MIDTERM #1
PHYS 3511 (Biological Physics)
DATE/TIME: February 27, 2020 (10:30 a.m. - 12:45 p.m.)
PLACE: CB4122
Only non-programmable calculators are allowed.

Name: ___________________________________

ID: ___________________________________

Please read the following instructions:

- This midterm has 7 pages. Make sure none are missing.
- There are 2 parts.
- Part I consists of 5 multiple-choice question, each worth 3 points for a total of 15 points. Do all questions.
- Part II consists of 3 full-answer questions, each worth 15 points for a total of 35 points. Do all works on the space provided below the questions.
- Rough work can be done on the back pages.
- Write your name and student ID in the provided space above.

Read questions carefully before attempting the solutions.

PART I: MULTIPLE CHOICE (question 1 to 5): Circle the **one** correct answer.

1. (**3 points**) An artery is partially clogged by a deposit on its inner wall, so that the radius around that area is smaller than in adjacent part of the artery. Which of the following statements best describes the processes that occur when blood rushes through this constriction? Treat blood as a **Newtonian** fluid.
   A) Blood will rush faster through the constriction due to the equation of continuity, causing additional wear and tear on the nearby blood vessel walls.
   B) Bernoulli's law and the equation of continuity predict a variation of the blood pressure in the constricted zone, but the blood vessel walls prevent any adverse effect due to the pressure variation.
   C) The blood pressure in the constriction zone is lower than in the adjacent blood vessel, causing the blood vessel to temporarily collapse at the constriction (vascular flutter).
   D) The blood pressure in the constriction zone is higher than in the adjacent blood vessel, causing a ballooning effect of the blood vessel at the constriction (aneurysm).
   ANSWER: C
2. (**3 points**) Circle the **one true statement** on Brownian Motion, who observed motions of pollen grains suspended in water.
   a) After a long period the motions stopped due to dissipation.
   b) The size of the pollens is similar to that of atoms and molecules ~ 0.1 nm.
   c) Brown observed that other lifeless particles, such as soot in water, do not move in the same way. Hence pollen grains must be alive.
   d) Multiple Collisions with water molecules, sometimes lead to motions by pollens that are visible to a light microscope.
   e) A single collision between water and pollen often leads to pollen moving far enough to be visible by a human looking through a light microscope.

ANSWER D

3. (**3 points**) A bacterium chromosomes have of DNA with molar mass $2.3 \times 10^9 \frac{g}{mole}$ with average nucleotide basepair (bp) molar mass $618 \frac{g}{mole}$. On average, the proteins in the bacterium are made of 300 amino acid (AA) residues. Assume that **three** nucleotide bp is a code for on AA, and that there's no junk gene (the whole DNA is coding). The number of gene in this bacterium is closest to:
   A) 2.5×10^6 **B)** 8×10^5 **C)** 12000 **D)** 4130

Estimated number of bp in the DNA is $\frac{2.3\times10^9 \frac{g}{mole}}{618 \frac{g}{mole\cdot bp^{-1}}} = 3.72 \times 10^6 bp$. The estimated number of genes (proteins) is $\frac{3.72\times10^6 bp}{3\frac{bp}{AA}\times 300\frac{AA}{protein}} = 4135\, protein.$ **ANSWER D**

4. (**3 points**) Use the data on the equation sheet to calculate the "resistance" of venule system. Using the viscosity of blood, the resistance of the venule system, $R_{venule\ system}$ is closest to:
   A) $1.04 \times 10^6 \frac{pa\cdot s}{m^3}$; **B)** $2.6 \times 10^{23} \frac{pa\cdot s}{m^3}$; **C)** $3.88 \times 10^{15} \Omega$; **D)** $5.21 \times 10^{14} \frac{pa\cdot s}{m^3}$

$R_{venule\ one} = \frac{8\times2\times10^{-3}m\times2.5\times10^{-3}pa\cdot s}{\pi(1.25\times10^{-5}m)^4} = 5.21 \times 10^{14} \frac{pa\cdot s}{m^3}$, $R_{venule\ system} = \left(\frac{5\times10^8}{R_{venule\ one}}\right)^{-1} = 1.04 \times 10^6 \frac{pa\cdot s}{m^3}$ **ANSWER: A**

5. (**3 points**) Circle the **one false statement** below:
   A) Humming Bird hovers by helicopter-like motion of the wings that pushes air downward.

B) When E. Coli swims it induces laminar flow of water.
C) The swimming of E. Coli can be explained by the momentum-disc theory
D) When human swims in water, turbulent flow is induced.
ANSWER: C

PART I: Do Question 6 to 8: Show work and calculations

6. (**15 points**) Let the genome of HIV be encoded on RNA of 8000 nucleotides (nt), and that when an RNA is being copied, **a copy error** occurs **every** 10^4 letters.

A) –Calculate the number of distinct two-base mutations, N_{2-base}^{tot}, and the probability, P_2, that when an a given viral particle has **two-base** copy error from the previous generation.

$N_{2-base}^{tot} = 3^2 \binom{8000}{2}, \binom{8000}{2} = \frac{8000!}{2!(7998)!} = \frac{8000 \times 7999}{2} = 31996000$

$N_{2-base}^{tot} = 3^2 \binom{8000}{2} = 287964000$ (**2 points**)

$P_2 = \binom{8000}{2} p^2 (1-p)^{8000-2}, p = \frac{1}{10^4}$ (**1 point**)

$P_2 = 31996000 \times (0.0001)^2 (0.9999)^{7998} = 0.14379$ (**2 points**)

B) Assume that 1.5×10^{10} new virus particle are formed in **one day**, and that 0.1% (1/1000) of the viruses go on to infect white blood cells, which then make copies of new RNA. Use this data and P_2 from part a), to calculate the number of new 2-base RNA mutants, N_2, produced in 1 day. Hence calculate the time it would take for HIV to mutate to a form that is resistant to 2 distinct anti-viral drugs is $t_2 = \frac{N_{2-base}^{tot}}{N_2}$.

$N_i = number\ of\ newly\ copied\ RNA\ per\ day \times P_i$

$N_2 = 1.5 \times 10^{10} \times \frac{1}{1000} \times P_2 \frac{1}{day} = 2.16 \times 10^6 day^{-1}$ (**2 points**)

$t_2 = \frac{N_{2-base}^{tot}}{N_2} = \frac{287964000}{2.16 \times 10^6 day^{-1}} = 133 days$ (**1 point**)

C) Repeat the calculation of part **A)** and **B)** for a 3-base mutation of RNA to find the time it would take for HIV to mutate to a form that is resistant to 3 distinct anti-viral drugs is $t_3 = \frac{N_{3-base}^{tot}}{N_3}$. Compare this to the answer of part **B)**, and briefly comment on the medical significance of the results.

$N_{3-base}^{tot} = 3^3 \binom{8000}{3}, \binom{8000}{3} = \frac{8000!}{3!(7997)!} = \frac{8000 \times 7999 \times 7998}{3!} = 8.53 \times 10^{10}$

$N_{3-base}^{tot} = 3^3 \times 8.53 \times 10^{10} = 2.303 \times 10^{12}$ (**2 points**)

$P_3 = \binom{8000}{3} p^3 (1-p)^{8000-3}, p = \frac{1}{10^4}$

$P_3 = 8.53 \times 10^{10} \times (0.0001)^3 (0.9999)^{7997} = 0.0383$ (**2 points**)

$N_3 = 1.5 \times 10^{10} \times \frac{1}{1000} \times P_3 \frac{1}{day} = 5.745 \times 10^5 day^{-1}$ (**2 points**)

$t_2 = \frac{N_{3-base}^{tot}}{N_3} = \frac{2.303 \times 10^{12}}{5.745 \times 10^5 day^{-1}} = 4 \times 10^6 days$ (**1 point**)

This is more than 10000 years. The virus will never attain resistance to the drug.

7. (**15 points**) A spherical bacterium of radius $r = 100 \mu m$ swims in water at a constant terminal speed of $3 \times 10^{-3} \frac{m}{s}$.

A) Calculate the minimum power produced by the bacteria to swim at this rate. **NOTE:** Power is the rate of work, i.e. energy per unit time.

$F_{drag} = \zeta v_t, \zeta = 6\pi\eta r, F_{drag} = 6\pi\eta r v_t = 6\pi \times 10^{-3} pa \cdot s \times 100 \times 10^{-6} m \times 3 \times 10^{-3} \frac{m}{s}$

$F_{drag} = 5.6 \times 10^{-9} N$. Power $= F_{drag} v_t = 1.7 \times 10^{-11} \frac{J}{s} = 1.7 \times 10^{-11} W$ (**4 points**)

B) If the bacterium lives for 1 day, calculate the minimum number of ATP needed. **Note:** Power is the energy produced per unit time (i.e. per second).

Energy needed in 1 day, $E_{day} = 1.7 \times 10^{-11} \frac{J}{s} \times 24 \frac{hr}{day} \times 3600 \frac{s}{hr} = 1.46 \times 10^{-6} J$

$ATP \rightarrow ADP$ releases 29kJ/mol of heat: Energy released per ATP, $E_{ATP} = 29 \frac{kJ}{mol} \times 1000 \frac{J}{kJ} \times \frac{1}{6.023 \times 10^{23} \frac{ATP}{mol}} = 4.81 \times 10^{-20} \frac{J}{ATP}$

Number of ATP needed, $N_{ATP} = \frac{1.46 \times 10^{-6} J}{4.81 \times 10^{-20} \frac{J}{ATP}} = 3 \times 10^{13} ATP$ (**4 points**)

C) If the bacterium in this question stop swimming, its speed will decrease according to $v(t) = v_t e^{-\lambda t}$, $\lambda = \frac{\zeta}{m}$, where m is the mass of the bacterium. By direct integration, $\Delta x = \int_0^\infty v dt$,show that the distance traveled is $\Delta x = \frac{m v_t}{\zeta}$.

$$\Delta x = \int_0^\infty v dt = \int_0^\infty v_t e^{-\lambda t} dt = -\frac{v_t}{\lambda}\left[e^{-\lambda t}\right]_0^\infty = -\frac{v_t}{\lambda}\left(e^{-\lambda\infty} - e^{-\lambda 0}\right) = \frac{v_t}{\lambda}$$

But $\lambda = \frac{\zeta}{m} \rightarrow \Delta x = \frac{m v_t}{\zeta}$ (**3 points**)

D) Use the result of part C to calculate the distance traveled Δx, before stopping.

Since the bacterium is spherical, its volume is $V = \frac{4}{3}\pi r^3$, and assuming that it has the same mass density as water, $\rho_{water} = 1000 kg \cdot m^{-3}$, its mass is

$$m = \frac{4}{3}\pi r^3 \times \rho_{water} = \frac{4}{3}\pi (10^{-4} m)^3 \times 1000 kg \cdot m^{-3} = 4.19 \times 10^{-9} kg$$

$\Delta x = \frac{m v_t}{6\pi\eta r} = \frac{4.19 \times 10^{-9} kg \times 3 \times 10^{-3} \frac{m}{s}}{6\pi \times 10^{-3} pa \cdot s \times 100 \times 10^{-6} m} = 6.7 \times 10^{-6} m$ (**4 points**)

8. (**15 points**) **Osmosis and the Wilson Syndrome** If we put pure water on both sides of a membrane where one side has a pressure drop of Δp, the volume flux out of the high-pressure side is $j_v = L_p \Delta p$, where $L_p$is the filtration coefficient. For this question assume $L_p = 7 \times 10^{-6} cm \cdot s^{-1} \cdot atm^{-1}$. In class we discussed starvation edema, where the normal blood plasma's osmotic pressure (inside the blood vessel) of $P_{osm} = 3800 Pa$ is reduced by 10% due to malnutrition. If we assume that initially the **osmotic pressures** due to various **osmolytes** (naturally occurring organic molecules) inside and outside are equal $P_{in} = P_{out} = 3800 Pa$ so that $\Delta p = 0$, and there is no net flow of water into or out of the cell. Due to a reduction in protein concentration, Δc, the change in pressure equals the change in osmotic pressure, $\Delta p = \Delta p_{osm} = \Delta c k_B T = \frac{1}{10} 3800 Pa$. This results in a flow of 6.2 L of water per day from the interior of the blood vessel to the interstitium outside the blood vessel.

A) In the **Wilson Temperature Syndrome**, a person body temperature is too low. Consider a person whose body temperature is **decreased** from the normal 37 C to 30 C. In no more than three sentences, use a **physics-type argument** to explain why water should flow out of the blood vessel into the interstitium.

The interior of the blood vessel has a higher protein concentration than in the interstitial region, outside the vessels. This help maintains the balance of the total pressure = hydrostatic pressure + osmotic pressure, $P_{osm} = ck_BT$. When the temperature is reduced the overall osmotic pressure is reduced, especially inside the vessels, which induce flow of water out of the vessels. (**3 points**)

B) Calculate the change in osmotic pressure $\Delta p = \Delta p_{osm}$ due to the temperature change. Use the equation above to calculate the volume flow (in litres) of water per day. **HINT:** i) use temperature conversion Celsius (C) to Kelvin (K), $K = ℃ + 273^o$; ii) find c at 37 C; iii) total flow is $\Delta V = j_v At$; iv) assume that the surface area of veins, arteries, and capillaries is $\sim 250m^2$.

$P_{osm} = ck_BT = 3800Pa \rightarrow c = \dfrac{3800pa}{1.381 \times 10^{-23}\frac{J}{K} \times 310K} = 8.876 \times 10^{23}m^{-3}$

At T = 30 C, $P_{osm}^{30C} = 8.876 \times 10^{23}m^{-3} \times 1.381 \times 10^{-23}\frac{J}{K} \times 303K = 3714Pa$

$\Delta p = \Delta p_{osm} = 3800pa - 3714pa = 86pa$ (**6 points**)

$L_p = 7 \times 10^{-6}cm \cdot s^{-1} \cdot atm^{-1} \times 10^{-2}\dfrac{m}{cm}\dfrac{1}{1.01325 \times 10^5 Pa \cdot atm^{-1}} = 6.9 \times 10^{-13}\dfrac{m}{s \cdot Pa}$
(**2 points**)

$j_v = L_p\Delta p = 6.9 \times 10^{-13}\frac{m}{s \cdot Pa} \times 86pa = 5.9 \times 10^{-11}\frac{m}{s}$ (**2 points**)

In one day, $t = 24\frac{hr}{day}3600\frac{s}{hr} = 86400s$.

total flow is $\Delta V = j_v At = 5.9 \times 10^{-11}\frac{m}{s} \times 86400s \times 250m^2 = 1.27 \times 10^{-3}m^3$, or about 1.27 litres per day. (**2 points**)

Useful Equations

Kinematic: Velocity $v = \frac{dx}{dt}$; acceleration $a = \frac{dv}{dt}$. Constant Acceleration, $v = v_0 + at$, $v^2 = v_0^2 + 2a(x - x_0)$, $x = x_0 + v_0 t + \frac{1}{2}at^2$; **Newton's second law**, $\vec{F} = m\vec{a}$; Newton's third law, when an object A act on Object B with a force, $\vec{F}_{BbyA}$, object B act on object A with a force, $\vec{F}_{AbyB} = -\vec{F}_{BbyA}$, equal in magnitude and opposite in direction.

Momentum: $\vec{p} = m\vec{v}$; in a collision of **N isolated particles** total momentum is $\vec{P}_{total} = \sum_{i=1}^{N} \vec{p}_i = constant$, where $\vec{p}_i$ is the momentum of the ith particle ; Newton 2nd law in terms of momentum $\vec{F}_{net} = \frac{d\vec{p}}{dt}$. Kinetic Energy K = $\frac{1}{2}mv^2$; **momentum-disc theory** Force= $\frac{\Delta p}{\Delta t} = \rho A v^2$; **work**: $W = \vec{F} \cdot \vec{s} = F\, s\, cos\theta$; $W^{net} = \Delta K = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$ (W^{net} is net work); gravitational potential energy (PE),$U_{PE} = mgh$; conservation of energy $E_{total} = U_{PE} + K = constant$. **Power = work/time**; linear power P = Fv. **ATP hydrolysis** $ATP \to ADP$ releases 29kJ/mol of heat. Reverse $ADP \to ATP$ requires 29kJ/mol.

Fluid Drag Force of an object in fluid (liquid or gas).**Turbulent Flow:** $F_{drag} = 0.5\rho A c_d v^2$: v is the speed of the object; ρ is the fluid density ($\rho_{water} = 1000 kg \cdot m^{-3}$, $\rho_{seawater} = 1025 kg \cdot m^{-3}$, $\rho_{air} = 1.29 kg \cdot m^{-3}$); A is the cross-area (perpendicular to the direction of motion); c_d is the drag coefficient that depends on the shape of the object. **Laminar flow:** $F_{drag} = \zeta v$, v is the speed), ζ is drag coefficient; Stokes relation $\zeta = 6\pi\eta r$, r is the radius, and η is the viscosity coefficient (for water $\eta = 10^{-3} pa \cdot s$, for blood $\eta = 2.5 \times 10^{-3} pa \cdot s$). **Bacterium** starting from rest to terminal speed, $v(t) = v_t(1 - e^{-\lambda t})$, where $v_t = \frac{F_{swim}}{\zeta}$ is the terminal speed, F_{swim} is the swimming force of the bacteria, $\lambda = \frac{\zeta}{m}$ and m is the mass of the bacteria. **Bacterium** swimming at terminal speed $v_t = \frac{F_{swim}}{\zeta}$, suddenly stop swimming, $v(t) = v_t e^{-\lambda t}$. Distance travel before it stops moving $\Delta x = \int_0^\infty v dt \to \Delta x = \frac{v_t}{\lambda}$

Osmotic pressure $p_{osm} = c k_B T, c = \frac{N}{V}$ is the number concentration (in m^{-3}) of solutes.

Osmotic flow: If we put pure water on both sides of a membrane where one side has a pressure drop of Δp, the volume flux out of the high-pressure side is $j_v = L_p \Delta p$, where L_p is the filtration coefficient $j_v = L_p \Delta p$

Fluid Flow equations: Equation of continuity $\frac{\Delta V}{\Delta t} = Av = constant$, v is the average fluid speed; **Bernoulli's law** for an **ideal fluid** $p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2 = constant$;

Poiseuille's law for **Newtonian fluid** in a cylindrical tube $Q = \frac{\pi}{8\eta} r_{tube}^4 \frac{\Delta p}{l}$, (unit $\frac{m^3}{s}$), where $Q = \frac{\Delta V}{\Delta t}$ the volume flow rate in unit of $m^3 \cdot s^{-1}$; $\eta = 10^{-3} pa \cdot s$for water, and for blood $\eta = 2.5 \times 10^{-3} pa \cdot s$. **Kirchoff's Law:** Pressure drop $\Delta p = QR$, where the "resistance" to flow (Q) is $R = \frac{8l\eta}{\pi r_{tube}^4}$; **blood vessels in series**, flow rate (Q) are the same, $\Delta p_1 = R_1 Q$, $\Delta p_2 = R_2 Q$, … gives total pressure drop $\Delta p = \Delta p_1 + \Delta p_2 + \cdots = R_{equivalent} Q$, with the equivalent "resistance", $R_{equivalent} = R_1 + R_2 + \cdots$; **blood vessels in parallel**, Pressure drop (Δp) are the same, $\Delta p = R_1 Q_1$, $\Delta p = R_2 Q_2$, … with total flow $Q = Q_1 + Q_2 + \cdots \to \Delta p = R_{equivalent} Q$, with the equivalent "resistance", $\frac{1}{R_{equivalent}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots$

DATA $e = 1.6 \times 10^{-19}C$; $1M = 1\frac{mole}{litre}$; $1litre = 10^{-3}m^3$; Avogardo Number $N_A = 6.023 \times 10^{23}\frac{particles}{mole}$; pressure unit $1atm = 1.01325 \times 10^5 pa$, $k_B = 1.381 \times 10^{-23}\frac{J}{K}$.
Geometrical Relations: 1) **sphere**, surface area $A = 4\pi r^2$ and volume $V = \frac{4}{3}\pi r^3$; 2) cylinder surface area $A = 2\pi rL$ and volume $V = \pi r^2 L$.

<u>Data of Arteries/Veins/Capillaries</u>

Gauge pressure is the pressure above or below the atmospheric pressure $1atm = 1.01325 \times 10^5 Pa, Pa = \frac{N}{m^2}$; Pressure starts at 3.3 kPa at artery to 1.3 kPa at the veins;
Oxygenated/Deoxygenated blood flow from/to heart through arteries/veins; Deoxygenated/Oxygenated blood flow from/to heart through pulmonary arteries/pulmonary veins; Arteries are divided into aorta, large arteries, small arteries, and arterioles; Veins are divided into venae cavae, large veins, small veins, venules; Capillaries are the smallest type of blood vessels.

	Number	Length of one	Radius of one
Aorta	1	34 cm	1.3cm
Large Arteries	40	12.5 cm	0.4 cm
Small Arteries	280	12.5 cm	0.15 cm
Arterioles	1.6×10^8	2.5×10^{-3}m	0.001 cm
Capillaries	3.2×10^9	8.5×10^{-4}m	0.00045 cm
Venules	5×10^8	0.2 cm	0.00125 cm
Small Veins	5600	1.55 cm	0.075 cm
Large Veins	1	30 cm	0.8 cm
Venae cavae	2	13.9 cm	1.6 cm

EQUATIONS for 1D Random Walk: For a1D step, system can make k different kinds of steps of length L_j, where j is the index for the kind of steps ($j = 1,2 \ldots k$), for the j^{th} kind of step there is a probability of P_j of occurring, with normalization $\sum_{j=1}^{k} P_j = 1$.
For **any step**, the **mean** (average) $\langle \Delta x \rangle = \sum_{j=1}^{k} P_j L_j = P_1 L_1 + P_2 L_2 + \cdots P_k L_k$. (E5A)
The **mean** (average) **squared** $\langle (\Delta x)^2 \rangle = \sum_{j=1}^{k} P_j L_j^2 = P_1 L_1^2 + P_2 L_2^2 + \cdots P_k L_k^2$. (E5B)
The **variance** $var(\Delta x) = \langle (\Delta x - \langle \Delta x \rangle)^2 \rangle = \sum_{j=1}^{k} P_j \left(L_j - \langle \Delta x \rangle\right)^2 = P_1 (L_1 - \langle \Delta x \rangle)^2 + P_2 (L_2 - \langle \Delta x \rangle)^2 + \cdots P_k (L_k - \langle \Delta x \rangle)^2$. (E5C)
FOR N STEPS: This involves N random steps of $\Delta x \rightarrow x_N = \Delta x_1 + \Delta x_2 + \cdots + \Delta x_N$
MEAN (Average) $\langle x_N \rangle = \langle \Delta x_1 + \Delta x_2 + \cdots + \Delta x_N \rangle \rightarrow \langle x_N \rangle = N \langle \Delta x \rangle$. (E5D)
VARIANCE $var(x_N) = \langle (x_N - \langle x_N \rangle)^2 \rangle = N var(\Delta x)$. (E5E)
For E5D you must use E5A. For E5E you must use E5C.

<u>HIV Physics</u> For HIV encoded by RNA of **n** nucleotide (nt), the **number** of **distinct** *i-base* mutations is $N_{i-base}^{tot} = 3^i \binom{n}{i}$, with $\binom{n}{i} = \frac{n!}{i!(n-i)!} = \frac{n \times (n-1) \ldots (n-i+1)}{1 \times 2 \times \ldots \times i}$ where the 3^i is due to the fact that an original base can mutate to one of the three (3) remaining bases (3 of G, U, A or C). The probability of a i-base error is $P_i = \binom{n}{i} p^i (1-p)^{n-i}$, where p is the probability of a copy error per letter (G U A C) copied (for example for one error in $-3 \times 10^4, p = \frac{1}{3 \times 10^4}$). Number of i-base mutants per day is $N_i = number\ of\ newly\ copied\ RNA\ per\ day \times P_i$. . Usually the first term is the number of new viruses per day × fraction of viruses that infects blood cells.