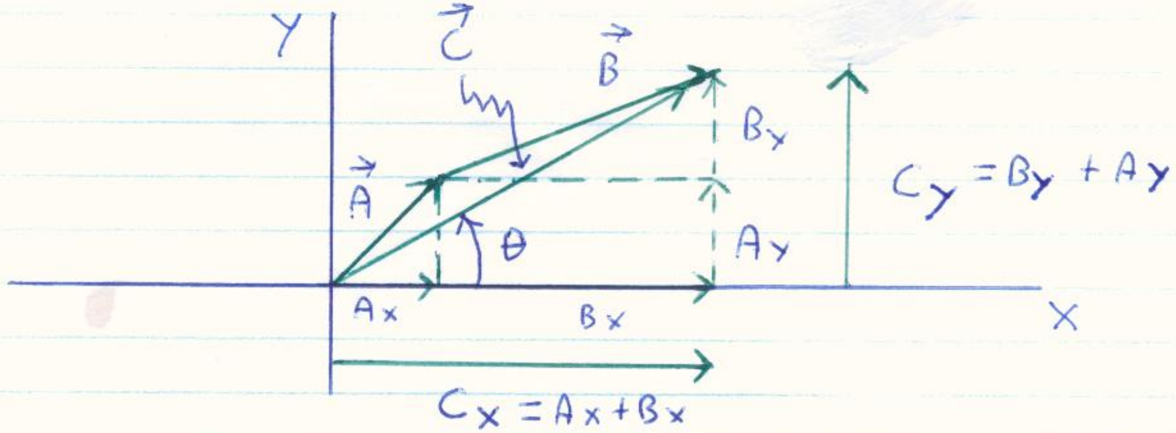
ADDING TWO VECTORS BY THE COMPONENT METHOD

- ADD TWO VECTORS SIMPLY BY ADDING THEIR COMPONENTS

$\boxed{\text{EXAMPLE}}$

FIND $\vec{C} = \vec{A} + \vec{B}$



MAGNITUDE OF $\vec{C}$ IS $|\vec{C}| = C = \sqrt{C_x^2 + C_y^2}$

$$\theta = \arccos\left(\frac{C_x}{C}\right)$$

$$\theta = \arcsin\left(\frac{C_y}{C}\right)$$

$$\theta = \arctan\left(\frac{C_y}{C_x}\right)$$

BEWARE: In calculating θ of a vetor from its components, the value of θ must be within the ALLOWED RANGE of the QUADRANT.

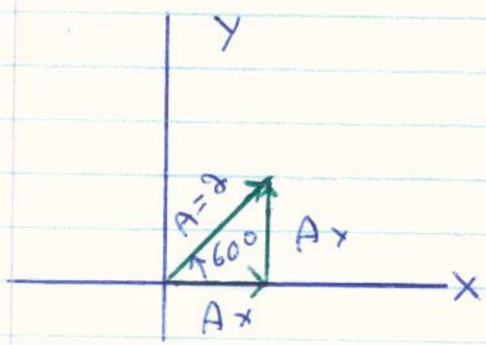
EXAMPLE

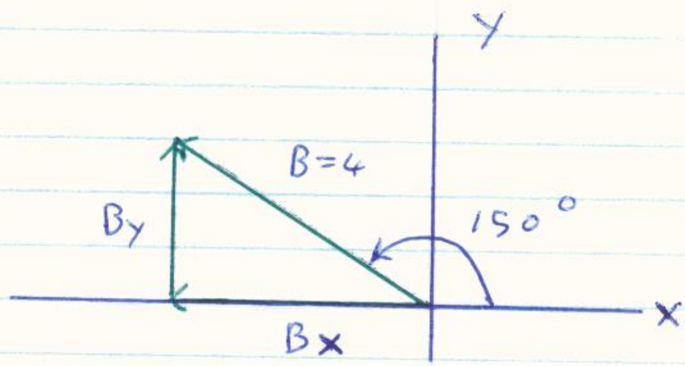
Consider two vectors: $\vec{A}$ ($A = 2.0$, $\theta = 60°$) and $\vec{B}$ ($B = 4.0$, $\theta = 150°$).
FIND $\vec{C} = \vec{A} + \vec{B}$

IDENTIFY & SETUP

- FIND COMPONENTS OF $\vec{A}$, $\vec{B}$.
- ADD components to FIND COMPONENTS OF $\vec{C}$.
- FIND C & θ

EXECUTE

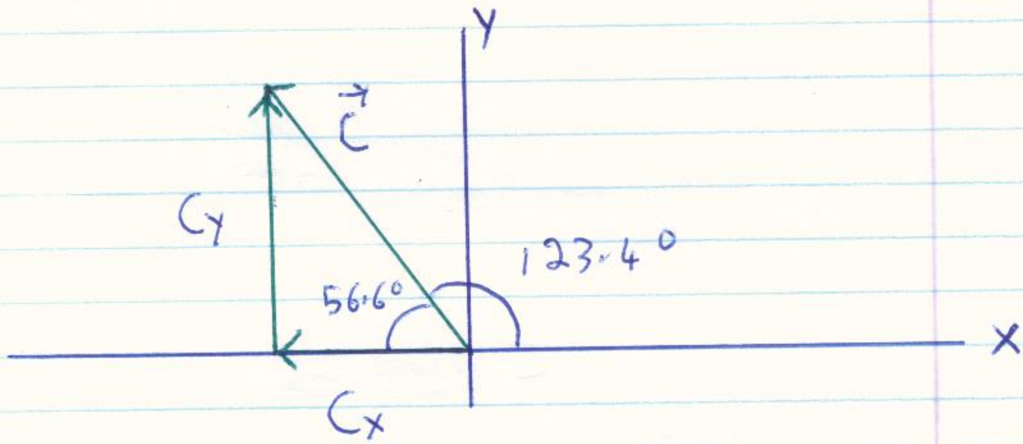




$A_x = A\cos 60° = 1.0$
$A_y = A\sin 60° = 1.732$

$B_x = B\cos 150° = -3.464$
$B_y = B\sin 150° = 2.0$

$$C_x = A_x + B_x = 1.0 - 3.464 = -2.464$$
$$C_y = A_y + B_y = 1.732 + 2.0 = 3.732$$



$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(-2.464)^2 + (3.732)^2}$$
$$= 4.472$$

FINDING θ

$$\theta = \arccos\left(\frac{C_x}{C}\right) = \arccos\left(\frac{-2.464}{4.472}\right)$$
$$= \cancel{+}\,123.4^\circ$$

$\theta = 123.4^\circ$ consistent, since $\vec{C}$ is in 2^{nd} QUDRANT ! $90^\circ < \theta < 180^\circ$.

FINAL ANSWER $\quad C = 4.5,\ \theta = 123.4^\circ$

Note

i) FINAL ANSWER has 2 S.F.

(ii) For accuracy more than 2 S.F. are USED DURING CALCULATIONS

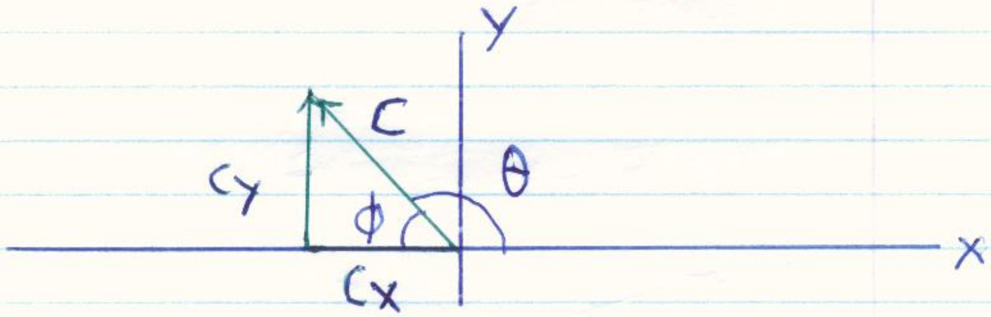
(iii) Don't worry about S.F of θ.

BEST METHOD TO DETERMINE θ

WE KNOW $C_x = -2.464 < 0$

$C_y = 3.732 > 0$

HENCE $\vec{C}$ IS IN 2nd QUADRANT



FIND BY $\phi = \arccos\left(\frac{|C_x|}{C}\right)$

ϕ - is MEASured from $-x$ axis IN the clockwise direction

$|C_x|$ - ABSOLUTE VALUE OF C_x

$$\phi = \arccos\left(\frac{2.464}{4.472}\right) = 56.6^\circ$$

$$\theta = 180^\circ - \phi = 180^\circ - 56.6^\circ = 123.4^\circ$$

or

$\theta = 123.4^\circ$ MEASured counterclockwise with respect to the $+x$ AXIS.

ADDING 3 OR MORE VECTORS

SUM the X & Y COMPONENTS

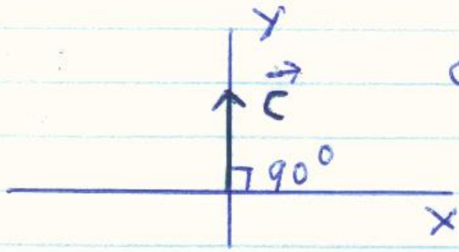
$\boxed{\text{EXAMPLE}}$

$\vec{A}\,(A = 2.0,\ \theta = 60^\circ)$
$\vec{B}\,(B = 4.0,\ \theta = 150^\circ)$
$\vec{C}\,(C = 2.366,\ \theta = 90^\circ)$

FIND $\vec{D} = \vec{A} + \vec{B} - 2\vec{C}$

X-Y COMPONENTS of $\vec{A}$ & $\vec{B}$ FOUND EARLIER

$A_x = 1.0\ ,\quad A_y = 1.732$
$B_x = -3.464\ ,\quad B_y = 2.0$



$C_x = 0\ ,\quad C_y = C = 2.366$

$$D_x = A_x + B_x - 2C_x$$
$$= 1.0 - 3.464 - 0$$
$$= -2.464$$

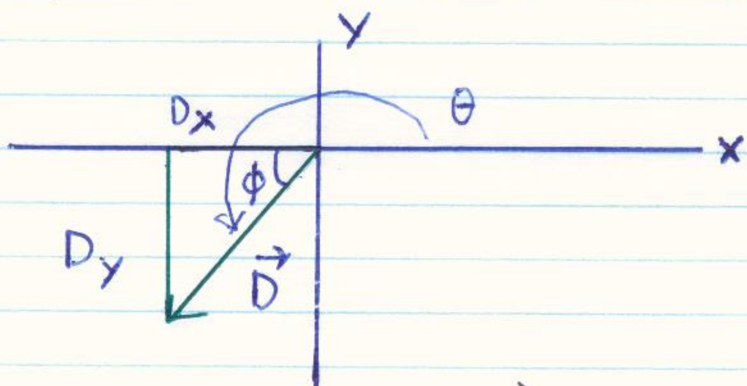
$$D_y = A_y + B_y - 2C_x$$
$$= 1.732 + 2.0 - 2(2.366)$$
$$= -1.0$$

$D_x = -2.464 \qquad D_y = -1.0$

$D = \sqrt{D_x^2 + D_y^2} = \sqrt{(-2.464)^2 + (-1.0)^2}$

$D = 2.659$

$D_x, D_y < 0$ SO $\vec{D}$ IS IN 3^{rd} QUADRANT



FIND $\phi = \arccos\left(\frac{|\vec{D}_x|}{D}\right)$

$\phi = \arccos\left(\frac{2.464}{2.659}\right) = 22.08^\circ$

$\boxed{\text{ANSWER}}$ $\vec{D}$: $D = 2.7$, $\phi = 22.08^\circ$ as DRAWN IN DIAGRAM

ALTERNATIVELY

$\theta = \phi + 180^\circ = 202.1^\circ$

$\vec{D}$: $D = 2.7$, 202.1° counterclockwise with respect to $+x$ axis