MIDTERM #1, PHYS 1211 (INTRODUCTORY PHYSICS),
AT 1001, 27 September 2017, 10:30 AM to 11:20 AM

INSTRUCTOR: Apichart Linhananta

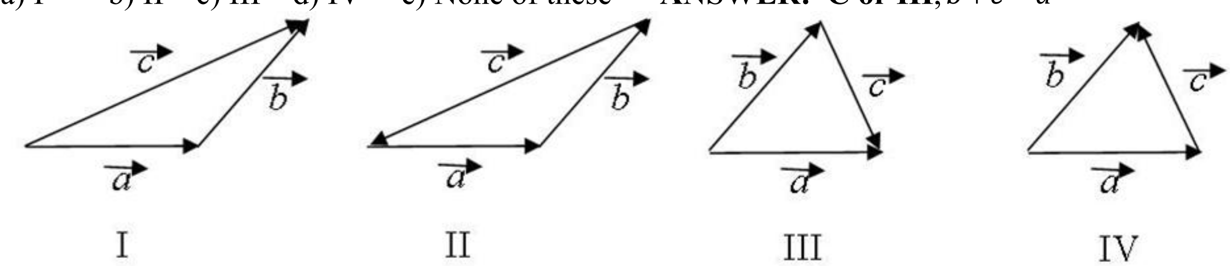
Student Name: **STUDENT ID:**

This exam book has 5 pages including an equation sheet on page 5. All works must be done on this exam paper. Only one **non-programmable calculator** is allowed.

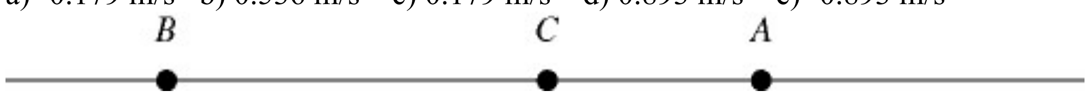
PART I: MULTIPLE CHOICE QUESTIONS (question 1 to 4)

For each question **circle** the correct answer (a,b,c,d or e).

1. (**2.5 points**) For the vector $\vec{A} = (-25m)\hat{i} + (45m)\hat{j}$, the angle measured (in degrees) counterclockwise from the x-axis to vector $\vec{A}$ is closest to:
   a) 209 b) 29 c) 61 d) 119 e) 151
   $A_x < 0$, $A_y > 0$ must be in second quadrant, $90^\circ < \theta < 180^\circ$ $|A_y| > |A_x| \rightarrow \theta < 135^\circ$.
   Answer d, should verify!
2. (**2.5 points**) The vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are related by $\vec{c} = \vec{a} - \vec{b}$. Which diagram below illustrates this relationship?
   a) I b) II c) III d) IV e) None of these **ANSWER: C or III**, $\vec{b} + \vec{c} = \vec{a}$



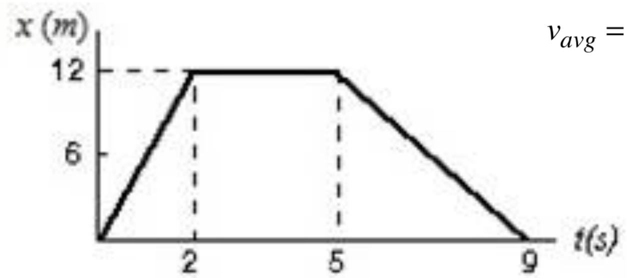
3. (**2.5 points**) A cat runs in a straight line (the x-axis) from point A to point B to point C, as shown below. The distance between point A and C is 5.00m, between point B and C is 10.0m, and the positive direction of the x-axis points to the right. The time to run from A to B is 20.0s and from B to C is 8.00s. The **average speed** for the whole trip is closest to
   a) -0.179 m/s b) 0.536 m/s c) 0.179 m/s d) 0.893 m/s e) -0.893 m/s



$\overline{AB} = 15m$, $\Delta t_{AB} = 20s$; $\overline{BC} = 10m$, $\Delta t_{BC} = 8s$. Average speed

$S_{avg} = \dfrac{\overline{AB} + \overline{BC}}{\Delta t_{AB} + \Delta t_{BC}} = 0.893\dfrac{m}{s}$. **ANSWER D.** Note that the speed must always be positive.

4. (**2.5 points**) The graph below shows the position of a particle as a function of time. What is its average velocity between t = 5s and t = 9s?
   3 m/s b) -3m/s c) 12 m/s d) -12 m/s e) need additional information



$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x(9s) - x(5s)}{9s - 5s} = \frac{0m - 12m}{4s} = -3\frac{m}{s}$$

ANSWER: B

PART II: FULL ANSWER QUESTIONS (question 5 to 7)

Do all four questions on the provided space. Show all work.

5. (**10 points**) Let $x(t) = At^4 - Bt^3 + Ct$, where $A = 1.2m/s^4$, $B = 2.0m/s^3$, and $C = 3.0m/s$ describes an object's position as a function of time:

a) Find the positions at t = 1.0 s and t = 2.0 s. Hence find the average velocity for time interval between t = 1.0 s to t = 2.0s.

At t = 1.0 s, $x(t) = (1.2m/s^4)(1.0s)^4 - (2.0m/s^3)(1.0s)^3 + (3.0m/s)(1.0s) = 2.2m$

(**0.5 point**)

At t = 2.0 s, $x(t) = (1.2m/s^4)(2.0s)^4 - (2.0m/s^3)(2.0s)^3 + (3.0m/s)(2.0s) = 9.2m$

(**0.5 point**)

$v_{av-x} = \dfrac{x(2.0s) - x(1.0s)}{2.0s - 1.0s} = \dfrac{9.2m - 2.2m}{1.0m} = 7.0m/s$ (**2 points**)

b) Find the velocity at time t = 0 s and at t = 2.0s.

$v_x = \dfrac{dx}{dt} = 4At^3 - 3Bt^2 + C$ (**2 points**)

At t = 0 s, $v_x = 4(1.2m/s^4)(0s)^3 - 3(2.0m/s^3)(0s)^2 + (3.0m/s) = 3.0m/s$ (**0.5 point**)

At t = 2.0 s, $v_x = 4(1.2m/s^4)(2.0s)^3 - 3(2.0m/s^3)(2.0s)^2 + (3.0m/s) = 17.4m/s$

(**0.5 point**)

c) Find the average acceleration for the time interval between t = 0 s to t = 2.0s

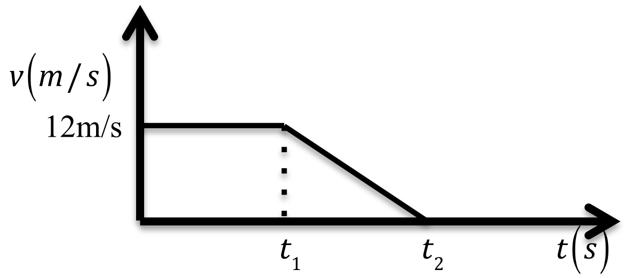
$a_{av-x} = \dfrac{v_x(2.0s) - v_x(0s)}{2.0s - 0s} = \dfrac{17.4m/s - 3.0m/s}{2.0s} = 7.2m/s$ (**1 points**)

d) Find the acceleration at time t = 1 s.

$a_x = \dfrac{dv_x}{dt} = 12At^2 - 6Bt$ (**2 points**)

At t = 1.0 s, $a_x = \dfrac{dv_x}{dt} = 12(1.2m/s^4)(1s)^2 - 6(2m/s^3)1s = 2.4m/s^2$ (**1 point**)

6. (**10 points**) The diagram below is a velocity$\left(v\right)$ vs time $\left(t\right)$ plot of an **athlete running** the **last 100m** of a marathon. From t = 0 to t_1, he is running with a **constant speed** of 12 m/s, till he covers 64 m. He then **decelerates** (slows down) for 6 s till he comes to a **stop** (v = 0) at the finish line.



a) From the graph and data above, calculate t_1. Determine t_2. Use the appropriate 1d kinematic equation to calculate the **constant acceleration** from time t_1 to t_2.

$x_1 = 64m = x_0 + v_0 t_1 = 0 + \left(12m \bullet s^{-1}\right)t_1 \rightarrow t_1 = 5.33s$. (**2 points**)

$t_2 = t_1 + 6s = 11.33s$ (**2 points**)

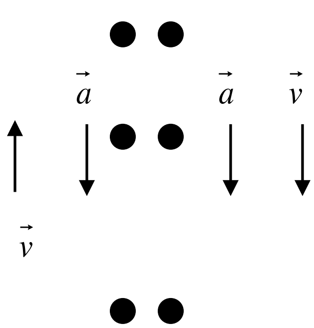
b) Use the method of **graphical integration**, $x - x_0 = \int_{t_0}^{t_2} v\,dt =$ area , to determine the distance traveled from time t = t_0 to t = t_2. Assume **initially** $t_0 = 0, x_0 = 0$. **NOTE:** You must use graphical integration to receive full credit.

$x - x_0 = x - 0 = \int_0^{t_2} v\,dt =$ area of rectangle + area of triangle. (**2 points**)

$x = \left(12m \bullet s^{-1}\right)\left(5.33s\right) + \frac{1}{2}\left(12m \bullet s^{-1}\right)\left(6s\right) = 100.m$ (**4 points**)

7. (**10 points**) Jack throws an egg straight up with a speed of 8.4 m/s. The egg is released at the same height as his head. It rises and then falls down and hits his head

a) Draw a **motion diagram** of the path of the egg from when it was released to the time it lands on his head. In the diagram, indicate the direction of the velocity and acceleration of the egg.



In the diagram the dots represent the positions of the egg at equal time interval. To obtain full marks the spacing between dot must decreases on the way up and increases on the down, in order to illustrate that the velocity is changing. Further the direction of the velocity and acceleration on the way up/down must be clearly indicated as shown. (**2 points**)

b) Find the velocities of the egg at the height of 2.0 m above Jack's head. Note that the egg will be 2.0 m above Jack's head twice, once on the way up, the other on the way down. Find the **times**, on the **way up** and **down**, when the egg is 2.0 m above Jack's head.

Easiest method

$v_y^2 = v_{0y}^2 - 2g\left(y - y_0\right) \Rightarrow v_y = \pm\sqrt{v_{0y}^2 - 2g\left(y - y_0\right)}$

v_{0y} = 8.4 m/s, y_0 = 0, y = 2.0m, v_y = velocity at 2.0 m (**1 points**)

$v_y = \pm\sqrt{v_{0y}^2 - 2g\left(y - y_0\right)} = \pm\sqrt{\left(8.4m/s\right)^2 - 2\left(9.8m/s^2\right)\left(2.0m\right)} = \pm 5.6m/s$ (**1 point**)

v_y = + 5.6 m/s, on the way up, v_y = -5.6 m/s, on the way down. (**1 point**)

way up $v = v_0 - gt \rightarrow t = \dfrac{\left(v_0 - v\right)}{g} = \dfrac{\left(8.4m\bullet s^{-1} - \left(5.6m\bullet s^{-1}\right)\right)}{9.8m\bullet s^{-2}} = 1.43s$.

way down $v = v_0 - gt \rightarrow t = \dfrac{\left(v_0 - v\right)}{g} = \dfrac{\left(8.4m\bullet s^{-1} - 5.6m\bullet s^{-1}\right)}{9.8m\bullet s^{-2}} = 0.286s$.(**1 point**)

Hard method

$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$, v_{0y} = 8.4 m/s, y_0 = 0, y = 2.0m, t = time when height is 2.0 m

$2.0m = \left(8.4m/s\right)t - \frac{1}{2}\left(9.8m/s^2\right)t^2 \Rightarrow 4.9t^2 - 8.4t + 2 = 0$ (**2 points**)

$t = \dfrac{8.4 \pm \sqrt{\left(8.4\right)^2 - 4\left(4.9\right)2}}{9.8} = .857s \pm 571s \Rightarrow t = 1.43s(down)$ and $t = 0.286s(up)$ (**1 point**)

at t = 1.43 s, $v = v_0 - gt = 8.4\frac{m}{s} - 9.8\frac{m}{s^2}1.43s = -5.6\frac{m}{s}$, **down**

at t = 0.286 s, $v = v_0 - gt = 8.4\frac{m}{s} - 9.8\frac{m}{s^2}0.286s = 5.6\frac{m}{s}$, **up** (**1 point**)

c) Find the maximum height of the egg. Find the time at the maximum height. What is the acceleration of the egg at its maximum height?

At Maximum height the velocity is zero $v_y = 0$, so use

$$v_y^2 = v_{0y}^2 - 2g(y - y_0) \rightarrow y = \frac{v_{0y}^2}{2g} = \frac{\left(8.4\frac{m}{s}\right)^2}{2\left(9.8\frac{m}{s^2}\right)} = 3.6m \text{ (**2 points**)}$$

Time, $v = v_0 - gt \rightarrow 0 = 8.4\frac{m}{s} - 9.8\frac{m}{s^2}t \rightarrow t = 0.86s$. (**1 point**)

The acceleration is $a_y = -g = -9.8m/s^2$. The direction is down. (**1 point**)

Useful Equations

$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$, unit vector notation.

$x = x_0 + v_0 t + \frac{1}{2}at^2$, $v = v_0 + at$, $v^2 = v_0^2 + 2a(x - x_0)$

$v_{avg} = \frac{x(t_2) - x(t_1)}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$, $a_{avg} = \frac{v(t_2) - v(t_1)}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$

$v = \frac{dx}{dt}$, $a = \frac{dv}{dt}$, $a = \frac{d^2x}{dt^2}$

Graphical Method:

$x = x_0 + \int_{t_0}^{t_1} v\,dt = \text{area}$, where x_0 and t_0 are the initial position and time, respectively, and x is the position at time t_1. Area is area under v vs. t plot.

$v = v_0 + \int_{t_0}^{t_1} a\,dt = \text{area}$, where v_0 and t_0 are the initial velocity and time, respectively, and v is the velocity at time t_1. Area is area under a vs. t plot.

average speed $s_{avg} = \frac{\text{total distance}}{\Delta t}$

Free Fall $g = 9.8 m/s^2$, with +y up, $y = y_0 + v_0 t - \frac{1}{2}gt^2$, $v = v_0 - gt$, $v^2 = v_0^2 - 2g(y - y_0)$

Solution of quadratic equation, $ax^2 + bx + c = 0 \rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Unit $1\frac{m}{s} = 3.6\frac{km}{h}$