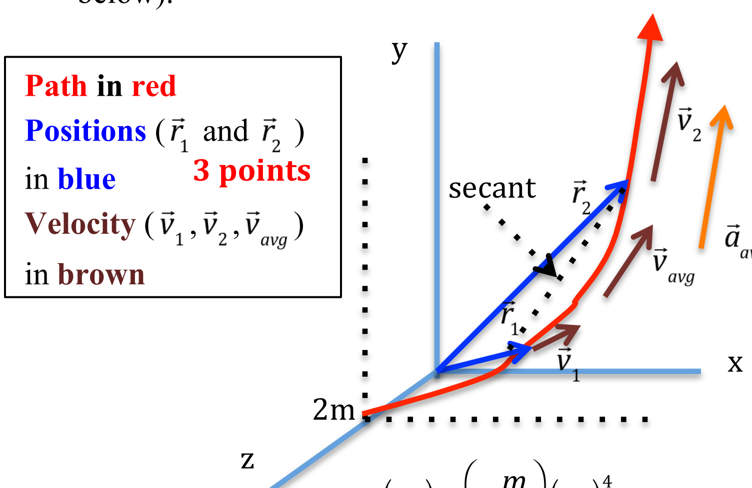
PHYS 1211, QUIZ 3, September 28, 2017

TOTAL 20 POINTS

An object follows a path described by the equation, $\vec{r}(t)=x(t)\hat{i}+y(t)\hat{j}+z(t)\hat{k}$, with $x(t)=\left(1\frac{m}{s^2}\right)t^2$, $y(t)=\left(1\frac{m}{s^4}\right)t^4$, z = 2 m.

A) Plot the path of the object on the **3D Cartesian coordinate** below. Calculate the position at t = 1s, $\vec{r}_1=\vec{r}(t=1s)=x(1s)\hat{i}+y(1s)\hat{j}+z(1s)\hat{k}$, and at t = 2s, $\vec{r}_2=\vec{r}(t=2s)$. Plot the positions on the diagram. Calculate the **average velocity**, $\vec{v}_{avg}$ during the **interval**, and plot on the path.

The x-component of the position is $x=t^2$, and the y-component is $y=t^4=(t^2)^2=x^2$. Hence the path traces a **parabola** (as shown in **red** below).



Position: 2 points

t₁ = 1s, $x_1=x(1s)=\left(1\frac{m}{s^2}\right)(1s)^2=1m$

$y_1=y(1s)=\left(1\frac{m}{s^4}\right)(1s)^4=1m$

$z_1 = z(1s) = 2m$ $\vec{r}_1=1m\hat{i}+1m\hat{i}+2m\hat{i}$

t₂ = 2s, $x_2=x(2s)=\left(1\frac{m}{s^2}\right)(2s)^2=4m$

$y_2=y(2s)=\left(1\frac{m}{s^4}\right)(2s)^4=16m$, $z_2 = z(2s) = 2m$, $\vec{r}_2=4m\hat{i}+16m\hat{i}+2m\hat{i}$

Average velocity: $v_{avg}=\frac{\vec{r}_2-\vec{r}_1}{t_2-t_1}=\frac{x_2-x_1}{t_2-t_1}\hat{i}+\frac{y_2-y_1}{t_2-t_1}\hat{j}+\frac{z_2-z_1}{t_2-t_1}\hat{k}=3\frac{m}{s}\hat{i}+15\frac{m}{s}\hat{j}$. See **brown arrow** that is parallel to the secant above. **2 points**

B) Calculate the **velocity**, $\vec{v}$, at t = 1s and t =2s, and draw it on the above graph. Find the **speed** (magnitude of velocity). Find the average acceleration $\vec{a}_{avg}$ during the interval, and plot of the graph.

$$\vec{v}=\frac{d\vec{r}}{dr}=\frac{dx}{dt}\hat{i}+\frac{dy}{dt}\hat{j}+\frac{dz}{dt}\hat{k},$$

x-component, $x(t)=\left(1\frac{m}{s^2}\right)t^2\rightarrow v_x=\frac{dx}{dt}=\left(2\frac{m}{s^2}\right)t$. **1 point**

y-component, $y(t)=\left(1\frac{m}{s^4}\right)t^4\rightarrow v_y=\frac{dy}{dt}=\left(4\frac{m}{s^4}\right)t^3$. **1 point** z-component, $z(t) = 2m \rightarrow v_z = \frac{dz}{dt} = 0$. **1 point**

$$\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k} = \left(2\frac{m}{s^2}\right)t\hat{i} + \left(4\frac{m}{s^4}\right)t^3\hat{j}$$

t = 1s, $\vec{v}_1 = \left(2\frac{m}{s^2}\right)(1s)\hat{i} + \left(4\frac{m}{s^4}\right)(1s)^3\hat{j} = 2\frac{m}{s}\hat{i} + 4\frac{m}{s}\hat{j}$, **1 point**

t = 2s, $\vec{v}_2 = \left(2\frac{m}{s^2}\right)(2s)\hat{i} + \left(4\frac{m}{s^4}\right)(2s)^3\hat{j} = 4\frac{m}{s}\hat{i} + 32\frac{m}{s}\hat{j}$. **1 point**

The velocities are plotted as above (brown arrows)

$v = |\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$; $v_1 = \sqrt{\left(2\frac{m}{s}\right)^2 + \left(4\frac{m}{s}\right)^2} = 4.47\frac{m}{s}$;

$v_2 = \sqrt{\left(4\frac{m}{s}\right)^2 + \left(32\frac{m}{s}\right)^2} = 32.2\frac{m}{s}$. **1 point**

Average Acceleration $\vec{a}_{avg} = \frac{v_{2x} - v_{1x}}{t_2 - t_1}\hat{i} + \frac{v_{2y} - v_{1y}}{t_2 - t_1}\hat{j} + \frac{v_{2z} - v_{1z}}{t_2 - t_1}\hat{k}$.

$$\vec{a}_{avg} = \frac{4m\bullet s^{-1} - 2m\bullet s^{-1}}{2s - 1s}\hat{i} + \frac{32m\bullet s^{-1} - 4m\bullet s^{-1}}{2s - 1s}\hat{j} + \frac{0 - 0}{t_2 - t_1}\hat{k}$$

$\vec{a}_{avg} = 2\frac{m}{s^2}\hat{i} + 28\frac{m}{s^2}\hat{j}$. $\vec{a}_{avg}$ is the orange arrow above. **3 point**

Note that $a_{avg} = |\vec{a}_{avg}| = 28\frac{m}{s^2}$, and angle wrt +x in x-y plane is $\theta = \tan^{-1}\frac{28}{2} = 86^\circ$.

For the $\vec{v}_{avg}$ in same interval, $v_{avg} = 15.3m\bullet s^{-1}$, and $\theta = 78^\circ$. That's why $\vec{a}_{avg}$ is drawn longer and steeper than $\vec{v}_{avg}$.

C) Find acceleration, $\vec{a}$, at t = 1.5 s. Calculate the **magnitude** of the **acceleration**.

$\vec{a} = \frac{d\vec{v}}{dr} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k}$, $v_x = \left(2\frac{m}{s^2}\right)t \rightarrow a_x = \frac{dv_x}{dt} = 2\frac{m}{s^2}$;

$v_y = \left(4\frac{m}{s^4}\right)t^3 \rightarrow a_y = \frac{dv_y}{dt} = 12\frac{m}{s^4}t^2$, and $v_z = 0 \rightarrow a_z = \frac{dv_z}{dt} = 0$

$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k} = \left(2\frac{m}{s^2}\right)\hat{i} + \left(12\frac{m}{s4}\right)t^2\hat{j}$ **2 points**

At t = 1.5s, $\vec{a} = \left(2\frac{m}{s^2}\right)\hat{i} + \left(12\frac{m}{s^4}\right)(1.5s)^2\hat{j} = \left(2\frac{m}{s^2}\right)\hat{i} + 29\frac{m}{s^2}\hat{j}$. **1 point**

Magnitude $a = |\vec{a}| = \sqrt{\left(2m\bullet s^{-2}\right)^2 + \left(27m\bullet s^{-2}\right)} = 27m\bullet s^{-2}$ **1 point**