INSTRUCTION
1. This exam booklet has eleven pages. Make sure none are missing
2. An equation sheet will be handed out prior to the beginning of the exam
3. There are two parts to the exam:
   • Part I has twelve multiple choice questions (1 to 12), where you must circle the one correct answer (a,b,c,d,e). Rough work can be done on the backside of the sheet opposite the question page
   • Part II includes nine full-answer questions (13 to 21). Do eight of nine questions. If you do all nine, I will take the best eight in calculating the exam mark. All works must be done on the blank space below the questions. If you run out of space, you may write on the backside of the sheet opposite the question page.
4. Calculators are allowed
PART I: MULTIPLE CHOICE QUESTIONS (question 1 to 12)
For each question circle the one correct answer (a,b,c,d or e).

1. (2.5 point) A cat runs in a straight line (the x-axis) from point A to point B to point C, as shown below. The distance between point A and C is 5.00m, between point B and C is 10.0m, and the positive direction of the x-axis points to the right. The time to run from A to B is 20.0s and from B to C is 8.00s. The average speed for the whole trip is closest to
a) -0.179 m/s   b) 0.536 m/s    c) 0.179 m/s    d) 0.893 m/s    e) -0.893 m/s

Definition of average speed \( v_{av} = \frac{\text{total distance}}{\text{total time}} = \frac{AB + BC}{t_{AB} + t_{BC}} = \frac{15.0m + 10.0m}{20.0s + 8.0s} = 0.893 m/s \).

Answer d) Note that average speed must be positive!!

2. (2.5 point) A ball is projected upward at time \( t = 0.0 \) s, from a point on a roof 90 m above the ground. The ball rises, then falls and strikes the ground. The initial velocity of the ball is 82.4 m/s. Consider quantities as positive in the upward direction. The velocity of the ball when it is 73 m above the ground is closest to:

a) -101 m/s   b) -68 m/s   c) -34 m/s    d) -51 m/s   e) -84 m/s

Use \( v_y^2 = v_{oy}^2 - 2g(y-y_o) \) \( v_y = \sqrt{v_{oy}^2 - 2g(y-y_o)} \), with \( y_o = 0 \), \( v_{oy} = 82.4 \) m/s and \( y = -17 \) m. Note \( y \) is the final position above the ground, so if we take \( y_o = 0 \) to be the initial position (on the roof) then the position of the ground is \( -90 \) m, it follows that 73 m above the ground is \( y = -17 \) m. Hence \( v_y = -\sqrt{82.4 \frac{m}{s}} - 2 \left( 9.8 \frac{m}{s^2} \right) (-17m - 0) = -84.4 \frac{m}{s} \)

ANSWER: e, the answer is negative since at that point the ball is falling.

3. (2.5 point) A golf ball is hit so that it leaves the ground at 60° above the horizontal and feels no air resistance as it travels. Which of the following statement about the subsequent motion of the ball while it is in the air is true? Only one correct answer!

A) Its speed is zero at its highest point.    B) Its velocity is zero at its highest point.
C) Its acceleration is always 9.8 m/s² downward.    D) Its forward acceleration is 9.8 m/s².
E) Its acceleration is zero at its highest point.

Answer c)

4. (2.5 point) A 50.0-N box is sliding on a rough horizontal floor, and the only horizontal force acting on it is friction. You observe that at one instant the box is sliding to the right at 1.75 m/s and that it stops in 2.25 s with uniform acceleration. The force that friction exerts on this box is closest to:
a) 3.97 N   b) 490 N   c) 50.0 N   d) 8.93 N   e) 38.9N

\[ f_k \ 
\begin{array}{c}
 m \\
 \hline
 50N + 9.8 m/s^2 = 5 \\
 v_{0x} = 1.75 m/s \\
 a_x \end{array} \]

Final velocity (x) \( v_x = 0 \)
\[ v_x = v_{0x} - a_x t, v_x = 0 \]
\[ a_x = \frac{v_{0x}}{t} = \frac{1.75m/s}{2.25s} = 0.778 m/s \]
2nd law taking left as positive
\[ F_{x}^{net} = f_k = ma_x = (5.1kg)(.778 m/s^2) = 3.97N \]

answer a)
5. (2.5 point) A 0.54-kg block is held in place against the spring by a 65-N horizontal external force. The external force is removed, and the block is projected with a velocity $v_1 = 1.2\, \text{m/s}$ upon separation from the spring. The block descends a ramp and has a velocity $v_2 = 1.4\, \text{m/s}$ at the bottom. The track is frictionless between points A and B. The block enters a rough section at B, extending to E. The coefficient of kinetic friction is 0.36. The velocity of the block is $v_3 = 1.4\, \text{m/s}$ at C. The block moves on to D, where it stops. In the figure below the initial compression of the spring, in cm, is closest to:

\begin{itemize}
  \item a) 0.64
  \item b) 0.43
  \item c) 1.6
  \item d) 1.2
  \item e) 2.4
\end{itemize}

**HINT:** Using Hooke’s law the force that holds the spring obeys the relation $kx = 65N$!

To answer the question we only need to consider all data in the vicinity where the box is in contact with the spring. Using the fact that the box start from rest and has a “final” speed of $v_1 = 1.2\, \text{m/s}$, and using conservation of energy (no friction) we obtain

$\frac{1}{2}mv_1^2 = \frac{1}{2}kx^2$, where $x$ is the maximum (initial) compression of the spring. Rearranging we obtain $x = \frac{mv_1^2}{k}$. Also we can use the hint $kx = 65N$,

\[ x = \frac{0.54\, \text{kg} \times (1.2\, \text{m/s})^2}{65\, \text{N}} = 0.01196m = 1.2\, \text{cm} \quad \text{ANSWER d} \]

6. (2.5 point) A block is placed on a rough wooden plane. It is found that when the plane is tilted 30° to the horizontal, the block will slide down at constant speed. The coefficient of kinetic friction of the block with the plane is closest to:

\begin{itemize}
  \item a) 0.5
  \item b) 0.577
  \item c) 1.73
  \item d) 0.866
  \item e) 4.90
\end{itemize}

Refer to Figure 5-15 on page 102 to 103 of textbook. It is easy to see that the x-component of the gravitational force is $mg\sin\theta$ **down the incline**. The normal force is $F_N = mg\cos\theta$, giving a friction force of $f = \mu_k mg\cos\theta$ directed up the incline, opposing the motion of the block down the incline. Since the block moves down at constant velocity, it is in equilibrium, so the two forces must be of **equal magnitude**:

$mg\sin\theta = \mu_k mg\cos\theta \rightarrow \mu_k = \tan\theta = \tan 30° = 0.577$ **ANSWER:** b

7. (2.5 points) The force on a particle is conservative if and only if:

\begin{itemize}
  \item a) the particle moves exactly once around any closed path.
  \item b) It is not a frictional force.
  \item c) the work done on the particle is independent of the path taken by the particle
  \item d) it obeys Newton’s second law
  \item e) it obeys Newton’s third law
\end{itemize}

**ANSWER:** c

8. (2.5 points) A 1.0 kg-ball moving toward and perpendicular to a wall at 2.0 m/s, rebounds from the wall at 1.5 m/s. The change in the momentum of the ball is closest to:
a) zero  b) 0.5kg·m/s away from wall  c) 0.5kg·m/s toward wall  

d) 3.5kg·m/s away from wall  e) 3.5kg·m/s toward wall  

Take toward wall as positive. **Initial momentum:** \( p_i = mv_i = 1kg \times 2m/s = 2kg·m/s \);  
**Final momentum:** \( p_f = mv_f = 1kg \times (-1.5m/s) = -1.5kg·m/s \) (note the negative sign). 

**Change:** \( \Delta p = p_f - p_i = -3.5kg·m/s \), negative sign means it is away from wall  

**ANSWER:** d

9. **(2.5 points) POWER** A 50-N force (the only force!) acts on a 2-kg crate that starts from rest. After 2 s the instantaneous power due to the force (rate of work by force) is:  

<table>
<thead>
<tr>
<th>Option</th>
<th>Power (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 75 W</td>
<td>b) 100 W</td>
</tr>
<tr>
<td>c) 1000 W</td>
<td>d) 2500 W</td>
</tr>
<tr>
<td>e) 5000 W</td>
<td></td>
</tr>
</tbody>
</table>

Since the 50-N force is the only force on the 2-kg crate, its acceleration is \( a = \frac{50N}{2kg} = 25m/s^2 \). If it starts from rest after two seconds it velocity will be \( v = at = 25m/s \times 2s = 50m/s \). The instantaneous power is \( P = Fv = 50N \times 50m/s = 2500W \) **ANSWER** d

10. **(2.5 points)** A machinist turns the power on to a grinding wheel, at rest, at time \( t = 0 \) s. The wheel accelerates uniformly for 10 s and reaches the operating angular velocity of 42 rad/s. The wheel is run at that angular velocity for 39 s and then power is shut off. The wheel decelerates uniformly at 2.6 rad/s² until the wheel stops. In this situation, the angular acceleration of the wheel between \( t = 0 \) s and \( t = 10 \) s is closest to:  

<table>
<thead>
<tr>
<th>Option</th>
<th>Angular Acceleration (rad/s²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 5.9 rad/s²</td>
<td>b) 7.6 rad/s²</td>
</tr>
<tr>
<td>c) 4.2 rad/s²</td>
<td>d) 5.0 rad/s²</td>
</tr>
<tr>
<td>e) 6.7 rad/s²</td>
<td></td>
</tr>
</tbody>
</table>

Use \( \alpha = \omega_0 + \alpha t \to \alpha = \frac{\omega - \omega_0}{t} \) with \( \omega_0 = 0 \) and \( \omega = 42 \frac{rad}{s} \), which gives \( \alpha = \frac{42rad/s}{10s} = 4.2 \frac{rad}{s²} \)  

**Answer:** c

11. **(2.5 points)** A solid uniform sphere \((I = (2/5)MR^2)\) of mass 1.85 kg and radius 0.225 m spins about an axle through its center. Starting with an angular velocity of \( \omega_0 = 15.1 \text{ rad/s} \), it stops after turning through \( \theta = 114.4 \text{ rad} \), due to a uniform angular acceleration \( \alpha \). The magnitude of the net torque acting on this sphere as it is slowing down is closest to:  

<table>
<thead>
<tr>
<th>Option</th>
<th>Torque (N·m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 0.0620 N·m</td>
<td>b) 0.0466 N·m</td>
</tr>
<tr>
<td>c) 0.149 N·m</td>
<td>d) 0.0372 N·m</td>
</tr>
<tr>
<td>e) 0.00593 N·m</td>
<td></td>
</tr>
</tbody>
</table>

Start with \( \omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0) \to \alpha = \frac{\omega^2 - \omega_0^2}{2(\theta - \theta_0)} \) with \( \omega_0 = 15.1 \text{ rad/s}, \omega = 0, \) and \( \theta - \theta_0 = 114.4 \text{ rad} \), which gives \( \alpha = \frac{0 - (15.1 \text{ rad/s})^2}{2(114.4 \text{ rad})} = -0.997 \text{ rad/s}^2 \).  

Using Newton’s second law \( \tau = I\alpha \) with \( I = \frac{2}{5}MR^2 \) gives \( \tau = \frac{2}{5}MR^2\alpha = \frac{2}{5}(1.85kg)(0.225m)^2(-0.997 \text{ rad/s}^2) = -0.0372N·m \)  

**Answer:** d, the implicit assumption here is that torque is simply the magnitude of the torque.

12. **(2.5 points)** In an interesting lecture demonstration a student sits in a swivel chair holding a spinning bicycle wheel oriented with its axis vertical. Initially the chair is motionless. Now the student rotates the axis of the spinning wheel by 180°, so that it is again vertical, but now the wheel is spinning in the opposite sense with respect to the
room. When this is done the student and chair begin to rotate also. Which of the following is an accurate statement concerning this process?

a) Assuming the swivel bearing of the chair is frictionless, rotational kinetic energy is conserved.
b) In the final state the chair will be rotating in the opposite sense of the wheel.
c) Total kinetic energy (linear plus rotational) is conserved in this process, independent of whether or not friction is present.
d) Angular velocity is conserved

e) In the final state the chair will be rotating in the same sense of the wheel.

Answer: b) 2.5 point        e) 1.5 point

**PART II: FULL ANSWER QUESTIONS (question 13 to 21)**

Do all **eight of nine** questions on the provided exam booklets. Show all works.

13. *(10 points)* A ball is thrown off a 9.8 m high cliff with an initial speed of $v_0 = 9.8 \text{ m/s}$ at $30^\circ$, as shown in the diagram below.

![Diagram of a ball thrown off a cliff](image)

a) Find the x- and y- components of the initial velocities.

$v_{ox} = v_0 \cos 30^\circ = (9.8 \text{ m/s}) \cos 30^\circ = 8.49 \text{ m/s}$ *(0.5 point)*

$v_{oy} = v_0 \sin 30^\circ = (9.8 \text{ m/s}) \sin 30^\circ = 4.9 \text{ m/s}$ *(0.5 point)*

b) Find the **maximum height** of the ball, $y_{max}$, from the ground.

As shown in the diagram, the maximum height the y-component of the velocity is zero, $v_y = 0$.

Using $v_y = v_{oy} - gt \rightarrow 0 = v_{oy} - gt \rightarrow t = \frac{v_{oy}}{g} = \frac{4.9 \text{ m/s}}{9.8 \text{ m/s}^2} = 0.5 \text{ s}$ *(2 point)*

Hence $y = v_{oy}t - \frac{1}{2}gt^2 = (4.9 \text{ m/s})(0.5 \text{ s}) - \frac{1}{2}(9.8 \text{ m/s}^2)(0.5 \text{ s})^2 = 1.23 \text{ m}$. *(1.5 point)*

And $y_{max} = y + 9.8 \text{ m} = 11.03 \text{ m} = 11 \text{ m}$ *(0.5 point)*

**Alternatively** use $v_y^2 = v_{oy}^2 - 2gy \rightarrow 0 = v_{oy}^2 - 2gy$ *(2 point)*

$y = \frac{v_{oy}^2}{2g} = \frac{(4.9 \text{ m/s})^2}{2(9.8 \text{ m/s}^2)} = 1.23 \text{ m}$. *(1.5 point)*
And \( y_{\text{max}} = y + 9.8 \text{ m} = 11.03 \text{ m} = 11 \text{ m} \) (0.5 point)

c) Find the time it takes for the ball to hit the ground. **ANSWER:** \( t = 2.0 \text{ s} \), but you must calculate this. **Note** the solution of \( ax^2 + by + c = 0 \rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \)

Using \( y = y_0 + v_{0y}t - \frac{1}{2}gt^2 \) (0.5 point), and setting \( y_0 = 0 \), we obtain

\[-9.8m = (4.9m/s^2)t - \frac{1}{2}(9.8m/s^2)t^2 \rightarrow (4.9m/s^2)t^2 - (4.9m/s^2)t - 9.8m = 0 \)(2 point)

To solve this quadratic equation we rewrite it as

\[t^2 - t - 2.0 \text{ s} = 0 \rightarrow (t - 2.0 \text{ s})(t + 1.0 \text{ s}) = 0 \text{ (1 point)}, \text{ giving the solution } t = 2.0 \text{ s} \text{ and } 1.0 \text{ s}. \text{ Only } t = 2.0 \text{ s} \text{ is the physical solution.} \text{ (0.5 point)}

d) Find the range (see diagram) of the trajectory.

The range is \( \text{range} = v_{0x}t = (8.49m/s)(2.0s) = 17.0m \) \text{ (1 point)}

14. (10 points). A 10.0 kg object, at rest on a surface, is acted on by external forces \( (F_1 = 30.0N \text{ and } F_2 = 20.0N) \), as shown below. The coefficient of kinetic friction between the surfaces is \( \mu_k = 0.2 \) and the coefficient of static friction is \( \mu_s = 0.3 \).

![Diagram](image)

a) Draw a free-body diagram of all the forces acting on the object.

b) Find the vertical and horizontal components of the net force acting on the object. Show that the box will accelerate.

Vertical \( n + F_{1x} + F_{2x} - mg = 0 \rightarrow n = mg - F_{1x} - F_{2x} \) \text{ (1 point)}

\( n = (10.0kg)(9.8m/s^2) - (30N \sin 31^\circ) - (-20.0N \sin 31^\circ) = 84N \) \text{ (1 point)}

Static Friction \( f_s \leq \mu_s n = (0.3)(84N) = 25.2N \) \text{ (0.5 point)}

Kinetic Friction \( f_k = \mu_k n = (0.2)(84N) = 16.8N \) \text{ (0.5 point)}
\[ F_{1x} + F_{2x} = (30.0N) \cos 53.1^\circ + (20.0N) \cos 30^\circ = 35.3N > 25.2N = \text{max.} f_x, \]

so the crate will move. \( \text{(1 point)} \)

**Horizontal** \[ F_{\text{net},x} = F_{1x} + F_{2x} - f_k = (30.0N) \cos 53.1^\circ + (20.0N) \cos 30^\circ - 16.8 = 18.5N \]

\( \text{(2 points)} \)

\[ \text{c) Find the acceleration of the object.} \]

Using Newton’s second law

\[ F_{\text{net},x} = ma \rightarrow a_x = \frac{F_{\text{net},x}}{m} = \frac{18.5N}{10.0kg} = 1.85 \text{m/s}^2. \] \( \text{(2 points)} \)

The acceleration is to the right.

15. \( \text{(10 points)} \)

In the **figure** to the left, block A (mass 1.2 kg) is connected to a wall by a rope. Block A lies on block B (mass 3.6 kg). For all surfaces, the coefficient of kinetic friction is 0.3. A force \( \vec{F} \) pulls block B as shown. All objects moves at constant velocity, which could be zero.

\( \text{a) Draw free-body diagrams that includes all force on block A. Hence find tension in the rope. **Hint:** Find friction forces between surface A and B.} \)

System is at equilibrium so all components of the net force must be zero.

\( \text{(2 points)} \)

\[ +y \]

\[ \begin{array}{c}
F_N \\
F_{\text{net},} \\
\text{Friction} \\
f_{AB} \\
F_g = m_A g
\end{array} \]

\[ \text{+x} \]

\[ \begin{array}{c}
F_{\text{net},x} \\
T
\end{array} \]

**Friction** \( f_{AB} \) on A due to B. Since box B is moving right, it will try to pull A with it! This is a kinetic friction

\[ T = F_N \mu_k = 11.8N \times 0.3 = 3.54N \] \( \text{(1 point)} \)

\[ x \text{-com } F_{\text{net},x} = T - f_{AB} = 0 \rightarrow T = f_{AB} = 3.54N \] \( \text{(1 point)} \)

b) Draw a free body diagram of all the forces acting on block B. Hence find the magnitude of the force, \( F \), acting on block B. **NOTE:** There are more than one friction forces. Newton’s third law is useful.

\( \text{(2 points)} \)

\[ +y \]

\[ \begin{array}{c}
F_N \\
f_{BA} \\
F_s \\
F_{\text{net},y} = F_N - m_A g = 0 \rightarrow F_N = m_A g = 11.8N \] \( \text{(1 point)} \)

\[ +x \]

\[ \begin{array}{c}
F_{\text{net},x} = F_s - m_B g = 0 \\
F = (m_A + m_B) g = 47N \] \( \text{(1 point)} \)

Friction Due to ground surface \( f_s = F_N \mu_k = 47N(0.3) = 14.1N \)

Due to B on A, use Newton third law \( f_{BA} = f_{AB} = 3.54N \) \( \text{(1 point)} \)

\[ x \text{-com } F_{\text{net},x} = f_s + f_{BA} - F = 0 \rightarrow F = 14.1N + 3.54N = 17.6N \] \( \text{(1 point)} \)

16. \( \text{(10 points)} \)

A hockey puck B at rest on a frictionless ice surface is struck by a second puck A, which was initially traveling at 15 m/s as shown in the diagram. Both pucks are
wrapped in Velcro and stick together after the collision and move off as shown below. Puck A has a mass of \( m_A = 2.5 \text{ kg} \), and puck B has a mass of \( m_B = 1.5 \text{ kg} \).

a) Use conservation of momentum to find speed of A+B after the collision.

Use conservation of momentum:

Initial \( m_A v_A = (m_A + m_B) v_f \) Final \( (2 \text{ points}) \)

Of course \( v_A = 15 \text{ m/s} \) and \( v_f \) is the unknown speed of A + B after the collision.

\[
v_f = \frac{m_A v_A^0}{m_A + m_B} = \frac{(2.5 \text{ kg})(15 \text{ m/s})}{2.5 \text{ kg} + 1.5 \text{ kg}} = 9.37 \text{ m/s} \quad (2 \text{ points})
\]

b) Calculate the change of kinetic energy due to the collision. Is the collision elastic? Why?

Initial \( K_i = \frac{1}{2} m_A v_A^2 = \frac{1}{2}(2.5 \text{ kg})(15 \text{ m/s})^2 = 281 J \quad (1.5 \text{ points}) \)

Initial \( K_f = \frac{1}{2}(m_A + m_B) v_f^2 = \frac{1}{2}(4.0 \text{ kg})(9.375 \text{ m/s})^2 = 176 J \quad (1.5 \text{ points}) \)

Change in kinetic energy \( \Delta K = K_f - K_i = 176 J - 281 J = -105 J \quad (1 \text{ point}) \)

No, there is a loss in kinetic energy of 105 J. Hence kinetic energy is not conserved. \( (2 \text{ points}) \)

17. \( (10 \text{ points}) \) A 14.0 kg rock slides down a hill, leaving point A with a speed of 12 m/s. There is no friction between point A and B. There is friction on the level surface between point B and the wall (where the spring is located).

\[\text{This equation may be useful}\]
\[ax^2 + by + c = 0 \rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]

\[\text{Rough surface with friction}\]
\[\mu_k = 0.2 \text{ and } \mu_s = 0.7\]

\[\text{Spring at equilibrium:}\]
\[k = 2.00 \text{ N/m}\]

a) What is the speed of the rock when it reaches point B (bottom of the hill).

Use conservation of mechanical energy. Take gravitational potential energy to be zero at the bottom of the hill \( U_B = 0 \), then at the top of the hill \( U_A = mgh \) (h=25m). \( (1.5 \text{ points}) \)
\[ K_A + U_A = K_B + U_B \rightarrow \frac{1}{2} mv_A^2 + mgh = \frac{1}{2} mv_B^2 + 0 \quad (1.5 \text{ points}) \]

\[ v_B = \sqrt{v_A^2 + 2gh} = \sqrt{(12m/s)^2 + 2(9.8m/s^2)(25m)} = 25.2m/s \quad (1 \text{ point}) \]

b) After reaching B, the rock travels 100 m on the level surface until it hits a very long light spring (k = 2.00 N/m). How far will the rock compress (maximum compression) the spring?

Friction is present for this part of the problem.

The rock travels a distance of 100 m + x, where x is the maximum compression of the spring. The dissipative work done by friction is

\[ W_f = -mg\mu_s (100m + x) = -(14kg)(9.8m/s^2)(0.2)(100m + x) = -2744W - (27.44N)x \quad (1 \text{ point}) \]

Here use conservation of mechanical energy.

\[ K_B + U_{el,B} + W_f = K_{max} + U_{el,max} \quad (1 \text{ points}) \]

At the maximum compression \( K_{max} = 0, U_{el,max} = \frac{1}{2} kx^2 = (1.00N/m)x^2 \quad (1 \text{ point}) \)

Of course \( U_{el,B} = 0 \quad (0.5 \text{ point}) \)

This gives \[ \frac{1}{2}(14kg)(25.2m/s^2) - 2744W - (27.44N)x = (1.00N/m)x^2 \quad (1 \text{ point}) \]

This gives the quadratic equation \[ x^2 + 27.44x - 1701 = 0 \quad (0.5 \text{ point}) \]

The solution is \[ x = \frac{-27.44 \pm \sqrt{(27.44)^2 - 4(-1701)}}{2} = 29.7m, -57.2m \quad (0.5 \text{ point}) \]

The physical solution is a maximum compression of 29.7m. (0.5 point)

18. (10 points)

In the figure to the left, a stuntman of mass 80.0 kg swings on a rope from a 5.0 m high ledge towards a 70.0 kg villain standing on the ground. Assume stuntman is initially at rest. Also assume that the stuntman velocity is horizontal just before he collides with the villain.

a) What is the speed of the stuntman just before he hits the villain?

This is found by using conservation of mechanical energy: basically the stuntman’s energy at the top must equal his energy just before he hits the villain.

Take the gravitational PE to be zero at the bottom, \( U_{grav} = 0 \) and \( U_{grav} = m_sgh \) (\( m_s = 80.0kg, h = 5.0 \text{ m} \)) at the top.

Conservation of energy \[ m_sgh = \frac{1}{2} m_s v_s^2 \], where \( v_s \) is the stuntman’s speed just before he hits the villain.

\[ m_sgh = \frac{1}{2} m_s v_s^2 \rightarrow v_s = \sqrt{2gh} = \sqrt{2(9.8m/s^2)(5.0m)} = 9.9m/s \quad (4 \text{ points}) \]

b) What is the horizontal component of the velocity of the stuntman just before he hits the villain? Hint: Look carefully at the diagram.
Obviously, at the bottom just before the collision the stuntman’s velocity is horizontal, so the horizontal component of the velocity is 9.9 m/s. (2 points)

c) Just after they (stuntman + villain) collide, and become entangled, what is their speed as they slide on the floor? Assume the floor is frictionless.

Use conservation of momentum of the horizontal component before and after the stuntman + villain collision.

\[ m_\text{s}v_\text{s} = (m_\text{s} + m_\text{v})v_f, \] m_s = 70.0 kg and v_f the final speed of the entangled pair.

\[ v_f = \frac{m_\text{s}v_\text{s}}{m_\text{s} + m_\text{v}} = \frac{(80.0 \text{ kg})(9.9 \text{ m/s})}{80.0 \text{ kg} + 70.0 \text{ kg}} = 5.28 \text{ m/s} \] (4 points)

19. (10 points) A Consider the system below where the pulley is a uniform thin-walled cylinder with a radius of R = 0.220 m, a mass M = 2.0 kg, and moment of inertia of \( I = MR^2 \). A box of mass \( m_B = 3.0 \text{ kg} \) hangs from the pulley by a massless rope that does not slip over the pulley’s rim, as the box descends.

a) Draw a free body diagram of the box showing all the forces acting on it. Draw a free body diagram of the pulley showing all the forces (including those due to the hinge) acting on it.

\[ F_y = T - m_Bg \]

b) Use Newton’s 2nd law for translation and rotation to determine the linear acceleration, \( a \), of the box, and the angular acceleration of the pulley, \( \alpha \).

Refer to the figure in part a) (LHS) for translation \( F_y = T - m_Bg = -m_Ba \)

Refer to the figure in part a) (RHS) for rotation \( \tau = TR = I\alpha \) with \( I = MR^2 \) which gives \( TR = MR^2\alpha \rightarrow T = MR\alpha \). Using the condition for rotation without slipping \( a = \alpha R \) gives \( T = Ma \).

Combining the equations gives \( T - m_Bg = -m_Ba \rightarrow Ma - m_Bg = -m_Ba \), which gives

\[ a = \frac{m_Bg}{M + m_B} = \frac{3 \text{ kg} \times 9.8 \text{ m/s}^2}{2 \text{ kg} + 3 \text{ kg}} = 5.88 \text{ m/s}^2 \] and \( \alpha = \frac{a}{R} = \frac{5.88 \text{ m/s}^2}{0.220 \text{ m}} = 26.7 \text{ rad/s}^2 \) (4 points)

c) If the box starts from rest, how far does the box descend after 1.2 seconds?
\[ y = \frac{1}{2} a t^2 = \frac{1}{2} \left( \frac{5.88 \text{ m}}{s^2} \right) (1.2 \text{s})^2 = 4.23 \text{m} \] (1 point)

d) Using the diagram drawn in part a) determine the force of the hinge (holding the pulley up) on the pulley.

Referring to the diagram \( F_{\text{net}} = F_{\text{hinge}} - Mg - T = 0 \), since the net force on the pulley must be zero. Hence \( F_{\text{hinge}} = T + Mg \). From part c) \( T = Ma = 2 \text{kg} \times \frac{5.88 \text{m}}{s^2} = 11.76 \text{N} \).

Finally \( F_{\text{hinge}} = 11.76 + 2 \text{kg} \times 9.8 \text{m/s}^2 = 31.4 \text{N} \) (2 points)

20. (10 points) In figure below, a 2.0 g bullet is fired into a block (mass 0.5 kg) attached to a non-uniform rod of length 0.60 m, and moment of inertia \( I_{\text{rod}} = 0.060 \text{kg} \cdot \text{m}^2 \). The speed of the bullet just before collision is \( v_{\text{bullet}} = 500 \text{m/s} \). The system (bullet+block+rod) rotates about an axis of rotation passing through point A, perpendicular to the page.

a) Before the collision, what is the **magnitude** of the angular momentum of the bullet, \( |L_{\text{bullet}}| \), with respect to **point A**? What is the **direction** of \( L_{\text{bullet}} \)? Directions (+x, +y, +z, -x, -y, -z) are as indicated in the above figure.

b) After the collision calculate the moment of inertia of the system (bullet+block+rod) about an axis through point A, perpendicular to the page. Treat the block and bullet as point particles.

\[
I_{\text{total}} = I_{\text{rod}} + Mr^2 + m_{\text{bullet}}r^2 = 0.06 \text{kg} \cdot \text{m}^2 + 0.5 \text{kg} \left( 0.6 \text{m} \right)^2 + 2 \times 10^{-3} \text{kg} \left( 0.6 \text{m} \right)^2 = 0.241 \text{kg} \cdot \text{m}^2
\]
(2 points)
c) Using the appropriate conservation law, calculate the angular speed \( \omega_{\text{final}} \) of the bullet+block+rod about axis through point A, after the collision. What is the direction and the angular velocity, \( \dot{\omega}_{\text{final}} \)? Use same direction definition as for part a).

Use conservation of angular momentum:

\[
L_{\text{initial}} = L_{\text{final}} \rightarrow I_{\text{bullet}} \omega_{\text{initial}} = I_{\text{bullet}} \omega_{\text{final}} = \frac{L_{\text{bullet}}}{I_{\text{total}}} = \frac{0.6 \text{ kg} \cdot \text{m}^2 / \text{s}}{0.241 \text{ kg} \cdot \text{m}^2} = 2.49 \text{s}^{-1} \quad \text{(3 points)}
\]

d) Using result of part c calculate the magnitude and direction of the angular momentum of the bullet+block+rod about point A.

**Magnitude:** \( L_{\text{final}} = I_{\text{total}} \omega_{\text{final}} = 0.241 \text{ kg} \cdot \text{m}^2 \times 2.49 \text{s}^{-1} = 0.6 \text{ kg} \cdot \text{m}^2 / \text{s} \). (1 point)

**Direction:** Using the right hand rule on the original of image, we can see that after the collision the system rotates ccw, resulting in a +z direction (out of the page). (1 point)

Basically \( \dot{L}_{\text{final}} = \dot{L}_{\text{bullet}} \), as required by conservation of angular momentum.

21. (10 points) In the diagram below a solid marble rolls from rest without slipping, down a mountain of height \( h = 100 \text{ m} \).

a) Find the **linear** and **angular speed** of the marble at the bottom of the hill 100 m below top.

**Hint:** 1) Use conservation of energy; 2) It is not necessary to know the mass \( M \) of marble.

Use conservation of energy between point 1 and point 2, assuming that the system from rest, and that the gravitational potential energy is zero at position 2.

\[
Mgh + \frac{1}{2} Mv_1^2 + \frac{1}{2} I\omega_1^2 = Mg(0) + \frac{1}{2} Mv_2^2 + \frac{1}{2} I\omega_2^2 \rightarrow Mgh = \frac{1}{2} Mv_2^2 + \frac{1}{2} I\omega_2^2 \quad \text{(2 points)}
\]

Now assume \( I = \frac{2}{5} MR^2 \)

\[
Mgh = \frac{1}{2} Mv_2^2 + \frac{1}{2} I\omega_2^2 = \frac{1}{2} Mv_2^2 + \frac{1}{2} \frac{2}{5} MR^2 \omega_2^2 \rightarrow gh = \frac{1}{2} v_2^2 + \frac{1}{5} (\omega_2 R)^2,
\]

and rolling without slipping \( v_2 = R\omega_2 \)

\[
gh = \frac{1}{2} v_2^2 + \frac{1}{5} (\omega_2 R)^2 \rightarrow gh = \frac{7}{10} v_2^2 \rightarrow v_2 = \sqrt{\frac{10}{7} \times 9.8 \text{ m/s}^2 \times 100 \text{ m}} = 37.4 \text{ m/s} \quad \text{(2 points)}
\]

\[
\omega_2 = \frac{v_2}{R} = \frac{37.4 \text{ m/s}}{0.1 \text{ m}} = 374 \text{ s}^{-1} \quad \text{(1 point)}
\]
b) Will the marble fall into the pit? Use kinematics equations to show this. **HINT:** 1) How long does it take to fall to the ground, just to cross the pit; 2) Just before it flies off the edge assume the velocity is horizontal.

This is a pure trajectory problem. Assume that when it flies off the edge, its velocity is horizontal $v_{2x} = 37.4 \text{ m/s}$.

**VERTICAL:** If it crosses the pit, then the time it takes to fall down 20 m is simply

$$y = 20m = \frac{1}{2} gt^2 \rightarrow t = \sqrt{\frac{2 \times 20m}{9.8 \text{ m/s}^2}} = 2.02 \text{ s}.$$  

**HORIZONTAL:** During that period it will travel horizontally 

$$x = v_{2x}t = 37.4 \text{ m/s} \times 2.02 \text{ s} = 75.6 \text{ m}.$$ This is more than enough to travel 36m before falling into the pit. (3 points)

c) Find the **linear** and **angular speed** of the marble, when it lands on the ground **in the pit** or **to the right of the pit**.

From the diagram, it is clear that to go from position 2 and position 3, the marble must fall 20m, so using conservation of energy and taking PE at point 3 to be zero:

$$Mg(20m) + \frac{1}{2} M v_2^2 + \frac{1}{2} I \omega_2^2 = Mg(0) + \frac{1}{2} M v_3^2 + \frac{1}{2} I \omega_3^2$$

Once the ball leaves the edge, there’s no longer any torque that can change the rotational energy of the marble so $\omega_2 = \omega_3$, which gives,

$$Mg(20m) + \frac{1}{2} M v_2^2 = \frac{I}{2} M v_3^2 \rightarrow v_2 = \sqrt{2g(20m) + v_3^2} = \sqrt{2 \left( \frac{9.8 \text{ m}}{\text{s}^2} \right) (20m) + \left( 37.4 \frac{\text{m}}{\text{s}} \right)^2} = \frac{423}{10} \text{ m/s}$$

Of course $\omega_3 = 374 \text{ s}^{-1}$ (2 points)