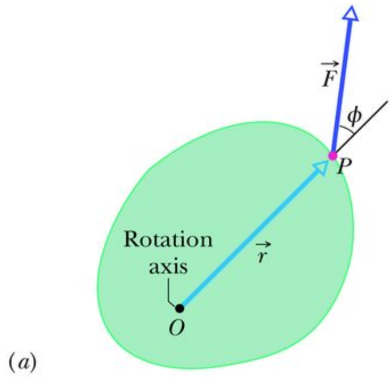
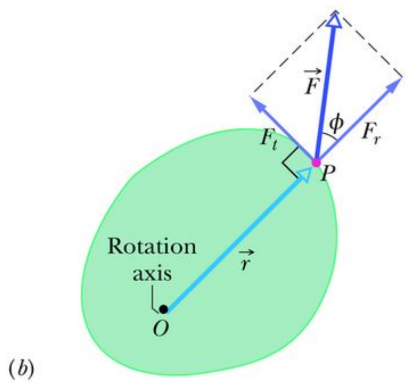
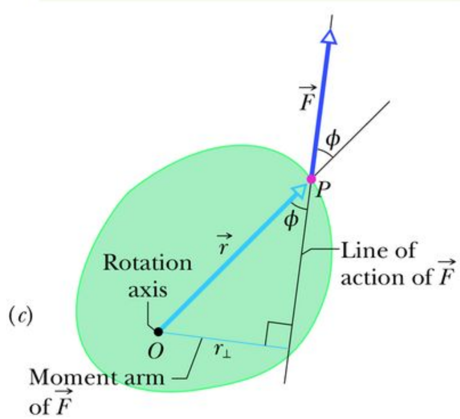


The torque due to this force causes rotation around this axis (which extends out toward you).



But actually only the *tangential* component of the force causes the rotation.



You calculate the same torque by using this moment arm distance and the full force magnitude.

Torque ($\vec{\tau}$), Force ($\vec{F}$), Moment Arm ($r_\perp$)

For this course we will deal with rotation about a single axis of rotation, aka 1D rotation. In figure (a) the **rotation axis** is **perpendicular** (⊥) to the **green object**, and passes through the origin O. If we imagine that the **green object** is attached at O, but is allowed to rotate about O.

The effect of force, $\vec{F}$, is to rotate about O

- **Qualitatively** the force $\vec{F}$ will rotate the **green object** about O counter-clockwise (ccw).
- More generally a rotation about an axis of rotation can be counterclockwise (ccw) or clockwise (cw).
- In general we will assume **ccw** to be **positive** (+), and **cw** to be **negative** ($-$).

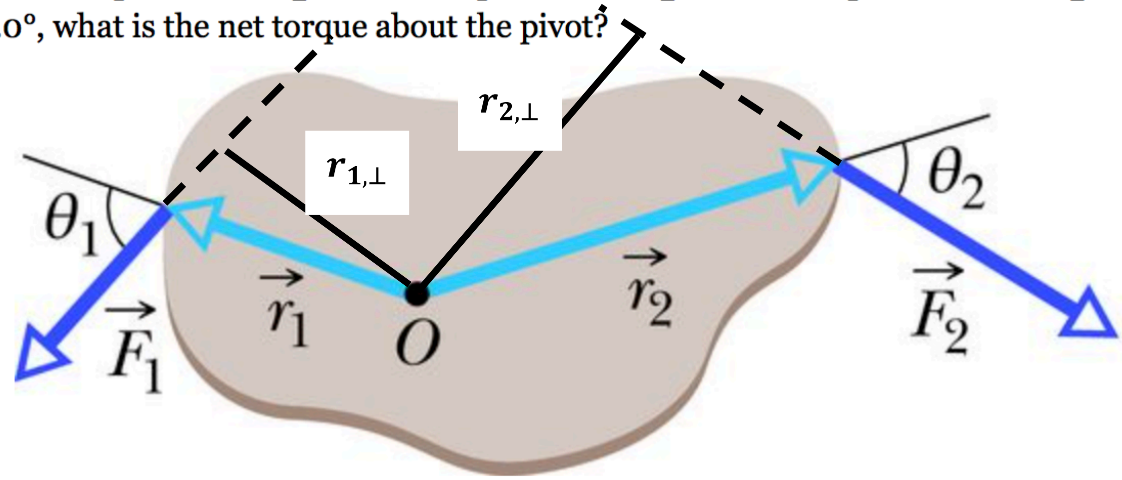
Torque ($\vec{\tau}$): Quantifying the Rotational effect of a force, $\vec{F}$, about rotational axis through origin (O)

- **Quantitatively** the force $\vec{F}$ will induce a torque ($\vec{\tau}$) on the **green object** counter-clockwise (ccw).
- In Figure b), the **torque** ($\vec{\tau}$) **about** the **rotational axis through origin O** is

$$\tau = +(Fr_\perp)$$

- $r_\perp$ is the **moment arm**, which is the perpendicular (⊥) distance from O to the **line of action** of the **force** $\vec{F}$.
- Note that the + sign in $\tau = +(Fr_\perp)$ indicates in figures a) and b) the **torque** will induce a **counterclockwise** (ccw), which was determined **qualitatively** above.
- Later we will have examples of **torques** that induce **clockwise** (cw) or + rotation
- Note that for 1D rotation the direction of $\vec{\tau}$ is indicated by $\tau > 0$ (ccw) or $\tau < 0$ (cw).

•45 SSM ILW The body in Fig. 10-39 is pivoted at O, and two forces act on it as shown. If $r_1 = 1.30$ m, $r_2 = 2.15$ m, $F_1 = 4.20$ N, $F_2 = 4.90$ N, $\theta_1 = 75.0°$, and $\theta_2 = 60.0°$, what is the net torque about the pivot?



Solution of Problem 45 (Chapter 10)

We use the method outlined in the previous page:

Torque, τ_1, due to force $\vec{F}_1$, magnitude F_1:

- $\tau_1 = (F_1 r_{1,\perp})$, where the minus $(+)$ sign indicates that this torque will "induce" a ccw rotation. Of course, whether the rotation will be cw or ccw will also depend on the other force (F_2).
- The moment arm $r_{1,\perp}$ is found by drawing the **line of action** of $\vec{F}_1$ (shown by the black dashed line above). Using geometry (see black solid line) $r_{1,\perp} = r_1 sin\theta_1 = 1.3m \times sin75^{\circ} = 1.256m$. Make sure you understand why we used $r_1 sin\theta_1$.

$$\tau_1 = (F_1 r_{1,\perp}) = 4.20N \times 1.256m = 5.27N \cdot m$$

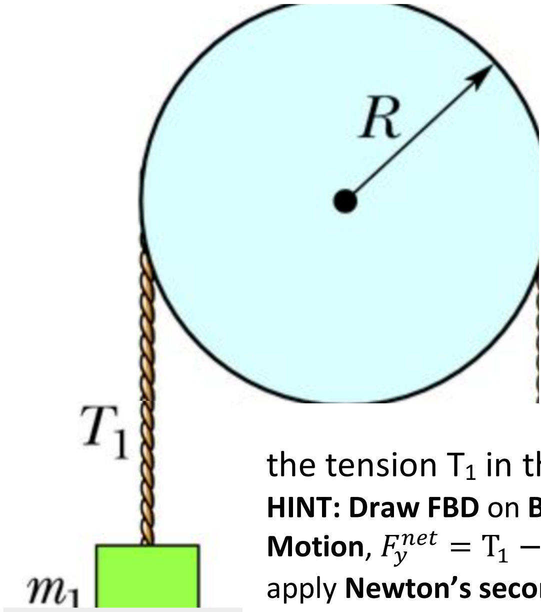
Torque, τ_2, due to force $\vec{F}_2$, magnitude F_2:

- $\tau_2 = -(F_2 r_{2,\perp})$, with $(-)$ sign indicates that this torque will "induce" a cw rotation.
- The moment arm $r_{2,\perp}$ is found by drawing the **line of action** of $\vec{F}_2$ (shown by the black dashed line above). Using geometry (see black solid line) $r_{2,\perp} = r_2 sin\theta_2 = 2.15m \times sin60^{\circ} = 1.86m$. Make sure you understand why we used $r_2 sin\theta_2$.

$$\tau_2 = -(F_2 r_{2,\perp}) = 4.90N \times 1.86m = -9.12N \cdot m$$

Net Torque, τ_{net}:

$\tau_{net} = \tau_1 + \tau_2 = 5.27N \cdot m - 9.12N \cdot m = 3.85N \cdot m$, which means that the object will rotate cw since $\tau_{net} < 0$.



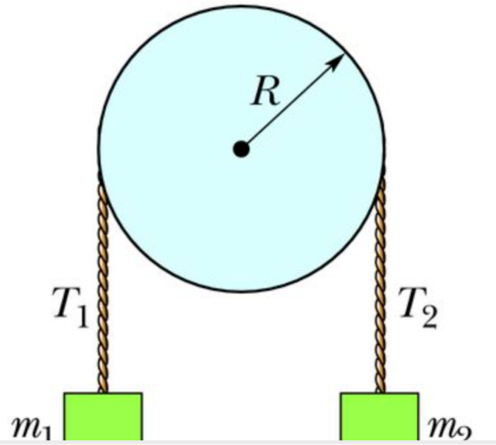
Example 2

In figure to the left, box 1 of mass m_1 = 5 kg, and the cylindrical pulley has mass M_p = 3 kg, and radius R = 0.3 m. Asuume that the disk is a solid cylinder with moment of inertia $I = \frac{1}{2} M_p R^2$The system is released from rest, and as box 1 descend the rope rotate the cylinder without slipping. Calculate the linear acceleration of the box, the angular acceleration of the pulley, and the tension T_1 in the rope.

HINT: Draw FBD on **Box 1** and apply **Newton's second law** for **linear Motion**, $F_y^{net} = \mathrm{T_1} - \mathrm{m_1 g} = -m_1 a$**. Draw FBD** on **Cylindrical pulley** and apply **Newton's second law** for **Rotational Motion**: $\tau_{net} = \mathrm{T_1 R} = \mathrm{I\alpha}$. Use the relation $a = R\alpha$. **Assume** that the **box will accelerate down.**

ANSWER: $a = 7.53 m \cdot s^{-2}$; $\alpha = 25.1 rad \cdot s^{-2}$; $T_1 = 11.35 N$.

••51 In Fig. 10-41, block 1 has mass $m_1 = 460$ g, block 2 has mass $m_2 = 500$ g, and the pulley, which is mounted on a horizontal axle with negligible friction, has radius $R = 5.00$ cm. When released from rest, block 2 falls 75.0 cm in 5.00 s without the cord slipping on the pulley. (a) What is the magnitude of the acceleration of the blocks? What are (b) tension T_2? and (c) tension T_1? (d) What is the magnitude of the pulley's angular acceleration? (e) What is its rotational inertia?



<u>Solution of Problem 51, Chapter 10</u>

Partly done in class

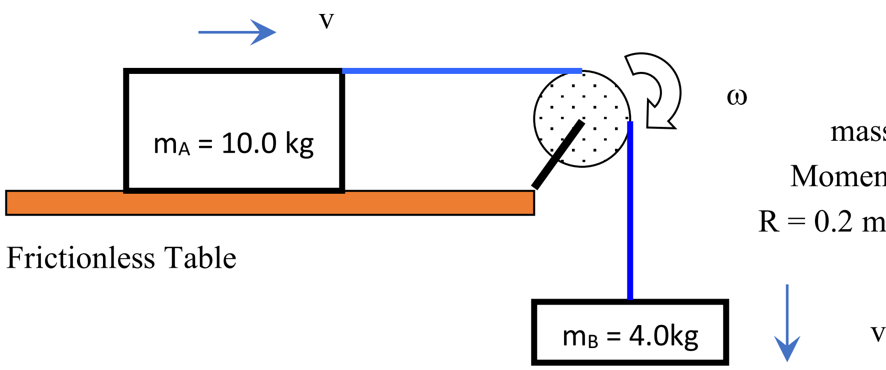
ANSWER BELOW:

ANSWER:

51. (a) 6.00 cm/s^2; (b) 4.87 N; (c) 4.54 N; (d) 1.20 rad/s^2; (e) 0.0138 kg$\cdot$ m^2

Question 4

In figure below, box A and B are connected by a rope-cylindrical pulley system. Box A is released from **rest**. When Box A is released, box B begin to fall, and the **friction** between the rope and pulley **rotates** the **cylindrical pulley**, **without slipping**. The pulley rotates **clockwise** with an angular velocity, ω. The data are shown in the figure. The ideal (no mass) rope is in **blue**.



mass of pulley is $M_P = 3.0$ kg

Moment of inertia $I = \frac{1}{2} M_p R^2$

R = 0.2 m is the radius of cylinder

Draw a FBD on box A and apply Newton's second law for linear motion; Draw a FBD on box B and apply Newton's second law for linear motion; Draw a FBD on cylinder and apply Newton's second law for rotational motion. Solve the equations to find the acceleration of the boxes, the angular acceleration of the cylinder. Find the two tensions in the rope.

ANSWER BELOW

ANSWER: $a = 2.52 m \cdot s^{-2}\ (down);\ \alpha = 12.6 \frac{rad}{s^2}\ (cw);\ T_A = 29.12N, T_B = 25.2N.$