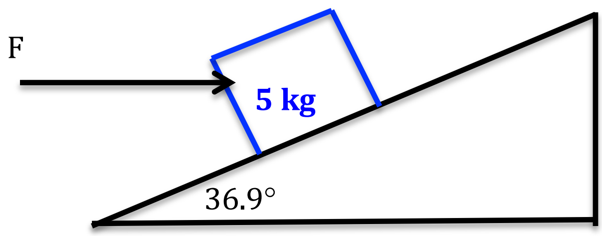
Solution, PHYS 1211, QUIZ 5, October 20, 2017 **17 points total**

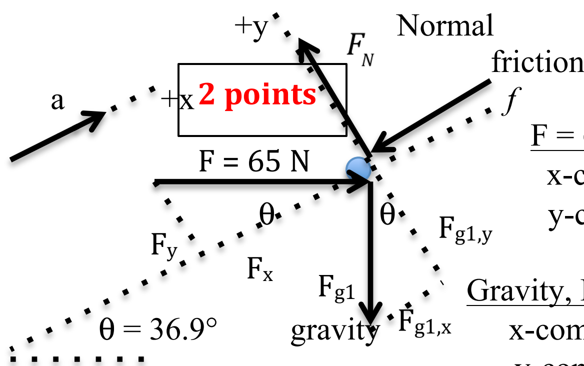
Below, a **5kg box** rests on a 36.9°, with **kinetic friction coefficient**, $\mu_k = 0.15$, and **static coefficient of friction**, $\mu_s = 0.25$, is pushed by a **horizontal force** of F = 65 N



A) Draw a **free body diagram** (FBD) of the **four forces** acting on the box. You will have to assume that the direction of the friction force is either up the incline, or down the incline. Calculate the magnitude normal force, F_N, which point up (+y) **normal** (perpendicular) to the surface. Determine the maximum static friction, $f_{s,\max} = \mu_s F_N$. Calculate the net force and determine whether the box will slide. If the answer is yes determine the **direction** of the slide, and the **acceleration**.

Solution:

Below we define, θ = 36.9°, and **assume** that the **acceleration** will be **up incline**, which means that the **friction**, $\vec{f}$, will be **down** incline. Assume +x is **up incline**.



F = 65N
x-com, F_x = F cos θ = 52 N
y-com, F_y = –F sin θ = –39 N

1 point

Gravity, $F_{g1} = m_1 g$ = 5kg×9.8m•s^{-2} = 49N
x-com, $F_{g1,x}$ = –F_g sin θ = –29.4N
y-com, $F_{g1,y}$ = –F_g sin θ = –39.2N

1 point

In the above, make sure you understand why $F_y < 0$, $F_{g1,x} < 0$ and $F_{g1,y} < 0$, but $F_x > 0$.

y-component $F_y^{Net} = F_N + F_y + F_{g1,y} = F_N - 39N - 39.2N = 0$, or F_N = 78.2N

Friction maximum static, $f_{s,\max} = \mu_s F_N = 0.25 \times 78.2N = 19.55N$

Kinetic $f_k = \mu_k F_N = 0.15 \times 78.2N = 11.73N$.

1 point

x-component $F_x^{Net} = F_x + F_{g1,x} - f = 0 \rightarrow 52N - 29.4N + f = 22.6N - f = 0$, where $-f$ assumes that acceleration is up hill (+x), so friction must be negative (–x).

Since $f < f_{s,max}$ = 19.55N, the 22.66N can overcome the maximum friction, so the box will slide up in up hill (+x) direction.

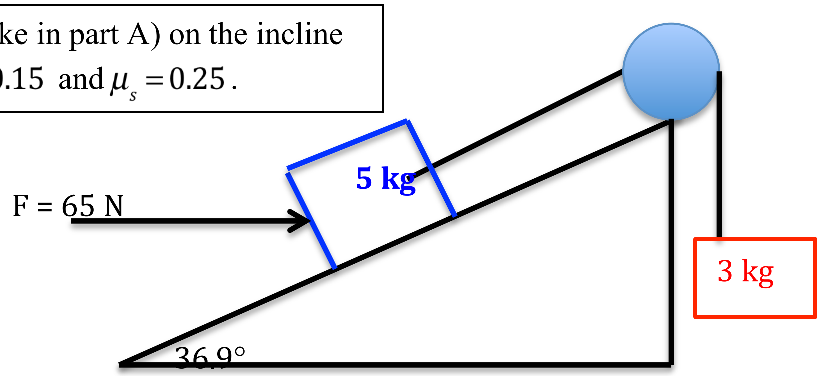
Use second law $F_x^{Net} = F_x + F_{g1,x} - f_k = m_1 a$, where f is replaced by f_k, since the box will slide up, and $F_x + F_{g1,x} - f_k = m_1 a \rightarrow 52N - 29.4N - 11.73N = (5kg)a$, or a = 2.2 m•s^{-2}.

a > 0, since box accelerates uphill (+x).

3 points

B) The **5 kg box** of part A is then attached to an **ideal rope**, and connected to a hanging **3kg box**, through a frictionless pulley system shown below. From your answer in part A, which direction do you think 5 kg will accelerate (up or down) incline. Justify your answer in one sentence. Draw a FBD of the **5 kg box**, and apply Newton's Law, then Draw a FBD of the **3 kg box**, and apply Newton's Law. Solve for the **acceleration**, a, and the **tension**, T, in the rope.

Just like in part A) on the incline $\mu_k = 0.15$ and $\mu_s = 0.25$.

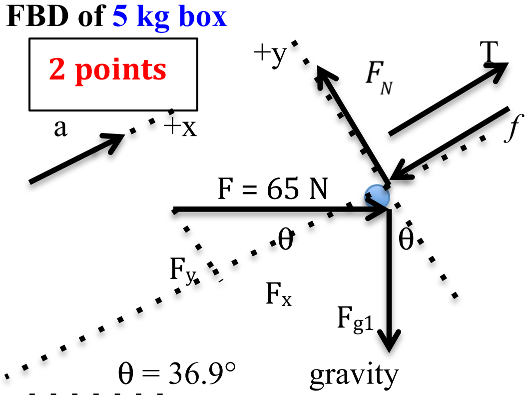


Solution:

Since in part A, it is already sliding up incline, adding a hanging weight will make it slide down even faster.

Below we define, θ = 36.9°, and **assume** that the **acceleration** will be **up incline**, which means that the **friction**, $\vec{f}$, will be **down** incline. Assume +x is **up incline**.

FBD of 5 kg box



There is no change from **part A** for y-component, and hence the normal force is still, F_N = 78.2N, and the kinetic friction is f_k = 11.73N

x-component

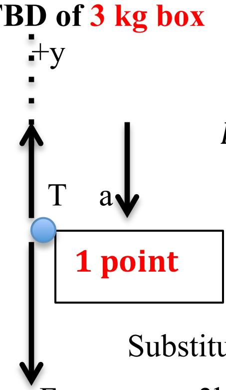
$$F_x^{Net} = F_x + F_{g1,x} - f_k + T = ma$$ **2 points**

Using data from part A)

52N – 29.4N – 11.73N + T = (5kg) a

10.87N + T = (5kg) a (1) **1 point**

FBD of 3 kg box Since acceleration of **5 kg box** is up incline, the acceleration of **3 kg box** must be **down**.



Using Newton's second law

$F_y^{Net} = T - m_2 g = -m_2 a$, where $-m_2a$ indicates the down acceleration

$T - 29.4N = -(3kg)a$ (2) **1 point**

Subtracting equation 1 by 2, (1) – (2)

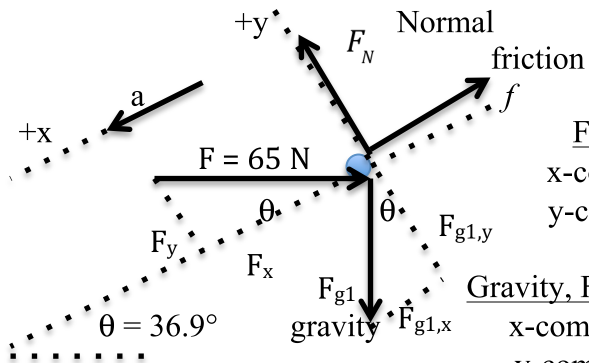
$40.27N = 8kg \times a \rightarrow a = 5.03 m \bullet s^{-2}$ **1 point**

Substituting into (2), $T = 29.4N - (3kg)a = 14.3N$ (or use equation 1)

$F_{g2} = m_2 g = 3kg \times 9.8\ m \bullet s^{-2} = 29.4$ N **1 point**

Alternative Solution to part A:

Below we define, θ = 36.9°, and **assume** that the **acceleration** will be **down** incline, which means that the friction, $\vec{f}$, will be up incline. Assume +x **down**.



F = 65N
x-com, $F_x = -F \cos\theta = -52$ N
y-com, $F_y = -F \sin\theta = -39$ N

Gravity, $F_g = m_1 g = 5\text{kg}\times 9.8\text{m}\bullet\text{s}^{-2} = 49\text{N}$
x-com, $F_{g,x} = F_g \sin\theta = 29.4\text{N}$
y-com, $F_{g,y} = -F_g \sin\theta = -39.2\text{N}$

In the above, make sure you understand why $F_y < 0$, $F_x < 0$ and $F_{g1,y} < 0$, but $F_{g1,x} > 0$.

y-component $F_y^{Net} = F_N + F_y + F_{g1,y} = F_N - 39N - 39.2N = 0$, or $F_N = 78.2\text{N}$

Friction maximum static, $f_{s,\max} = \mu_s F_N = 0.25\times 78.2N = 19.55N$

Kinetic $f_k = \mu_k F_N = 0.15\times 78.2N = 11.73N$.

x-component $F_x^{Net} = F_x + F_{g1,x} - f = 0 \rightarrow -52N + 29.4N - f = -22.6N - f = 0$, where $-f$ assumes that acceleration is downhill (+x), so friction must be negative (–x).
Since $f < f_{s,max}$ = 19.55N, the –22.66N can overcome the maximum friction, so the box will slide up in negative (–x) direction.
Use second law $F_x^{Net} = F_x + F_{g1,x} + f = -m_1 a$, where we replaced $-f$ by $+f_k$, and ma by –ma, since the box will slide up.
This gives $F_x + F_{g1,x} + f_k = -m_1 a \rightarrow -52N + 29.4N + 11.73N = -(5kg)a$, or a = – 2.2 $\text{m}\bullet\text{s}^{-2}$.
a < 0, since box accelerates up (–x).