrt{v_{0y}^2 - 2g(y - y_0)} = \pm\sqrt{(8.4m/s)^2 - 2(9.8m/s^2)(2.0m)} = \pm 5.6m/s$ (**1 point**)

v_y = + 5.6 m/s, on the way up, v_y = -5.6 m/s, on the way down. (**1 point**)

Hard method

$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$, v_{0y} = 8.4 m/s, y_0 = 0, y = 2.0m, t = time when height is 2.0 m (**1 point**)

$2.0m = (8.4m/s)t - \frac{1}{2}(9.8m/s^2)t^2 \Rightarrow 4.9t^2 - 8.4t + 2 = 0$ (**1 point**)

$t = \frac{8.4 \pm \sqrt{(8.4)^2 - 4(4.9)2}}{9.8} = .857s \pm 571s \Rightarrow t = 1.43s(down)$ and $t = 0.286s(up)$ (**1 point**)

Way Up t = 0.286 s

$v_y = v_{0y} - gt = 8.4m/s - (9.8m/s^2)(0.286) = 5.6m/s$ (**0.5 point**)

Way down t = 1.43 s

$v_y = v_{0y} - gt = 8.4m/s - (9.8m/s^2)(1.43s) = -5.6m/s$ (**0.5 point**)

(c) Find the time when the egg hits Jack's head.

$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$, v_{0y} = 8.4 m/s, y_0 = 0, y = 0m (**1.5 points**)

$0 = 0 + v_{0y}t - \frac{1}{2}gt^2 \Rightarrow -\frac{1}{2}gt\left(t - \frac{2v_{0y}}{g}\right) = 0$ (**1.5 points**)

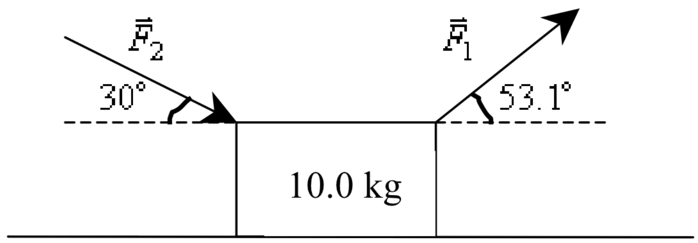
There are two solutions:

(1)t=0, not relevant since this is the time when the egg is thrown

(2) $t = \frac{2v_{0y}}{g} = \frac{2(8.4m/s)}{9.8m/s^2} = 1.71s$, correct solution

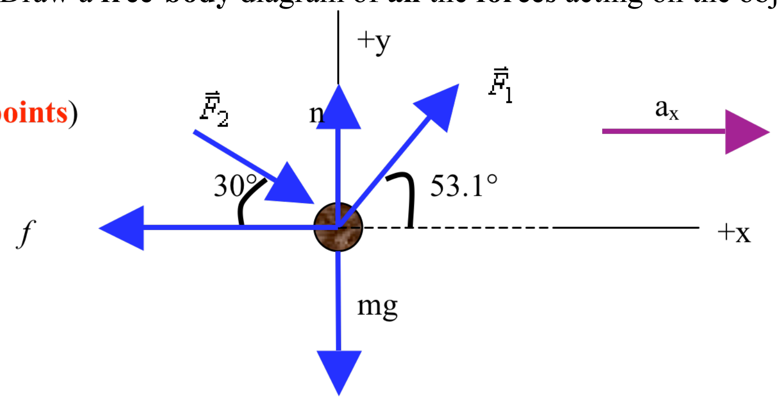
The egg will hit Jack's head 1.71 s after it was launched. (**1 point**)

11. (**10 points**) A **10.0 kg** object on a surface, is acted on by **external forces** ($F_1 = 30.0N$ and $F_2 = 20.0N$), as shown below. The **coefficient** of **kinetic friction** between the surfaces is $\mu_k = 0.2$ and the **coefficient** of **static friction** is $\mu_s = 0.3$.



(a) Draw a **free-body** diagram of **all** the **forces** acting on the object.

(**2 points**)



(b) Find the **vertical** and **horizontal components** of the **net force** acting on the object.

Vertical $n + F_{1x} + F_{2x} - mg = 0 \rightarrow n = mg - F_{1x} - F_{2x}$ (**1 point**)

$n = (10.0kg)(9.8m/s^2) - (30N \sin 53.1^\circ) - (-20.0N \sin 30^\circ) = 84N$ (**1 point**)

Static Friction $f_s \le \mu_s n = (0.3)(84N) = 25.2N$ (**0.5 point**)

Kinetic Friction $f_k = \mu_k n = (0.2)(84N) = 16.8N$ (**0.5 point**)

$F_{1x} + F_{2x} = (30.0N)\cos 53.1^\circ + (20.0N)\cos 30^\circ = 35.3N > 25.2N =$ max.f_s,

so the crate will move. (**1 point**)

Horizontal $F_x^{net} = F_{1x} + F_{2x} - f_k = (30.0N)\cos 53.1^\circ + (20.0N)\cos 30^\circ - 16.8 = 18.5N$

(**2 points**)

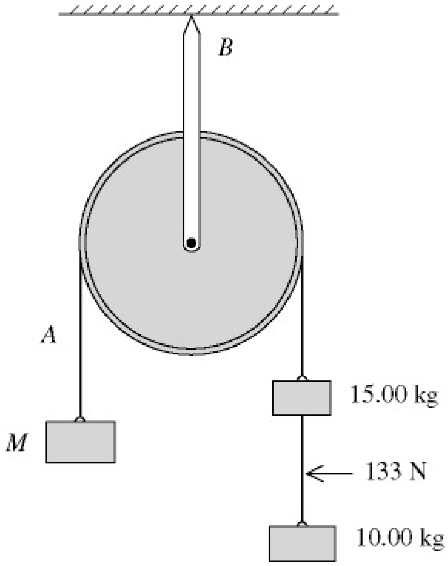
(c) Find the **acceleration** of the object.

Using Newton's second law

$F_x^{net} = ma_x \rightarrow a_x = \frac{F_x^{net}}{m} = \frac{18.5N}{10.0kg} = 1.85m/s^2$. (**1.5 points**)

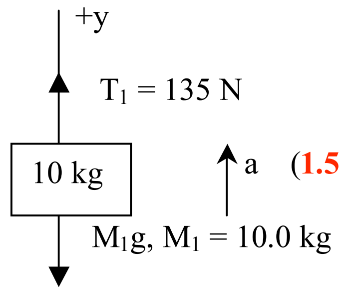
The acceleration is to the right. (**0.5 points**)

12. (**10 points**) Three objects are connected by massless wires over a massless frictionless pulley as shown below. The tension in the rope between the 10.0kg and 15.0-kg objects is measured to be 133N.



a   Assume 15-kg object accelerates upward

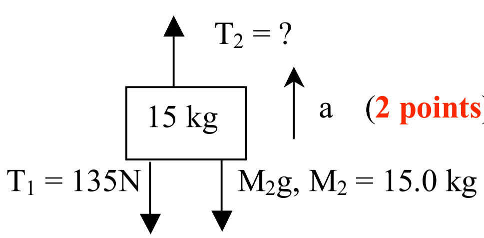
a) Draw a **free-body** diagram of all the forces acting on the 10.0-kg object. Hence determine the acceleration, a, of the 10.0-kg object.



(**1.5 points**)

$$F_y^{net} = T_1 - M_1 g = M_1 a \rightarrow a = \frac{T_1 - M_1 g}{M_1} = \frac{135N - 10.0kg \times 9.8m/s^2}{10.0kg} = 3.5\frac{m}{s^2}$$ (**1.5 points**)

b) Draw a **free-body** diagram of all the forces acting on the 15.0-kg object. Using the acceleration found in part (a) find the tension between the rope between the mass M and the 15.0-kg object. **Note: The answer is not 133N!**

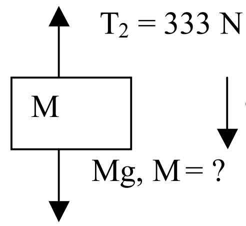


(**2 points**)

$$F_y^{net} = T_2 - T_1 - M_2 g = M_2 a \rightarrow T_2 = 133N + (15.0kg)\left(9.8\frac{m}{s^2} + 3.5\frac{m}{s^2}\right) = 333N$$

(**2 points**)

c) Draw a **free-body** diagram of all the forces acting on the mass M. Hence, determine the mass M.



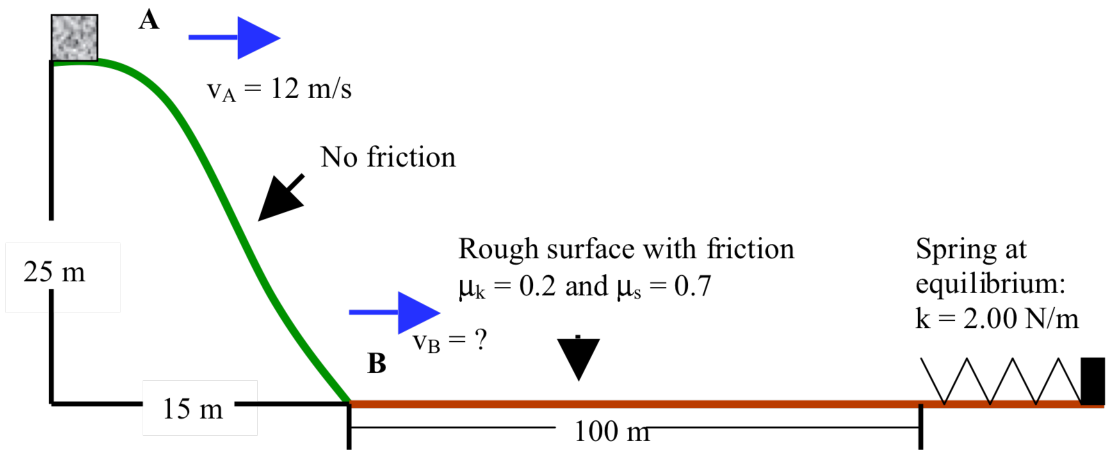
(**1.5 points**)

$$F_y^{net} = T_2 - Mg = -Ma \rightarrow M = \frac{T_2}{g-a} = \frac{333N}{6.3m/s^2} = 52.9kg$$ (**1.5 points**)

PART III: FULL ANSWER QUESTIONS (question 13 to 16)

Do all four questions on the provided exam booklets. Show all works.

13. (**10 points**) A 14.0 kg ball rock slides down a hill, leaving point A with a speed of 12 m/s. There is no friction between point A and B. There is friction on the level surface between point B and the wall (where the spring is located).



a) What is the speed of the rock when it reaches point B (bottom of the hill).
Use **conservation of mechanical energy**. Take gravitational potential energy to be zero at the bottom of the hill $U_B = 0$, then at the top of the hill $U_A = mgh$ (h=25m). (**1.5 points**)

$$K_A + U_A = K_B + U_B \rightarrow \frac{1}{2}mv_A^2 + mgh = \frac{1}{2}mv_B^2 + 0$$ (**1.5 points**)

$$v_B = \sqrt{v_A^2 + 2gh} = \sqrt{(12m/s)^2 + 2(9.8m/s^2)(25m)} = 25.2m/s$$ (**1 point**)

b) After reaching B, the rock travels 100m on the level surface until it hits a very long light spring (k = 2.00 N/m). How far will the rock compress (maximum) compression) the spring?

The rock travels a distance of 100 m + x, where x is the maximum compression of the spring. The dissipative work done by friction is

$W_f = -mg\mu_k(100m + x) = -(14kg)(9.8m/s^2)(0.2)(100m + x) = -2744W - (27.44N)x$

(**1 point**)

Here use **conservation of mechanical energy**.

$K_B + U_{el,B} + W_f = K_{\max} + U_{el,\max}$ (**1 points**)

At the maximum compression $K_{\max} = 0, U_{el,\max} = \frac{1}{2}kx^2 = (1.00N/m)x^2$ (**1 point**)

Of course $U_{el,B} = 0$ (**0.5 point**)

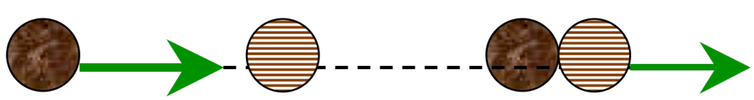
This gives $\frac{1}{2}(14kg)(25.2m/s)^2 - 2744W - (27.44N)x = (1.00N/m)x^2$ (**1 point**)

This gives the quadratic equation $x^2 + 27.44x - 1701 = 0$ (**0.5 point**)

The solution is $x = \frac{-27.44 \pm \sqrt{(27.44)^2 - 4(-1701)}}{2} = 29.7m, -57.2m$ (**0.5 point**)

The **physical solution** is a **maximum compression** of 29.7m. (**0.5 point**)

14. (**10 points**) A hockey puck B at **rest** on a **frictionless** ice surface is struck by a second puck A, which was initially traveling at 15 m/s as shown in the diagram. Both pucks are wrapped in Velcro and stick together after the collision and move off as shown below. Puck A has a mass of m_A = 2.5 kg, and puck B has a mass of m_B = 1.5 kg



a) Find the speed of the pucks after the collision.

Use conservation of momentum:

Initial $m_A v_A = (m_A + m_B)v_F$ Final (**2 points**)

Of course $v_A = 15m/s$ and v_F is the unknown speed of A + B after the collision.

$v_F = \frac{m_A v_A^0}{m_A + m_B} = \frac{(2.5kg)(15m/s)}{2.5kg + 1.5kg} = 9.37m/s$ (**2 points**)

b) Calculate the **change** in **kinetic energy** due to the collision.

Initial $K_i = \frac{1}{2}m_A v_A^2 = \frac{1}{2}(2.5kg)(15m/s)^2 = 281J$ (**1.5 points**)

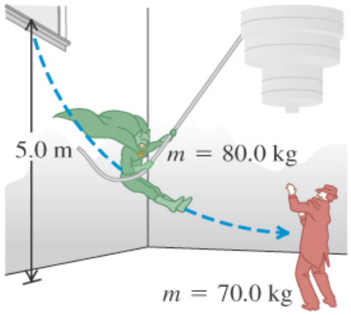
Initial $K_F = \frac{1}{2}(m_A + m_B)v_F^2 = \frac{1}{2}(4.0kg)(9.375m/s)^2 = 176J$ (**1.5 points**)

Change in kinetic energy $\Delta K = K_F - K_i = 176J - 281J = -105J$ (**1 point**)

c) Is the **collision elastic**? Briefly **justify** your answer.

No, there is a loss in kinetic energy of 105 J. Hence kinetic energy is not conserved. (**2 points**)

15. (**10 points**) A stuntman of mass 80.0 kg swings on a rope from a 5.0 m high ledge towards a 70.0 kg villain standing on the ground. Assume stuntman is **initially** at **rest.**



a) What is the speed of the stuntman just before he hits the villain?

This is done by using **conservation of mechanical energy**; basically the stuntman's energy at the top must equal his energy just before he hits the villain.

Take the **gravitational PE** to be zero at the bottom, $U_{bottom}^{grav} = 0$ and $U_{top}^{grav} = m_s gh$ (m_s = 80.0kg, h = 5.0 m) at the top.

Conservation of energy $m_s gh = \frac{1}{2} m_s v_s^2$, where v_s is the stuntman's speed just before he hits the villain.

$m_s gh = \frac{1}{2} m_s v_s^2 \rightarrow v_s = \sqrt{2gh} = \sqrt{2(9.8m/s^2)(5.0m)} = 9.9m/s$ (**4 points**)

b) What is the horizontal component of the velocity of the stuntman **just before** he hits the villain? **Hint:** Look carefully at the diagram.

Obviously, at the bottom just before the collision the stuntman's velocity is horizontal, so the horizontal component of the velocity is 9.9m/s. (**2 points**)

c) Just after they (stuntman + villain) collide, and **become entangled**, what is their speed as they slide on the floor?

Use **conservation of momentum** of the **horizontal component** before and after the stuntman + villain collision.

$m_s v_s = (m_s + m_v) v_f$, m_s = 70.0 kg and v_f the final speed of the entangled pair.

$v_f = \frac{m_s v_s}{m_s + m_v} = \frac{(80.0kg)(9.9m/s)}{80.0kg + 70.0kg} = 5.28m/s$ (**4 points**)

16. (**10 points**) An electric fan is turned off, and its angular velocity decreases uniformly from 460 rev/min to 150 rev/min in a time interval of length 3.75 s.

a) Find the **angular acceleration** in rev/s^2?

Use $\omega_z = \omega_{0z} + \alpha_z t$ with initial angular velocity $\omega_{0z} = 460 \frac{rev}{\min} \times \frac{1}{60s/\min} = 7.67 \frac{rev}{s}$ and final angular velocity $\omega_z = 150 \frac{rev}{\min} \times \frac{1}{60s/\min} = 2.50 \frac{rev}{s}$ and t = 3.75 s. This gives $\alpha_z = \frac{2.50 rev/s - 7.67 rev/s}{3.75s} = -1.38 \frac{rev}{s^2}$ (**4 points**)

b) Find the number of revolutions made by the motor in the time interval of length 3.75 s.

Use $\theta = \omega_{0z} t + \frac{1}{2}\alpha_z t^2 = \left(7.67 \frac{rev}{s}\right)(3.75s) + \frac{1}{2}\left(-1.38 \frac{rev}{s^2}\right)(3.75s)^2 = 19.1 rev$ (**3 points**)

c) How many seconds, after it is turn off, are required for the fan to come to rest? Assume the angular acceleration remain constant at the value calculated in part a).

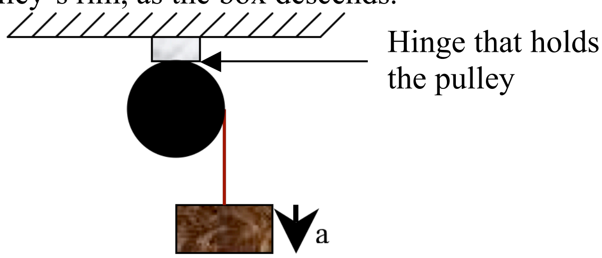
Use $\omega_z = \omega_{0z} + \alpha_z t$ with initial angular velocity $\omega_{0z} = 7.67\frac{rev}{s}$ and final angular velocity $\omega_z = 0$, which gives $t = -\frac{\omega_{0z}}{\alpha_z} = -\frac{7.67 rev/s}{-1.38 rev/s^2} = 5.56 s$ (**3 points**)

Note: 1 rev = 2π rad; 1 min = 60 s, but you **do not need** to change rev to rad.

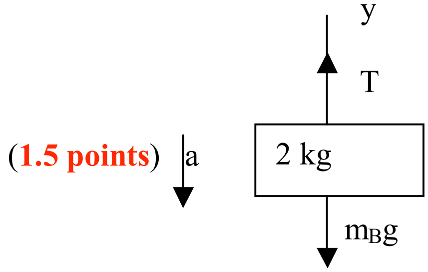
PART IV: FULL ANSWER QUESTIONS (question 17 to 19)

Do **two of three** questions on the provided exam booklets. Show all works.

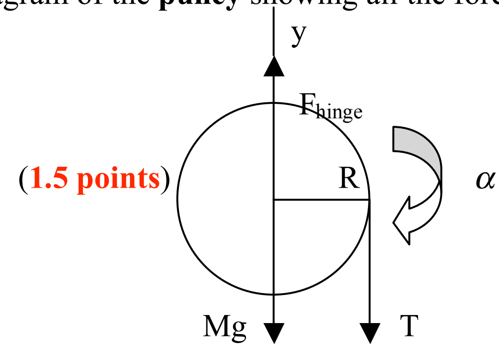
17. (**10 points**) A Consider the system below where the pulley is a uniform thin-walled cylinder with a radius of R = 0.220 m, a mass M = 2.0 kg, and moment of inertia of $I = MR^2$. A box of mass m_B = 3.0 kg hangs from the pulley by a **massless** rope that **does not slip** over the pulley's rim, as the box descends.



a) Draw a free body diagram of the **box** showing all the forces acting on it.



b) Draw a free body diagram of the **pulley** showing all the forces acting on it.



c) Use **Newton's 2nd law** for **translation** and **rotation** to determine the **linear acceleration**, a, of the box, and the **angular acceleration** of the pulley, α.

Refer to the figure in part a) for translation $F_y^{net} = T - m_B g = -m_B a$

Refer to the figure in part b) for rotation $\tau^{net} = TR = I\alpha$ with I = MR^2 which gives $TR = MR^2\alpha \rightarrow T = MR\alpha$. Using the condition for rotation without slipping $a = \alpha R$ gives $T = Ma$.

Combining the equations gives $T - m_B g = -m_B a \rightarrow Ma - m_B g = -m_B a$, which gives

$$a = \frac{m_B g}{M + m_B} = \frac{3kg \times 9.8m/s^2}{2kg + 3kg} = 5.88\frac{m}{s^2} \text{ and } \alpha = \frac{a}{R} = \frac{5.88m/s^2}{0.220m} = 26.7\frac{rad}{s^2}$$

(**4 points**)

d) How far does the box descend after 1.2 seconds?

$$y = \frac{1}{2}at^2 = \frac{1}{2}\left(5.88\frac{m}{s^2}\right)(1.2s)^2 = 4.23m$$ (**1 point**)

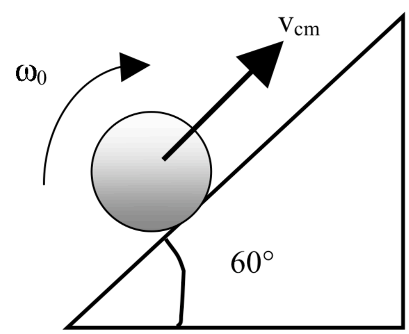
e) Using the diagram drawn in part b) determine the force of the hinge (holding the pulley up) on the pulley.

Referring to the diagram of part b $F_y^{net} = F_{hinge} - Mg - T = 0$, since the net force on the pulley must be zero. Hence $F_{Hinge} = T + Mg$. From part c) $T = Ma = 2kg \times 5.88m/s^2 = 11.76N$.

Finally $F_{Hinge} = 11.76 + 2kg \times 9.8/s^2 = 31.4N$

(**2 points**)

18. (**10 points**) A solid sphere of R = 0.250 m, a mass M = 3.0 kg, and moment of inertia of I = $\frac{2}{5}$MR^2 is rolling, **without slipping**, up a 60° incline

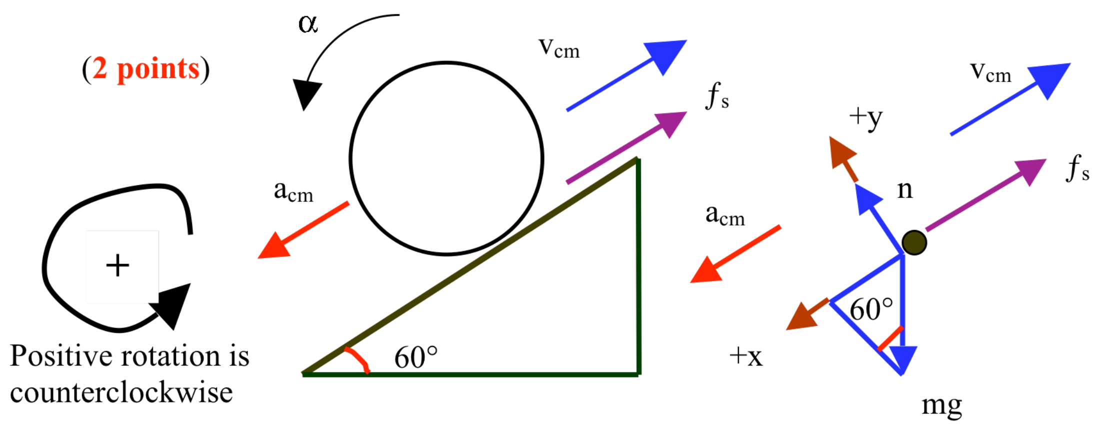


a) The initial center-of-mass velocity is $v_{cm} = 2.0m/s$. What is its initial angular velocity $\omega_\circ$ of the rolling sphere?

Using the no-slip condition

$$v_{cm} = R\omega_0 \rightarrow \omega_0 = \frac{v_{cm}}{R} = \frac{2.0m/s}{0.250m} = 8.0rad/s$$ (**1 point**)

b) Draw a free-body diagram of all the forces acting on the sphere. **Briefly explain** why the **friction force** must be directed **up** the **incline**.



Note that both the center-of-mass acceleration (a_{cm}) and the center-of-mass velocity (v_{cm}) are downhill. More importantly, the angular acceleration is counterclockwise. The force due to weight of the ball passes through the axis of rotation (the center of the ball), and contributes no torque. Only the static friction force, f_s, contributes a torque, which must be directed uphill to induce a counterclockwise angular acceleration. (**1 point**)

c) Determine the center-of-mass acceleration of the sphere. **HINT:** Use **Newton's 2nd law** for **translation** and **rotation**.

Center-of Mass Motion Newton's second law for translation

x-component

$Mg\sin 60^\circ - f_s = Ma_{cm} \to (3.0kg)(9.8m/s^2)\sin 60^\circ - f_s = (3.0kg)a_{cm}$

$25.5N - f_s = (3.0kg)a_{cm}$, (1) (**1 point**)

Rotation about center-of-mass Newton's second law for rotation

Counterclockwise rotation As mention, only f_s can induce a torque that in turn induces the angular acceleration.

$\tau = I\alpha \to f_s R = \frac{2}{5}MR^2\alpha \to f_s = \frac{2}{5}M(R\alpha) = \frac{2}{5}(3.0kg)(R\alpha)$ (**1 point**)

Using the **no-slip** condition $a_{cm} = R\alpha$ (**0.5 point**)

$f_s = (1.2kg)a_{cm}$, (2) (**0.5 point**)

Substituting (2) into (1) gives

$25.5N - (1.2kg)a_{cm} = (3.0kg)a_{cm} \to a_{cm} = \frac{25.5N}{4.2kg} = 6.07m/s^2$ (**1 point**)

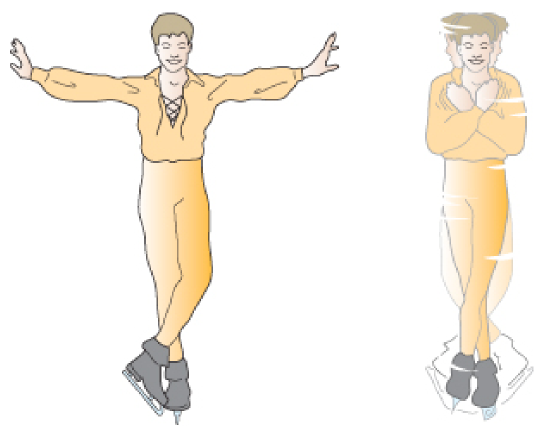
$a_{cm} = R\alpha \to \alpha = \frac{a_{cm}}{R} = \frac{6.07m/s^2}{0.250m} = 24.3rad/s^2$ (**0.5 point**)

d) **Briefly explain** why the **friction force** must be a **static force** of **friction**, and determine this **static force** of **friction** as the sphere rolls uphill.

Referring to the first diagram, we see that the center-of-mass of the wheel is moving uphill. Due to the clockwise rotation, ω_0, of the wheel, the point of contact between the wheel and the surface is moving downhill **relative** to the center of mass. If there is **no slippage**, the two movements cancel, and the point of contact is stationary with respect to the surface, and the **friction force** is **static**. (**1 point**)

Using equation (2) $f_s = (1.2kg)a_{cm} = (1.2kg)(6.07m/s^2) = 7.3N$ (**0.5 point**)

19. (**10 points**) **Spinning Figure Skater**. In the figure below, a skater with outstretched arm (situation 1) is spinning at angular speed $\omega_1 = 0.3rev/s$. He then folds his arm (situation 2), which **increases** his **angular speed** to a value of ω_2. In situation 1 (before) assume his body is a solid cylinder of mass M = 60.0 kg and radius R = 0.25m (moment of inertia $I = \frac{1}{2}MR^2$), and his outstretched arm is a thin rod of mass m = 10.0 kg and length L = 1.6 m (moment of inertia $I = \frac{1}{12}mL^2$). In scenario 2 (after) assume **his whole body** is a solid cylinder of mass M_{total} = 70.0 kg and radius R = 0.25m.



Before (situation 1) ω_1 After (situation 2) ω_2

a) Calculate the **total** moment of inertia in situation 1 (before). Assume that the two contributions discussed above can be added to find the **total moment of inertia**.

$$I_1^{total} = \frac{1}{2}MR^2 + \frac{1}{12}mL^2 = \frac{1}{2}(60kg)(0.25)^2 + \frac{1}{12}(10kg)(1.6)^2 = 4.01kg \bullet m^2$$ (**3 points**)

b) Calculate the **total moment of inertia** in situation 2 (after).

$$I_2^{total} = \frac{1}{2}M_{total}R^2 + \frac{1}{12}mL^2 = \frac{1}{2}(70kg)(0.25)^2 = 2.19kg \bullet m^2$$ (**3 points**)

c) Using the results of part a) and b) determine the final (after, situation 2) angular speed ω_2.

Using conservation of angular momentum $I_1^{total}\omega_1 = I_2^{total}\omega_2 \rightarrow \omega_2 = \frac{I_1^{total}}{I_2^{total}}\omega_1$

$$\omega_2 = \frac{4.01kg \bullet m^2}{2.19kg \bullet m^2}\left(0.3\frac{rev}{s}\right) = 0.549\frac{rev}{s}$$ (**4 points**)

Useful Equations

Kinematics $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$, $v_x = v_{0x} + a_x t$, $v_x^2 = v_{0x}^2 + 2a_x\left(x - x_0\right)$

$\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$, $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k}$

$x = x_0 + \int_{t_0}^{t} v_x\, dt$, $y = y_0 + \int_{t_0}^{t} v_y\, dt$, $z = z_0 + \int_{t_0}^{t} v_z\, dt$

$v_x = v_{0x} + \int_{t_0}^{t} a_x\, dt$, $v_y = v_{0y} + \int_{t_0}^{t} a_y\, dt$, $v_z = v_{0z} + \int_{t_0}^{t} a_z\, dt$

average velocity (x-com) $v_{ave,x} = \frac{x_2 - x_1}{t_2 - t_1}$, **average acceleration (x-com)** $a_{ave,x} = \frac{v_{2x} - v_{1x}}{t_2 - t_1}$

Newton's Law $\vec{F}^{net} = \sum \vec{F} = 0$ (Object in equilibrium), $\vec{F}^{net} = m\vec{a}$ (Nonzero net force).

Friction Force: $f_s \le \mu_s n$, $f_k = \mu_k n$

Work-Energy Theorem $W^{net} = \Delta K = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$ (W^{net} is the **net** or **total work done** on the object)

Conservation of Mechanical Energy (only **conservative forces** are present)

$W^{net} = -\Delta U = -\left(U_2 - U_1\right) = \Delta K = K_2 - K_1, U_1 + K_1 = U_2 + K_2, U^{grav} = mgy, U^{el} = \frac{1}{2}kx^2$

Non-Conservative Forces $U_1 + K_1 + W_{other} = U_2 + K_2$ (W_{other} work done by **non-conservative** forces)

Impulse-Momentum $\vec{P} = m\vec{v}$, $\vec{J} = \int_{t_1}^{t_2} \vec{F}dt = \vec{F}_{av}\left(t_2 - t_1\right)$, $\Delta\vec{J} = \vec{J}_2 - \vec{J}_1 = \Delta\vec{P} = \vec{P}_2 - \vec{P}_1$

Newton's Second Law in Terms of Momentum

$\vec{F}^{ext} = \frac{d\vec{p}}{dt}$. For $\vec{F}^{ext} = 0$, $\frac{d\vec{p}}{dt} = 0$ and **momentum** is **conserved**.

Rotational Kinematics Equations $s = r\theta$, $\omega_z = \frac{d\theta}{dt}$, $\alpha_z = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$, $v = r\omega_z$, $a_{\tan} = r\alpha_z$, $a_{rad} = \frac{v^2}{r}$.

$\omega_{av-z} = \frac{\theta_2 - \theta_1}{t_2 - t_1}$, $\alpha_{av-z} = \frac{\omega_{2z} - \omega_{1z}}{t_2 - t_1}$

For $\alpha_z =$ constant, $\omega_z = \omega_{0z} + \alpha_z t$, $\theta = \theta_0 + \omega_{0z}t + \frac{1}{2}\alpha_z t^2$, $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z\left(\theta - \theta_0\right)$

Moment of Inertia and Rotational Kinetic Energy $I = \sum_{i=1}^{N} m_i r_i^2$, $K_{rot} = \frac{1}{2}I\omega^2$

Torque and Newton's Laws for Rotating Body $\tau = F\ell$, $\vec{\tau} = \vec{r} \times \vec{F}$, $\sum_{i=1}^{N} \tau_{iz} = I\alpha_z$, ℓ-moment arm

Combined Rotation and Translation of a Rigid Body $K = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$, $\sum_{i=1}^{N} \vec{F}_i^{ext} = M\vec{a}_{cm}$,

$\sum_{i=1}^{N} \tau_{iz} = I_{cm}\alpha_z$. **Rolling without slipping** $s = R\theta, v_{cm} = R\omega_z, a_{cm} = R\alpha_z$.

Equilibrium conditions $\sum \vec{F} = 0$ **about all object,** $\sum \vec{\tau} = 0$ **about any point.**

Angular Momentum $L = I\omega$ where I is the moment of inertia about the axis of rotation.