PHYS3511-Biological Physics
Fall 2018, Assignment #1

Assigned on Monday September 17, 2018, 2007. Due on Monday September 24, 2018.

Read Chapter 2 of textbook before attempting the assignment

Exercise 1) Consider again the coin-tossing problem discussed in class. The coins can land as Heads (H) or Tails (T). For the case where there are N coins, the total number of microstates (i.e. the # of different arrangements of heads and tails) is 2^N. This is consistent with the result in class where the total number of microstates for 3 coins is $2^3 = 8$. It was shown that the number of **configurations**, also called the **microstates** with n heads (in a coin tossing experiment of N coins) is:

$$\Omega_N(n) = \frac{N!}{n!(N-n)!}.$$

Now consider a coin tossing experiment of 4 coins.

a) Just as in class, make a table to count all the possible microstates of the experiment. Count the total number of microstates. Is it consistent with the formula 2^N?

Microstate #	Coin 1	Coin 2	Coin 3	Coin 4	# of heads
1	H	H	H	H	4
2	H	H	H	T	3
3	H	H	T	H	3
4	H	T	H	H	3
5	T	H	H	H	3
6	H	H	T	T	2
7	H	T	H	T	2
8	H	T	T	H	2
9	T	H	H	T	2
10	T	H	T	H	2
11	T	T	H	H	2
12	H	T	T	T	1
13	T	H	T	T	1
14	T	T	H	T	1
15	T	T	T	H	1
16	T	T	T	T	0

In all there are 16 microstates, consistent with the formula $2^4 = 16$.

b) In such an experiment where 4 coins are tossed what **do you think** is the most **probable** number of heads? Briefly justify your response.
Of course, the answer is 2 H and 2T. Anytime you toss a coin it is equally probable that it is either head or tail.

c) Using the given formulas, calculate the probabilities that a toss of all four coins gives: i) 4H, 0T; ii) 3H, 1T; iii) 2H, 2T; iv) 1H 3T; v) 0H, 4T.

i) 4H, 0T, n = 4, $\Omega_4(n) = \frac{4!}{4!0!} = 1$, note $0! = 1$.

ii) 3H, 1T, n = 3, $\Omega_4(3) = \frac{4!}{3!1!} = 4$

iii) 2H, 2T, n = 2, $\Omega_4(2) = \frac{4!}{2!2!} = 6$

iv) 1H 3T, n = 1, $\Omega_4(1) = \frac{4!}{1!3!} = 4$

v) 0H, 4T, n = 0, $\Omega_4(0) = \frac{4!}{0!4!} = 1$

Note that the result is consistent with the table of part (a)

d) Using the result of c) what is the most probable outcome of a coin toss experiment? Is this consistent with your answer in b) ?

The n = 2 macrostate (2H and 2T) includes 6 microstates ($\Omega_4(2) = 6$), the most of any macrostates. All microstates having equal probabilities of occurring means that the macrostate with 2H and 2T (n = 2) is the most likely. This occur with the probability $\Omega_4(2)/16 = 6/16 = 3/8$, where we've used the fact that there are 16 microstates in total (part a). For comparisons, the other probabilities are $\Omega_4(4)/16 = 1/16$ for n =2, $\Omega_4(3)/16 = 4/16 = 1/4$ for n = 3, $\Omega_4(1)/16 = 4/16 = 1/4$ for n = 1, and $\Omega_4(0)/16 = 1/16$ for n = 0. Hence the macrostate with n = 2 has the largest probability of occurring. This is consistent with the result of part a.

Exercise 2) Consider the random partitioning of **one parent** E. Coli into two daughters E. Coli, and assume that the 2×10^6 proteins (see Table 2.1) is distributed into the daughters by a random process as illustrated in Figure 2.7. This is described by the **Binomial distribution** in equation 2.6 on page 45, with p = 0.5 and q = 1 – p = 0.5.

a) Calculate the probability that the 1×10^6 proteins is in one of the daughter cells.
b) Calculate the probability that the 2×10^4 proteins is in one of the daughter cells.

Note: Express the **probability** in the **form**, 10^x, for example $10^{-2.77}$ or 10^{-88899}. Use Stirling approximation, $N! \sim \sqrt{2\pi N}\left(\frac{N}{e}\right)^N = (2\pi N)^{1/2}\left(\frac{N}{e}\right)^N$, valid for large N.

Similar problem done in class.

Exercise 3) Problem 2.3 of Chapter 2

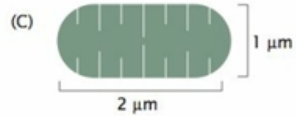


Fig 2.1 shows that it is a cylinder of radius r = 0.5μm and length L = 0.5μm

A) Area: $A_{Ecoli} = 2\pi RL = 6.28 \times 10^{-12} m^2 = 6.28 \mu m^2$

Volume: $V_{Ecoli} = \pi R^2 L = 1.57 \times 10^{-18} m^3 = 1.57 \mu m^3$. Since $1l = 10^{-3} m^3$, $V_{Ecoli} =$
$= 1.57 \times 10^{-18} m^3 = 1.57 \times 10^{-15} l = 1.57 fl$

B) Assume that a cell has the density of water $\rho = 1000\frac{kg}{m^3}$. For 3kg of bacteria in the intestine, the total number of cells is about $\frac{3kg}{1000\frac{kg}{m^3}\times 1.57\times 10^{-18}m^3} = 1.91\times 10^{15}$. For a human of 100 kg, $\frac{100kg}{1000\frac{kg}{m^3}\times 1.57\times 10^{-18}m^3} = 6.0\times 10^{16}$. This probably too large since the size of a mammal cell is probably larger. A more realistic estimate would use the dimension of a cell as $\sim 25\mu m$

C) Assume that in the top 100 m of the ocean there are 10^{28} prokaryotes. Assume that each occupy a volume of $V_{Ecoli} = 1.57\times 10^{-18}m^3 = 1.57\mu m^3$. This gives the volume occupied by the prokaryotes as $V_{total} = 10^{28}V_{Ecoli} = 1.57\times 10^{10}m^3 = 15.7km^3$.

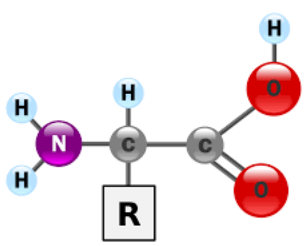
Exercise 4) <u>Problem 2.5 of Chapter 2</u>

Minimal growth medium for bacteria such as E. coli includes various salts with characteristic concentrations in the mM range and a carbon source. The carbon source is typically glucose and it is used at 0.5% (a concentration of 0.5 g/100 mL). For nitrogen, minimal medium contains ammonium chloride (NH4Cl) with a concentration of 0.1 g/100 mL.

(a) Make an estimate of the number of carbon atoms it takes to make up the macromolecular contents of a bacterium such as E. coli. Similarly, make an estimate of the number of nitrogens it takes to make up the macromolecular contents of a bacterium? What about phosphate?

We begin by estimating the number of carbon atoms in an E. Coli. This is done by considering the macromolecular content of the E. Coli in Table 2.1:
http://alinhana.lakeheadu.ca/MoleculeinEcoli.png

<u>Proteins</u>

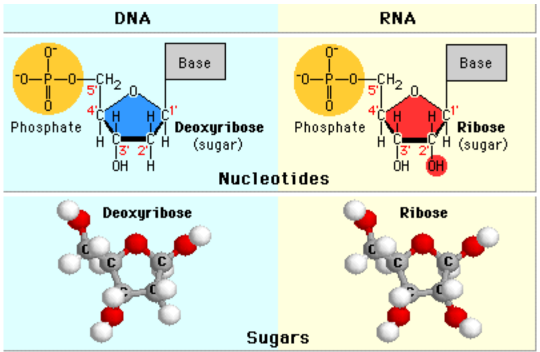


Proteins are comprised of **20 types** of **amino acids (aa)**, all with a **common backbone** that has **two carbon atoms**, distinguished by the **20 kinds** of **sidechains**, indicated by the letter **R** in the figure on the left. A sidechain can have **zero** to **9 carbon** atoms. For the purpose of this problem, let's assume that **one AMINO ACID** has **SIX CARBON ATOMS**, $6\frac{carbon}{aa}$ on average.

Table 2.1 states that the number of proteins in E. coli is 2×10^6. As stated in class and in the textbook, the average size of a protein is $\sim 300\ aa$, where aa stands for amino acids. Hence the number of carbon atoms of proteins in E. Coli is:

$$N_{carbon}^{protein} = 2.4\times 10^6 proteins \times 300\frac{aa}{protein}\times 5\frac{carbon}{aa} = 4.3\times 10^9 carbon$$

<u>RNA</u>



RNA/DNA is made of a 5-carbon sugar. The base is made of 4 or 5 carbons. We will assume that one **RNA** nucleotide (nt), aka one base pair (bp), has about 5 carbon atoms: $\sim 10\frac{carbon}{bp} = 10\frac{carbon}{nt}$.

<u>23S RNA, 16S RNA, SSRNA</u>
Table 2.1 indicates that there are about 1.9×10^4 23S RNA, 1.9×10^4 16S RNA, and 1.9×10^4 SS RNA. Online search reveals that 23S RNA is 2904 $\frac{nt}{RNA}$ long, and makes up 10.6 % of the dry weight of the E. Coli:

Carbon of 23S RNA: $\sim 1.9 \times 10^4 RNA \times 2904\frac{nt}{RNA} \times 10\frac{carbon}{nt} \approx 5.5 \times 10^8 carbon.$

16S RNA make up 5.5% of E. Coli dry weight, compare to 10.6% for 23S RNA. This gives

Carbon of 16S RNA: $\sim \frac{5.5}{10.6} \times 5.5 \times 10^8 \approx 2.9 \times 10^8$carbon.

SS RNA make up 0.4% of E.Coli dry weight, compare to 10.6% for 23S RNA. This gives

Carbon of SS RNA: $\sim \frac{0.5}{10.6} \times 5.5 \times 10^8 \approx 2.6 \times 10^7$carbon.

<u>Transfer RNA (4S)</u> There are 2×10^5transfer RNA, which is about 80 nt long.

Carbon of Transfer RNA: $\sim 2 \times 10^5 tRNA \times 80\frac{nt}{tRNA} \times 10\frac{carbon}{nt} \approx 1.6 \times 10^8 carbon.$

<u>Messenger RNA or mRNA</u> There are 1400 mRNA, which are on average 900 nt long (since an average protein is 300 aa)

Carbon of Transfer RNA: : $\sim 1400 mRNA \times 900\frac{nt}{mRNA} \times 10\frac{carbon}{nt} \approx 1.26 \times 10^7 carbon.$

Total number of Carbon in all RNA is:

$$N_{carbon}^{Rna} = 5.5 \times 10^8 + 2.9 \times 10^8 + 2.6 \times 10^7 + 1.6 \times 10^8 + 1.26 \times 10^7$$

$$N_{carbon}^{Rna} \approx 1.0 \times 10^9 carbon$$

<u>Phospholipids</u> Table 2.1 shows that an E. Coli has about 2.2×10^7 phospholipids. Information on the chemical structure of phospholipid is found on this link: https://alevelnotes.com/notes/biology/biological-molecules/biological-molecules/lipids

From the figure at the bottom let's estimate that there are 20 carbons per lipid: $30\frac{carbon}{lipid}$:

Carbon in lipids: $N_{carbon}^{lipid} = 2.2 \times 10^7 lipids \times 30\frac{carbon}{lipid} = 6.6 \times 10^8 carbon$

liposaccharide Table 2.1 shows that an E. Coli has about 1.2×10^6 liposaccharides. Information on liposaccharide can be found on this link: https://www.sigmaaldrich.com/technical-documents/articles/biology/glycobiology/lipopolysaccharides.html

I found one chemical structure C211H376N8O126P6. Let's assume that one liposaccharide has 200 carbons, $200\frac{carbon}{lipo}$:

Carbon in lipo: $N_{carbon}^{lipo} = 1.2 \times 10^6 lipo \times 200\frac{carbon}{lipid} = 2.4 \times 10^8 carbon$

DNA E. Coli has 2 copies of DNA (Table 2.1), with size of $4.6 \times 10^6 bp$:

Carbon in DNA: $N_{carbon}^{DNA} = 2DNA \times 4.6 \times 10^6 \frac{bp}{DNA} \times 20\frac{carbon}{bp} = 1.9 \times 10^8 carbon.$

In the above one DNA basepair (bp) has 20 carbons, since each nucleotide (nt) has 10 carbons.

GLYCOGEN Table 2.1 states that there are 4360 Glycogen. A glycogen is made of 8 to 12 glucose units, with glucose having the chemical formula $C_6H_{12}O_6$. We will assume that each glycogen has 60 carbons: $N_{carbon}^{Glycogen} = 4360 \times 60 carbons = 2.6 \times 10^5 carbons.$

Murien The structure of the Murien is not clear, but since it constitute only 2.5% of E. Coli, we will just neglect it.

$$N_{carbon}^{EColi} = N_{carbon}^{protein} + N_{carbon}^{RNA} + N_{carbon}^{lipid} + N_{carbon}^{lipo} + N_{carbon}^{DNA} + N_{carbon}^{Glycogen}$$

$$N_{carbon}^{EColi} = 6.7 \times 10^9 carbons \sim 10^{10} carbons$$

This value is similar to that stated in the book.

(b) How many cells can be grown in a 5 mL culture using minimal medium before the medium exhausts the carbon? How many cells can be grown in a 5 mL culture using minimal medium before the medium exhausts the nitrogen? Note that this estimate will be flawed because it neglects the energy cost of synthesizing the macromolecules of the cell. These shortcomings will be addressed in Chapter 5.

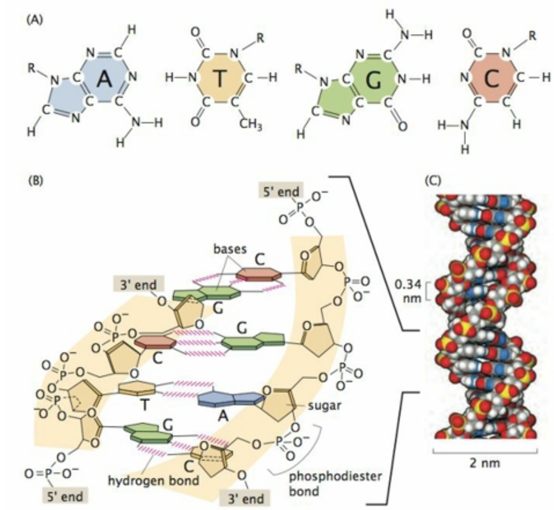
A minimal medium has a mass concentration of 0.5 g/100 mL = $5 \times 10^{-3}\frac{g}{ml}$ of glucose, which has a chemical formula $C_6H_{12}O_6$, with molecular weight $180\frac{g}{mol}$.The number of moles of glucose in a 5 ml minimal medium:

$N_{glucose}^{minimal} = 5 \times 10^{-3}\frac{g}{ml} \times 5ml \div 180\frac{g}{mol} = 1.39 \times 10^{-4} mol$, and since each glucose has 6 carbons, $N_{carbon}^{minimal} = 6 \times 1.39 \times 10^{-4} mol = 8.34 \times 10^{-4} mol$. Using Avogadro's number, $N_A = 6.023 \times 10^{23}\frac{particle}{mol}$, in this case the particle is carbon, which gives:

$N_{carbon}^{minimal} = 8.34 \times 10^{-4} mol \times 6.023 \times 10^{23}\frac{carbon}{mol} = 5 \times 10^{20} carbons.$

From part a) we found that one E. Coli has $\sim 10^{10} carbons$. Hence the 5 ml of minimal medium should be able to supports up to 5×10^{10}E. Coli.

To repeat for nitrogen, use the nucleotide structure belos:



Note that nitrogen are mainly in proteins, RNA and DNA.
Phosphate is mainly in DNA and RNA
https://en.wikipedia.org/wiki/Phosphate

Exercise 5) **<u>Problem 2.8 of Chapter 2</u>**