PHYS3511-Biological Physics
Fall 2018, Assignment #3

Due Monday October 15, 2018.Read Chapter 3 before attempting the assignment

Exercise 1) **Problem 3.7 of Chapter 3**

In rapidly dividing bacteria, the cell can divide in times as short as 1200s. make a careful estimate of the number of sugars (glucose) needed to provide the carbon for constructing the macromolecules of the cell during one cell cycle of a bacterium. Use the result to work out the number of carbon atoms that need to be taken into the cell each second.

The textbook states that approximately 10^{10} carbon (page 39). This has already been done in assignment 1. Glucose has a chemical formula $C_6H_{12}O_6$, with molecular weight $180\frac{g}{mol}$. Hence the number of glucose needed is about 1.66×10^9. The number of carbon atoms needed per second is about $8.3 \times 10^6 \frac{carbon}{s}$.

Exercise 2) Use Fick's law to estimate how long it would take for glucose from a minimal media to diffuse through the membrane of E. Coli in order for the cell to produce enough proteins, RNA and DNA (i.e. carbons) for it to divide into 2 daughter cells. Assume that the E. Coli is cylindrical with a length (L) of 2 μm and diameter 1 μm, and that the membrane is about 5 nm thick. Also assume that the **viscosity** of the membrane is the same as that of **A)** water $\eta = 10^{-3}\frac{kg}{m \cdot s}$; **B)** has value $\eta = 10^{-9}\frac{kg}{m \cdot s}$. **C)** do you think that the viscosity of the membrane will be more like the value of part A or B? Explain why. **HINT:** assume that **glucose** diffuses into the membrane through the **lateral area** of the cylinder ($area = 2\pi r L$). Size (i.e. radius) of glucose (sugar) can be found on table presented on PDF of October 3, 2018; see **exercise** 5 for how to calculate the **diffusion coefficient** D.

Use Fick's law $j = -D\frac{\Delta c}{\Delta \mathrm{x}} = -D\frac{c_{in} - c_{out}}{\Delta \mathrm{x}}$, with $\Delta \mathrm{x} = 3\mathrm{nm}$, being the approximate lipid membrane thickness, $c_{in} = 0$ the interior glucose concentration, assumed to be zero (unrealistic but ok for now), c_{out} is the glucose concentration outside, which should be equal to that of the minimal media. A minimal medium has a mass concentration of 0.5 g/100 mL = $5 \times 10^{-3}\frac{g}{ml}$ of glucose, which has a chemical formula $C_6H_{12}O_6$, with molecular weight $180\frac{g}{mol}$. This is converted to number concentration (1 litre = $1l = 10^{-3}m^3$): $c_{out} = \left(5 \times 10^{-3}\frac{g}{ml} \div 180\frac{g}{mol}\right) \times 6.023 \times 10^{23}\frac{glucose}{mol} \times \frac{10^{-3}kg \cdot g^{-1}}{10^{-6}m^3 \cdot ml^{-1}}$

$$c_{out} = 1.67 \times 10^{22} m^{-3}$$

A) For water $\eta = 10^{-3}\frac{kg}{m \cdot s}$, $D = \frac{k_B T}{\zeta} = \frac{k_B T}{6\pi\eta R} = \frac{(1.381 \times 10^{-23} J \cdot K^{-1})(300K)}{6\pi\left(10^{-3}\frac{kg}{m \cdot s}\right)(0.75 \times 10^{-9}m)}$

$D = 2.9 \times 10^{-10}\frac{m^2}{s}$, where we used the size of sugar (about 0.5 to 1 nm) as 0.75 nm. This gives $j = -D\frac{c_{in} - c_{out}}{\Delta \mathrm{x}} = -\left(2.9 \times 10^{-10}\frac{m^2}{s}\right)\frac{-1.67 \times 10^{22} m^{-3}}{3 \times 10^{-9} m}$

$$j = 1.6 \times 10^{21}\frac{1}{m^2 \cdot s}.$$

The current density is the flux, or the number of particles flowing through a unit area in one second. The total number of particles passing through a given area, A, in a time t is, $N = jAt$. From the previous question the number of glucose needed is $N = 1.66 \times 10^9$, and from the question, the lateral area of the E. Coli is $A = 2\pi rL$, where we will assume for E. Coli, $r = 0.5\mu m$ and $L = 2.0\mu m$ (similar values are ok), $A = 6.3 \times 10^{-12} m^2$. This gives a time $t = \frac{1.66\times10^9}{\left(1.6\times10^{21}\frac{1}{m^2\cdot s}\right)(6.3\times10^{-12}m^2)} = 0.164$ s.

This value is actually way too low. But don't forget that the glucose is diffusing through 3 nm of lipid membrane, not water.

B) For water $\eta = 10^{-9}\frac{kg}{m\cdot s}$, $D = \frac{k_B T}{\zeta} = \frac{k_B T}{6\pi\eta R} = \frac{(1.381\times10^{-23} J\cdot K^{-1})(300K)}{6\pi\left(10^{-9}\frac{kg}{m\cdot s}\right)(0.75\times10^{-9}m)}$

$D = 2.9 \times 10^{-4}\frac{m^2}{s}$, where we used the size of sugar (about 0.5 to 1 nm) as 0.75 nm. This gives $j = -D\frac{c_{in}-c_{out}}{\Delta x} = -\left(2.9 \times 10^{-4}\frac{m^2}{s}\right)\frac{-1.67\times10^{22}m^{-3}}{3\times10^{-9}m}$

$$j = 1.6 \times 10^{27}\frac{1}{m^2\cdot s}.$$

The current density is the flux, or the number of particles flowing through a unit area in one second. The total number of particles passing through a given area, A, in a time t is, $N = jAt$. From the previous question the number of glucose needed is $N = 1.66 \times 10^9$, and from the question, the lateral area of the E. Coli is $A = 2\pi rL$, where we will assume for E. Coli, $r = 0.5\mu m$ and $L = 2.0\mu m$ (similar values are ok), $A = 6.3 \times 10^{-12}m^2$. This gives a time $t = \frac{1.66\times10^9}{\left(1.6\times10^{27}\frac{1}{m^2\cdot s}\right)(6.3\times10^{-12}m^2)} = 1.64 \times 10^{-8}s.$

Again way too fast!!

I probably gave you the wrong values. I found a recent paper where the viscosity was experimentally found with values of $10^{-15}\frac{m^2}{s}$ to $10^{-12}\frac{m^2}{s}$. This will give a time that varies from 10 s to 10^5 s, which is actually quite realistic. However, we note the wrong assumption that the interior of the cell has no glucose, which is actually not the case once a number of glucose has migrated into the cell. This will decrease the flux density, j, and increase the total time for the cell to uptake all required glucose to divide.

Exercise 3) <u>Problem 3.8 of Chapter 3</u>

Assume that 1kg of bacteria burn oxygen at a rate of $A = 0.006\frac{mol}{s\cdot kg}$. The oxygen enters the bacterium by diffusion through the surface at a rate of $\Phi = 4\pi R D c_0$ with $D = 2 \times 10^{-9}\frac{m^2}{s}$; $c_0 = 0.2\frac{mol}{m^3}$.

A) Let's assume that the bacterium is a sphere of radius, $R = 10\mu m = 10^{-5}m$, with a density of water $\rho = 1000 kg\cdot m^{-3}$, then the mass of the bacterium is $M_{bacteria} = \rho\frac{4}{3}\pi R^3 = 4.19 \times 10^{-12}m^3$, the activity of this bacteria is $A_{one} = AM_{bacteria} = 2.51 \times 10^{-14}\frac{mol}{s}$.

We now use the equation to determine the O_2 diffusion rate $\Phi = 4\pi RDc_0$ for a bacterium of radius $R = 10^{-5}m$,

$$\Phi = 4\pi RDc_0 = 4\pi(10^{-5}m)\left(2\times10^{-9}\frac{m^2}{s}\right)\left(0.2\frac{mol}{m^3}\right) = 5\times10^{-14}\frac{mol}{s} > A_{one}$$

B)The activity, A, is equal to the oxygen consumed by the one kg of bacteria per second

$$\mathrm{A} = \frac{\Phi}{M_{bacteria}},$$

where the **mass of the bacteria** is $M_{bacteria} = \rho\frac{4}{3}\pi R^3$, with the density of the bacteria assumed to be that of water $\rho = 1000kg\cdot m^{-3}$, and the volume of the bacteria is $\frac{4}{3}\pi R^3$(assumed to be a sphere of radius R):

$$\mathrm{A} = \frac{4\pi RDc_0}{\rho\frac{4}{3}\pi R^3},$$

which gives a maximum radius of

$$R_{max} = \sqrt{\frac{3Dc_0}{\rho\mathrm{A}}}.$$

Using data from problem 3.8, $\mathrm{A} = 0.006\frac{mol}{s\cdot kg}$; $\mathrm{D} = 2\times10^{-9}\frac{m^2}{s}$; $c_0 = 0.2\frac{mol}{m^3}$, gives $R_{max} = 1.41\times10^{-5}m = 14.1\mu m.$

Exercise 4) For the previous Metabolic Rate problem 3.8, we assume that the number concentration of oxygen, $c(r)$, obeys the equation DE: $\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dc}{dr}\right) = 0$. Show that the solution that satisfies the boundary condition $c(\infty) = c_0$ and $c(R) = 0$ is

$$c = c_0\left(1-\frac{R}{r}\right), r \geq R;\ c = 0, r < R$$

and where c_0 is the **bulk oxygen concentration**, and R is the radius of the bacterium. Derive the current density of oxygen, $j(r)$, towards the cell, using the Fick's law equation $j = -D\frac{dc}{dr}$. Show that the total diffusion at fixed distance, r, from the center of the bacteria is a constant. The total diffusion is found by the equation:

$$\Phi = (area\ of\ sphere\ of\ radius\ r)\times j(r)$$

In a few sentences explain why Φ must be a **constant**. **Hint:** the **concentration** of **oxygen does not vary** with **time**, $\frac{\partial c}{\partial t} = 0$.

This is done by direct substitution of $c = c_0\left(1-\frac{R}{r}\right)$ into DE and verifying that the solution give the correct concentration at r = 0 and r = R.

$$j = -D\frac{dc_0\left(1-\frac{R}{r}\right)}{dr} = -Dc_0\frac{R}{r^2}.$$

$$\Phi = (4\pi r^2)\times j(r) = 4\pi RDc_0$$

Note that the total diffusion at a given distance r is constant. The total diffusion at r must be constant, otherwise the concentration will change, i.e, c(r) will vary.

Exercise 5) Brownian Motion. Here is a fictitious story on a modern-day Robert Brown, who observed a spherical object (object 1) of radius $R_1 = 1\mu m$ moves "randomly" by about $1\mu m$ every second. He also observed another spherical object (object 2) of radius $R_2 = 100\mu m$ moves "randomly" by about 1mm every second. Use the Einstein fluctuation-dissipation relation $D = \frac{k_B T}{\zeta}$, the Stokes relation $\zeta = 6\pi\eta R$, and the diffusion equation to determine which object is living. Assume a reasonable value for the temperature (say 300 K).

As explain in class each direction (x, y, or z) contributes *2Dt*, resulting in the *6Dt* in the 3D diffusion length. The diffusion length is the average distance the object moves due to random fluctuations.

Object 1 $D = \frac{k_B T}{\varsigma} = \frac{k_B T}{6\pi\eta R_1} = \frac{\left(1.381\times 10^{-23} JgK^{-1}\right)(300K)}{6\pi\left(10^{-3} kggs^{-1}gm^{-1}\right)\left(10^{-6} m\right)} = 2.2\times 10^{-13}\frac{m^2}{s}$. Hence in one second *t* = *1s*, the diffusion length is

$r_{\text{diffuse}} = \sqrt{\left\langle r^2\right\rangle} = \sqrt{6\times 2.2\times 10^{-13} m^2 gs^{-2}\times 1s} = 1.15\times 10^{-6} m = 1.15\mu m$. The fact that it is observed to move by about $1\mu m$every second means that the motion is probably due to random fluctuation and there is **no evidence that the object is alive**.

Object 2 $D = \frac{k_B T}{6\pi\eta R_2} = \frac{\left(1.381\times 10^{-23} JgK^{-1}\right)(300K)}{6\pi\left(10^{-3} kggs^{-1}gm^{-1}\right)\left(10^{-4} m\right)} = 2.2\times 10^{-15}\frac{m^2}{s}$. Hence in one second *t* = *1s*, the diffusion length is

$r_{\text{diffuse}} = \sqrt{\left\langle r^2\right\rangle} = \sqrt{6\times 2.2\times 10^{-15} m^2 gs^{-2}\times 1s} = 1.15\times 10^{-7} m = 1.15\times 10^{-4} mm$. That it moves by about 1mm every second means that the object is an organism capable of swimming at a rate much greater than random diffusion.