FALL TERM EXAM, PHYS 1211, INTRODUCTORY PHYSICS I
Tuesday, 10 December 2019, 9 AM to NOON, Field House Gym

NAME: ________________________________

STUDENT ID: _______________________

INSTRUCTION

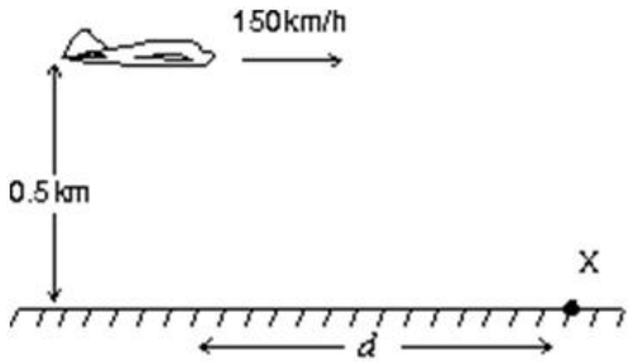
1. This exam booklet has 12 pages. Make sure none are missing
2. There is an equation sheet on page 12. You may tear the equation sheet off.
3. There are two parts to the exam:
    - Part I has twelve multiple choice questions (1 to 12), where you must circle the one correct answer (A, B, C, D, E). Rough work can be done on the backside of the sheet opposite the question page
    - Part II includes **eight** full-answer questions (13 to 20). Do **all eight** questions. All work must be done on the blank space below the questions. If you run out of space, you may write on the backside of the sheet opposite the question page.
4. **Non-Programmable calculators** are **allowed**
5. **Programmable calculators** are **NOT ALLOWED**.

PART I: MULTIPLE CHOICE QUESTIONS (question 1 to 12)

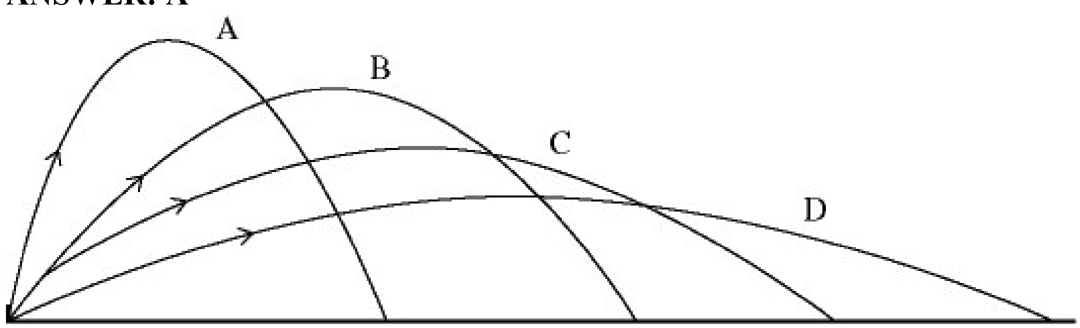
For each question **circle** the **one correct answer** (A, B, C , D or E).

1. (**2.5 point**) The airplane shown on the right, is in level flight at an altitude of 0.50 km and a speed of 150 km/h. At what distance d should it release a heavy bomb to hit the target X? Take $g = 10$ m/s^2.
   A) 150m; **B)** 295 m;

   C) 417 m; **D)** 1500 m;
   E) 15000 m. **ANSWER: C**,



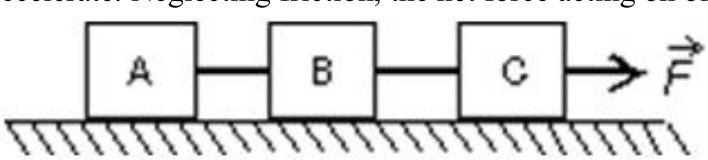
2. (**2.5 point**) The figure below shows trajectories of four artillery shells. Each fired with the same initial speed. Which trajectory remains in the **air** for the **longest time**? Circle the right answer. **Hint**: ask yourself how to throw a ball so that it remains in the air for the longest.
   A) **B)** **C)** **D)** **E)** All were in the air for the same amount of time.
   ANSWER: A



3. (**2.5 point**) A 5.0-kg crate is resting on a horizontal plank. The coefficient of static friction is 0.50 and the coefficient of kinetic friction is 0.40. After one end of the plank is raised so the plank makes an angle of 30° with the horizontal, the force of friction is:
   A) 0N **B)** 17N **C)** 20 N **D)** 25N **E)** 49N

ANSWER: B

4. (**2.5 point**) Three blocks (A, B, C), each having the same mass *M*, are connected by strings as shown. Block C is pulled to the right by a force that causes the entire system to accelerate. Neglecting friction, the net force acting on block B is:



   A) 0; **B)** $\vec{F}/3$; **C)** $\vec{F}/2$;

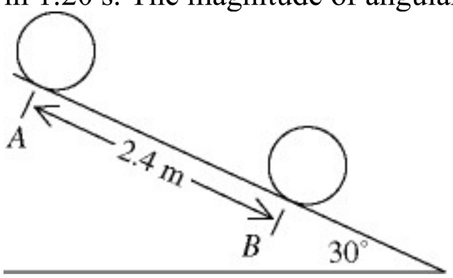
   D) $2\vec{F}/3$; **E)** $\vec{F}$ **ANSWER: B**

5. (**2.5 point**) A 100-kg piano rolls down a 20° incline. A man tries to keep it from accelerating, and manages to keep its acceleration to 1.2 m/s^2. If the piano rolls 5 m, what is the **net work done** on it by **all** the **forces** acting on it?
   A) 60J **B)** 100J **C)** 600J **D)** 1000J **E)** 490 J **ANSWER: C**
6. (**2.5 point**) The work done by gravity during the **descent** of a **projectile** is:
   A) positive **B)** negative **C)** zero **D)** depends on the direction of the *y* axis
   E) depends on the direction of both the *x* and *y* axis **ANSWER: A**

7. (**2.5 point**) A force of 10 N holds an ideal spring with a 20-N/m spring constant in compression. The potential energy stored in the spring is:
   A) 0.5J **B)** 2.5J **C)** 5J **D)** 10J **E)** 200J **ANSWER: B**
8. (**2.5 points**) Two boys with masses of 40 kg and 60 kg stand on a horizontal frictionless surface holding the ends of a light 10-m long rod. The boys pull themselves together along the rod. When they meet the 40-kg boy will have moved what distance?
   A) 4 m **B)** 5m **C)** 6m **D)** 10m **E)** Need to know the forces they exert

ANSWER: C, Both boys will end up at the COM.

9. (**2.5 points**) A 0.3 kg rubber ball is dropped from the window of a building. It strikes the sidewalk below at 30 m/s and rebounds up at 20 m/s. The **magnitude** of the **impulse** due to the collision with the sidewalk is:
   ANSWER: D, $\Delta p = (0.3kg \times 20m/s) - (-0.3kg \times 30m/s) = 15 \text{ kg} \cdot \text{m/s}$
   A) 3.0 kg·m/s **B)** 6.0 kg·m/s **C)** 9.0 kg·m/s **D)** 15 kg·m/s **E)** 29 kg·m/s
10. (**2.5 points**) Below, the radius of a 3.0-kg wheel is 6.0 cm. The wheel is **released** from **rest** at point A on a 30 incline. The wheel rolls **without slipping** and moves 2.4 m to point B in 1.20 s. The magnitude of angular acceleration of the wheel is closest to:



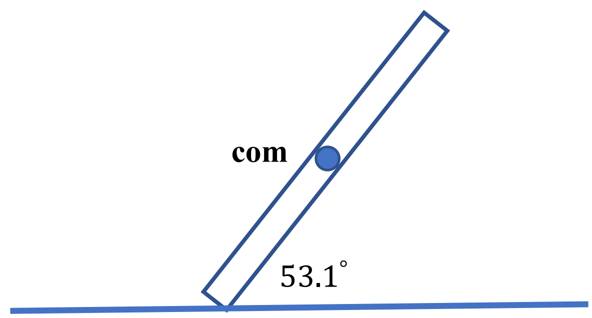
A) $56\frac{rad}{s^2}$; **B)** $48\frac{rad}{s^2}$; **C)** $65\frac{rad}{s^2}$

D) $73\frac{rad}{s^2}$; **E)** $82\frac{rad}{s^2}$

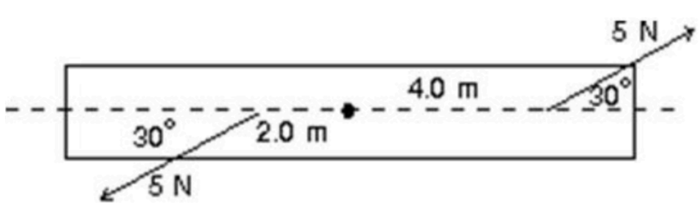
$2.4m = 0.5at^2 \rightarrow a = 3.33m \cdot s^{-2}$

$\alpha = \frac{a}{R} = 56\frac{m}{s^2}$. **ANSWER: A**

11. (**2.5 points**) The figure on the right shows a **meter stick** is held at angle, 53.1°to the horizontal with one end on the floor. It is then allowed to fall. Assuming that the end on the floor does **not slip**, the speed of the **other end** just before it hits the floor is **closest** to:



   A) $7.7\frac{m}{s}$; **B)** $5.42\frac{m}{s}$; **C)** $4.84\frac{m}{s}$
   D) $4.2\frac{m}{s}$ **E)** $4.43\frac{m}{s}$, **ANSWER: C**
   $Mg(L/2)sin53.1^\circ = (1/2)(ML^2/3)\omega^2$, with $v = \omega L$, L = 1m, v = 4.84 m/s
   Hint: Consider the stick to be a thin rod, $I = \frac{ML^2}{3}$ and use the conservation of energy.
12. (**2.5 points**) A rod is pivoted about its center. A 5-N force is applied 4 m from the pivot and another 5-N force is applied 2m from the pivot as shown. The **magnitude** of the **total torque** about the **pivot** is:



A) 0 N•m B) 5 N•m

C) 8.7 N•m D) 15 N•m

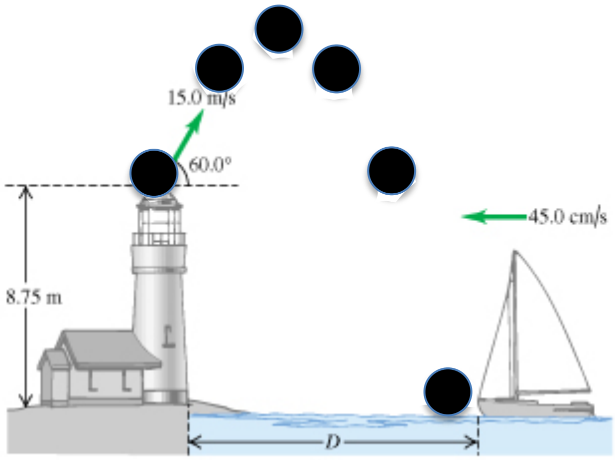
E) 26 N•m

Take CCW as +: $\tau^{net} = 5N \times (4m \times sin30^\circ) - 5N \times (2m \times sin30^\circ) = 5N \cdot m$
ANSWER: B

PART II: FULL ANSWER QUESTIONS (question 13 to 20)

Do **all eight** questions on the provided area below the questions. Show all work.

13. (**10 points)** In the figure on the right, a ship approaches the dock at 45 cm/s. An important piece of **equipment** is thrown from the top of a tower to the ship at 15.0 m/s at 60°above the horizontal. The top of the tower is 8.75m above water level.



A) Draw a motion diagram of the trajectory of the equipment. Calculate the x- and y-component of the initial velocity. See Above right (**1 point**)

$v_{0x} = 15\frac{m}{s} \times cos60^{\circ} = 7.5\frac{m}{s}$;

$v_{0y} = 15\frac{m}{s} \times sin60^{\circ} = 13\frac{m}{s}$ (**2 points**)

B) Calculate the time it takes for the equipment to reach water level (8.75 m below). The answer is t = 3.2 s, but **sufficient detail** is required for full marks. Calculate the **range** of the **equipment**. To find the time it takes to reach the bottom of the cliff look at the **vertical** (y-axis) **component**:

Use $y = y_0 + v_{0y}t - \frac{1}{2}gt^2 \rightarrow -8.75m = 0 + 13\frac{m}{s} \times t - 4.9\frac{m}{s^2} \times t^2$

$$4.9t^2 - 13t - 8.75 = 0 \rightarrow t = \frac{13 \pm \sqrt{13^2 + 4 \times -4.9 \times -8.75}}{9.8} \rightarrow t = 3.21s, -0.556s$$

Physical solution is t = 3.21s. (**4 points**)

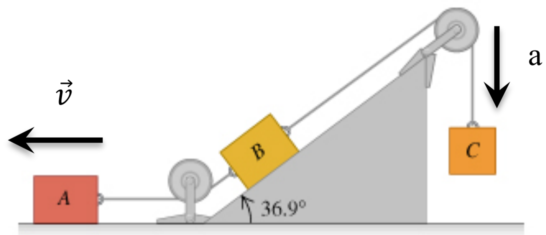
The **horizontal displacement of the equipment** is found by looking at the **horizontal** (x-axis) **component**, where the velocity is constant $v_{0x} = 7.5\frac{m}{s}$ and

$Range = x = v_{0x}t = 7.5\frac{m}{s} \times 3.21s = 24.07m$ (**2 points**)

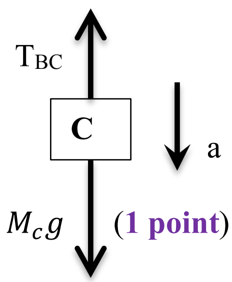
C) For the equipment to land at the front of the ship, at what distance D from the dock should the ship be when the equipment is thrown?

During the t = 3.21 s period it takes the LP to reach the surface of the lake, the boat must travels a distance $x_B = v_{0B}t = 0.45\frac{m}{s} \times 3.21s = 1.44m$ towards the lighthouse. Using the above diagram:$D = x_B + Range = 25.5m$. (**1 point**)

14. (**10 points**) On the right, **Box A**, is on a floor with **kinetic friction coefficient**, $\mu_k = 0.15$, and **static coefficient of friction**, $\mu_s = 0.25$. **Box A** is connected to a frictionless pulley system to **Box B**, M_B = 3 kg, on a 36.9° incline, with **no friction**. **Box B**, is connected by a frictionless pulley system, to **Box C**, M_C = 10 kg, as shown. Assume that **Box A** is **moving left** as shown in the diagram. The **acceleration** of **Box C** is $a = 5.55\frac{m}{s^2}$ **down**, as shown in diagram.



A) Draw a Free-Body Diagram (FBD) on **Box C**. Use Newton's law to find the tension in the rope connecting **Box C** and **Box B**, T_{BC}.



(**1 point**)

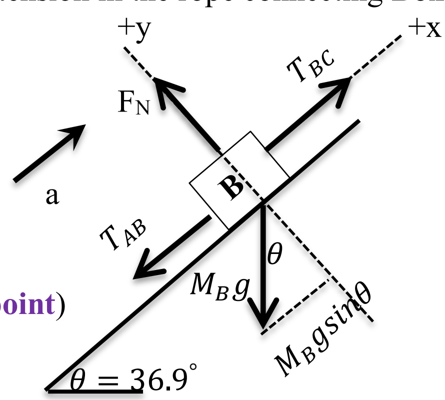
Newton's second Law

$$F_{net,y} = T_{BC} - M_C g = -M_C a$$

$$T_{BC} = M_C g - M_C a = 10kg \times 9.8\frac{m}{s^2} - 10kg \times 5.55\frac{m}{s^2}$$

$$T_{BC} = 42.5N$$ (**2 points**)

B) Draw a **free-body** diagram of all the forces acting on **Box B**. Use Newton's law to find the tension in the rope connecting **Box B** and **Box A**, T_{AB}.



(**1 point**)

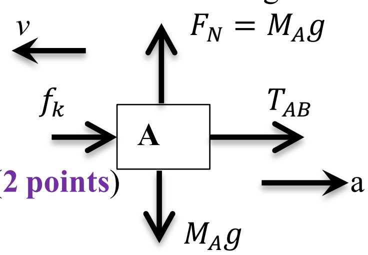
Newton's second law in x-component

$$F_{net,y} = T_{BC} - M_B g sin\theta - T_{AB} = M_B a$$

$$T_{AB} = 42.5N - 3kg \times 9.8\frac{m}{s^2} \times 0.6 - 3kg \times 5.55\frac{m}{s^2}$$

$$T_{AB} = 8.21N$$ (**2 points**)

C) Draw **FBD** of all forces acting on **Box A**. Use Newton's law to **find** the **mass** of box A, M_A.



(**2 points**)

Newton's second law

x-comp, $T_{AB} + f_k = M_A a$

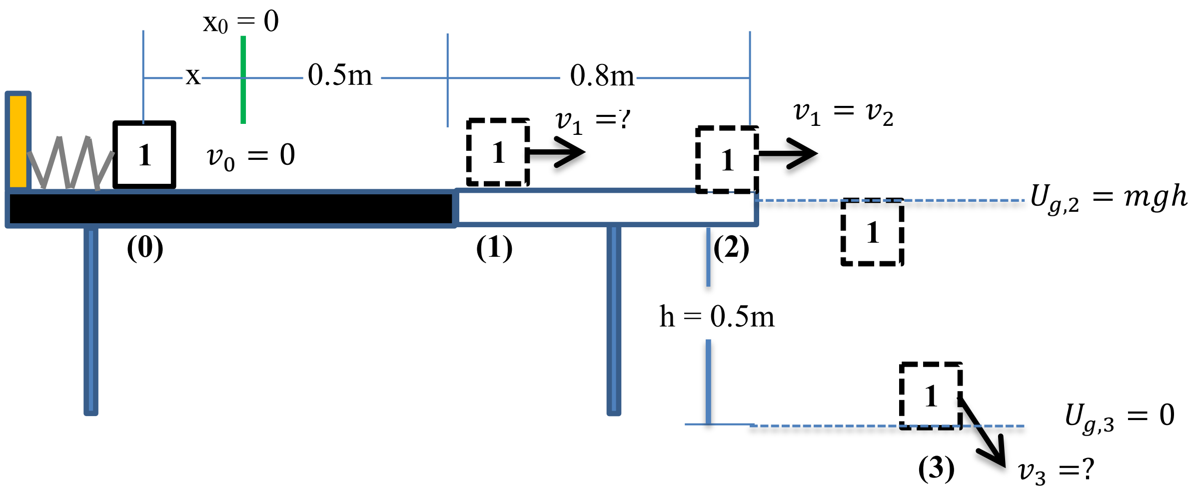
Friction, $f_k = M_A g \mu_k$

Combining, $T_{AB} + M_A g \mu_k = M_A a$

Solving, $M_A = \frac{T_{AB}}{a - g\mu_k} = \frac{8.21N}{5.55 m \cdot s^{-2} - 9.8 m \cdot s^{-2} \times .15} = 2kg$

(**2 points**)

15. (**10 points**) In the Figure below, **Box 1** (m_1 = 1.5 kg) is on a table. An unknown human **compresses** the box and spring ($k = 400\ N \cdot m^{-1}$) by x = 15 cm from **equilibrium** (indicated by the **line** with $x_0 = 0$). The **darkened portion** of the table has **friction** ($\mu_k = 0.3$ and $\mu_s = 0.55$), while the clear portion is **ice** (assumed **frictionless**). The dimension of the table is indicated in the diagram. The human then **releases** Box 1 (i.e. the box and spring is now allowed to move).



A) Use Hooke's law $F = -kx$ show that once box one is released the force of the spring will overcome static friction and **Box 1** will move.

$$F = -kx = -400\ N \cdot m^{-1} \times 0.15m = -60N$$

Maximum static friction, $f_{s,max} = m_1 g \mu_s = 8.1N$

Since 60N > 8.1 N, the spring force will overcome the static friction, and the box will move.

(**2 points**)

B) Find the **speed** of box 1 when it reaches the **ice**. **Hint:** Use Conservation of Energy.

Use conservation of energy, $W_{ext} = \Delta E_{mec} + E_{th} = \Delta K + \Delta U + f_k d$, where d = x+ 0.5m, or d = 0.65 m. $\Delta K = K_1 - K_0 = \frac{1}{2}m_1 v_1^2 - 0 = \frac{1}{2}m_1 v_1^2$; $\Delta U = U_1 - U_0 = 0 - \frac{1}{2}kx^2 = -\frac{1}{2}kx^2$. This gives $\frac{1}{2}m_1 v_1^2 - \frac{1}{2}kx^2 + f_k d = 0$, with

$f_k = m_1 g \mu_k = 1.5kg \times 9.8\frac{m}{s^2} \times 0.3 = 4.41N \rightarrow f_k d = 2.87J$

$$\frac{1}{2}m_1 v_1^2 - \frac{1}{2}kx^2 + f_k d \rightarrow v_1 = \sqrt{\frac{k}{m_1}x^2 - \frac{2f_k d}{m_1}}$$

$v_1 = \sqrt{\frac{400\ N \cdot m^{-1}}{1.5kg}(0.15m)^2 - \frac{2 \times 2.87J}{1.5kg}} = 1.47\frac{m}{s}$ (**5 points**)

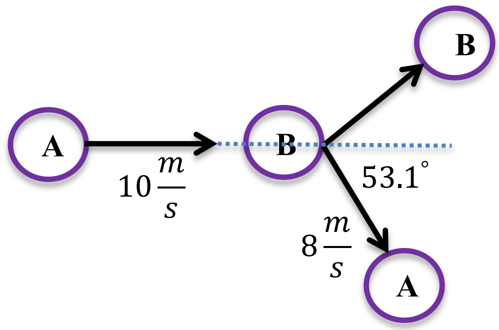
C) Find the **speed** of box 1 just before it hits the ground. **Hint:** Use Conservation of Mechanical Energy.

	KE	PE	KE	PE

Initial (2) $\frac{1}{2}m_1 v_2^2 + m_1 gh = \frac{1}{2}m_1 v_3^2 + 0$

$$v_3 = \sqrt{v_2^2 + 2m_1 gh} = \sqrt{\left(1.47\frac{m}{s}\right)^2 + 2 \times 9.8\frac{m}{s^2} \times 0.5m} = 3.45\frac{m}{s}$$ (**3 points**)

16. (**10 points**) A hockey **puck A** moves right at $10\frac{m}{s}$ on a **frictionless** ice surface strikes a second **puck B**, which is at rest. After the collision **puck A** moves off at speed of $8\frac{m}{s}$, at angle of 53.1°, **below** the **horizontal**. **Puck A** has a mass of $M_A = 2.0$ kg, and **puck B** has a mass of $M_B = 2.0$ kg.



A) Use conservation of momentum to find the x- and y- component of the **velocity** of **Puck B**. Find the **speed** of **Puck B**, after the collision.
Conservation of Momentum
x-comp, $M_A v_{A0} = M_A v_{A1} cos53.1^\circ + M_B v_{B1,x}$, with $M_A = M_B$

$$v_{A0} = v_{A1} cos53.1^\circ + v_{B1,x} \rightarrow v_{B1,x} = 10\frac{m}{s} - 8\frac{m}{s} \times 0.6 = 5.2\frac{m}{s}$$

y-comp, $0 = -M_A v_{A1} sin53.1^\circ + M_B v_{B1,y}$, with $M_A = M_B$

$$0 = -v_{A1} sin53.1^\circ + v_{B1,y} \rightarrow v_{B1,y} = 8\frac{m}{s} \times 0.8 = 6.4\frac{m}{s}$$

Speed: $v_{B1} = \sqrt{(v_{B1,x})^2 + (v_{B1,y})^2} = \sqrt{\left(5.2\frac{m}{s}\right)^2 + \left(6.4\frac{m}{s}\right)^2} = 8.24\frac{m}{s}$

(**4 points**)

B) Calculate the **change** of **kinetic energy** due to collision. Is the collision **elastic**? Why?

$$\Delta K = \left(\frac{1}{2}M_A v_{A1}^2 + \frac{1}{2}M_B v_{B1}^2\right) - \frac{1}{2}M_A v_{A0}^2$$

$$\Delta K = \left(\frac{1}{2}2kg \times \left(8\frac{m}{s}\right)^2 + \frac{1}{2}2kg \times \left(8.24\frac{m}{s}\right)^2\right) - \frac{1}{2}2kg \times \left(10\frac{m}{s}\right)^2 = 31.8J$$

This is not an elastic collision since $\Delta K \neq 0$.

(**2 points**)

C) Calculate the Impulse of Puck B, $\vec{J}_B$, due to the collision.

$$\vec{J}_B = \Delta\vec{P} = \vec{P}_{B,1} - \vec{P}_{B,0} = M_B \vec{v}_1 - 0 = 2kg \times (v_{B1,x}\hat{\imath} + v_{B1,y}\hat{\jmath})$$

$$\vec{J}_B = 10.4\frac{kg \cdot m}{s}\hat{\imath} + 12.8\frac{kg \cdot m}{s}$$

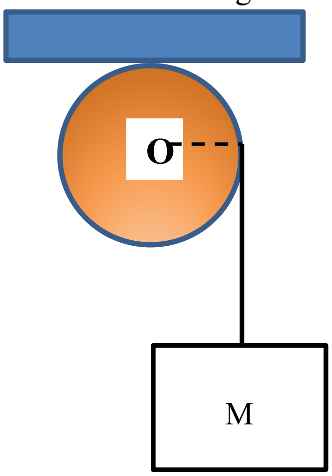
(**2 points**)

D) Find the Impulse of Puck A, $\vec{J}_A$, due to the collision. **Hint:** Use result of part C

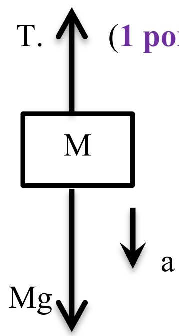
$$\vec{J}_A = -\vec{J}_B = -10.4\frac{kg \cdot m}{s}\hat{\imath} - 12.8\frac{kg \cdot m}{s}$$

(**2 points**)

17. (**10 points**) Box of Mass M = 4 kg hangs from a rope attached to a pulley of moment of Inertia, $I = 0.01 kg \cdot m^2$, and radius R = 10 cm. When the system is released from rest the box moves down, and the rope rotates the pulley about its center axis, indicated by **O**. Assume the **rope does not slip** on the **pulley**. The pulley is a solid cylinder $I = \frac{1}{2} m_p R^2$ of mass m = 2 kg.



A) Draw a free body diagram of all the forces acting on the Box. Use Newton's law to write an equation that is a function of the tension of the rope, T, and the acceleration, a, of the box.

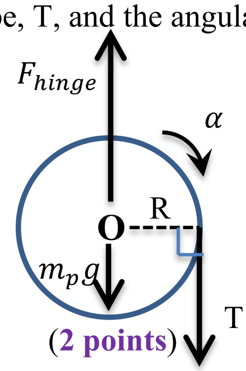


(**1 point**)

$F_{net,y} = T - Mg = -Ma$ **(1)**

(**1 point**)

B) Draw FBD of all the forces on the pulley (including the Hinge force, F_{Hinge}, on the pulley). Use Newton's law to write an equation that is a function of the tension of the rope, T, and the angular acceleration, α, of the pulley.



(**2 points**)

Take CW as positive (+)

$m_p g$ and F_{hinge} apply no torque since they pass through the axis of rotation at O.

Only T can apply torque, since it has a moment arm of R.

Second law of Newton for Rotation

$\tau_{net} = TR = I\alpha = \frac{1}{2} m_p R^2 \alpha$

$T = \frac{1}{2} m_p R\alpha$ **(2)** (**2points**)

C) Use the **no-slip condition** to solve the equations derived in part A and B, to find T, a, and α.

No-slip condition is $R\alpha = a$, **(3)**, which when substitute into (2) gives

$T = \frac{1}{2} m_p a$ **(4)**

Substitute (4) into (1) gives $\frac{1}{2} m_p a \quad - Mg = -Ma$

$a = \frac{Mg}{\frac{1}{2} m_p + M} = \frac{4kg \times 9.8\frac{m}{s^2}}{\frac{1}{2} 2kg + 4kg} = 7.84\frac{m}{s^2}$

Subs a into (4), $T = \frac{1}{2} 2kg \times 7.84\frac{m}{s^2} = 7.84N$

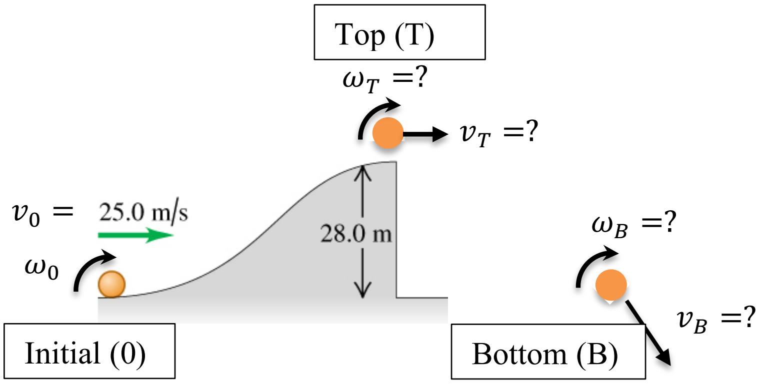
Using (3), $\alpha = \frac{7.84\frac{m}{s^2}}{0.1m} = 78.4\frac{rad}{s^2}$ (**3 points**)

D) Use the fact that the **net force** on the **pulley** must be **zero** to find the force of the hinge on pulley, F_{Hinge}. **Hint:** Hinge force must cancel tension and the pulley's weight.

Using the diagram of part B: $F_{net,y} = F_{hinge} - T - m_p g = 0$

$F_{hinge} = 7.84N + 2kg \times 9.8\frac{m}{s^2} = 27.44N.$ (**1 point**)

18. (**10 points**) In the diagram below, a solid uniform ball ($I = \frac{2}{5}MR^2$, R = 0.15 m) **rolls without slipping** with a linear speed of 25.0 m/s. It moves up the hill, and goes over a 28 meter high cliff.



A) Calculate the linear, v_T, and rotational, ω_T, speed at the top of the hill.
Use conservation of mechanical energy
Initial(0) $\frac{1}{2}Mv_0^2 + \frac{1}{2}I\omega_0^2 = \frac{1}{2}Mv_T^2 + \frac{1}{2}I\omega_T^2 + Mgh$ Final (T), (**2 points**)
with h = 28 m
Use $I = \frac{2}{5}MR^2 \rightarrow \frac{1}{2}Mv_0^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\omega_0^2 = \frac{1}{2}Mv_T^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\omega_T^2 + Mg$
Canceling the M, $\frac{1}{2}v_0^2 + \frac{1}{2}\left(\frac{2}{5}R^2\right)\omega_0^2 = \frac{1}{2}v_T^2 + \frac{1}{2}\left(\frac{2}{5}R^2\right)\omega_T^2 + gh$
Use no-slip, $v_0 = \omega_0 R$ and $v_T = \omega_T R$
$\frac{1}{2}v_0^2 + \frac{1}{5}v_0^2 = \frac{1}{2}v_T^2 + \frac{1}{5}v_T^2 + gh \rightarrow \frac{7}{10}v_0^2 = \frac{7}{10}v_T^2 + gh$, (**2 points**)
$v_T = \sqrt{v_0^2 + \frac{10}{7}gh} = \sqrt{\left(25\frac{m}{s}\right)^2 - \frac{10}{7} \times 9.8\frac{m}{s^2} \times 28m} = 15.26\frac{m}{s}$, (**1 point**)
$\omega_T = \frac{v_T}{R} = \frac{15.26\frac{m}{s}}{0.15m} = 101.73\frac{rad}{s}$ (**1 point**)

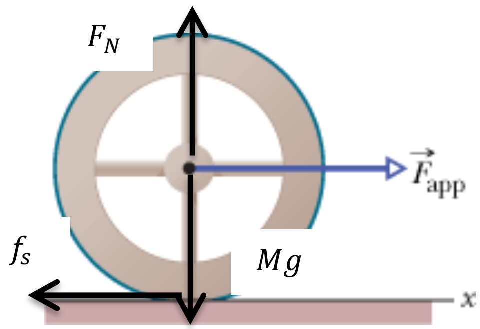
B) Calculate the linear, v_B, and rotational, ω_B, speed just before it lands on the ground.
Initial(T) $\frac{1}{2}Mv_T^2 + \frac{1}{2}I\omega_T^2 + Mgh = \frac{1}{2}Mv_B^2 + \frac{1}{2}I\omega_B^2$ Bottom (B)
With h = 28m. Once in the air, there's no torque to change the rotational speed, hence $\omega_T = \omega_B = 101.73\frac{rad}{s}$, which gives $\frac{1}{2}Mv_T^2 + Mgh = \frac{1}{2}Mv_B^2$ (**1 point**)
$v_B = \sqrt{v_T^2 + 2gh} = \sqrt{\left(15.26\frac{m}{s}\right)^2 + 2 \times 9.8\frac{m}{s^2} \times 28m} = 27.9\frac{m}{s}$, (**2 points**)

C) In part b, if you did the question correctly, the final linear speed of the ball should be greater than the initial speed (25 m/s). Does the fact that this speed is greater than the initial linear speed means that the ball gains energy?
No! The kinetic energy at position (0) is the same as at position (B): $K_{Linear,0} + K_{Rot,0} = K_{Linear,B} + K_{Rot,B}$. But at the bottom the linear kinetic energy is greater $K_{Linear,B} > K_{Linear,0}$ and the rotational energy is less, $K_{rot,B} < K_{Rot,0}$. (**1 point**)

Long Answer

It is noted that the linear speed at position B ($v_B = 27.9\frac{m}{s}$) is greater than at position 0 ($v_0 = 25\frac{m}{s}$), even though they are at the same height. This is explained by the fact that the kinetic energy includes rotational component. When the ball roll up the hill from 0 to T it gains PE of mgh and it loses mgh in KE, and since there is no slipping both the linear and rotational component lose energy so that $v_1 > v_2$ and $\omega_1 > \omega_2$. When it falls off the cliff from T to B it regains mgh in KE, but the rotational component cannot change so that $\omega_T = \omega_B$, meaning that the linear component regains the whole mgh so that $v_B > v_0$ but also $\omega_T = \omega_B < \omega_0$, so the rotational energy at position B is less than at 0. However it is noted that the total kinetic energy (rotational + linear) are the same at position 0 and B.

19. (**10 points**) In the figure below, an applied force $\vec{F}_{app}$ to the **right** is applied to the **center** of **mass** (**COM**) of a wheel. The wheel will rotate **clockwise** (**CW**) with an angular acceleration (α). At the same time the **COM** of wheel will **move right without slipping**.



Data on wheel: **mass** M = 2 kg;
Radius, R = 0.25 m
Moment of **inertia**, I = 0.0625 kg•m^2,

Friction, $\mu_k = 0.11$ and $\mu_s = 0.44$

A) Draw a **free-body diagram (FBD)** of **all** the **forces** acting on the **wheel**. **Briefly justify** the **direction** of the force of **friction**, $\vec{f}_s$, on the bottom of the wheel (**left** or **right**). **Briefly explain** why the force of friction is static, $\vec{f}_s$.

FBD above shows, normal force F_N, weight Mg, and f_s is the force of friction.
Since F_N and Mg passes through the com, they exert no torque. Only friction can apply a torque.
Since the wheel must rotate CW, the force of friction must be left to induce CW rotation.
If the wheel does not slip, the point of contact between the wheel and the ground must be stationary, and the force of friction is static, $\vec{f}_s$. (**4 points**)

B) The applied force is F_{app} = 10N. Use the **FBD** above, and Newton's **second law** for **translation** (linear motion) and **rotation**, to calculate the **linear**, a, and **angular** α, **acceleration**, and the **friction force**, f_s. **HINT:** The Newton's laws should give you **two equations** for **three unknowns**, a, α and f_s. Use the **no-slip** condition to eliminate **one** of the **unknowns**, then solve.

Newton's law for translation in the x-comp gives, $F_{app} - f_s = Ma$, **[1]**. (**1.5 points**)

Newton's law for rotation gives, $\tau_{net} = f_s R = I\alpha \rightarrow f_s = \frac{I}{R^2}(R\alpha) = \frac{I}{R^2}a$ [2] (**1.5 points**)

where we used the no-slip condition, $a = R\alpha$

Subs [2] into [1], $F_{app} - \frac{I}{R^2}a = Ma \rightarrow a = \frac{10N}{2kg + \frac{0.0625\ \text{kg•m2}}{(0.25m)^2}} = 3.33\frac{m}{s^2}$

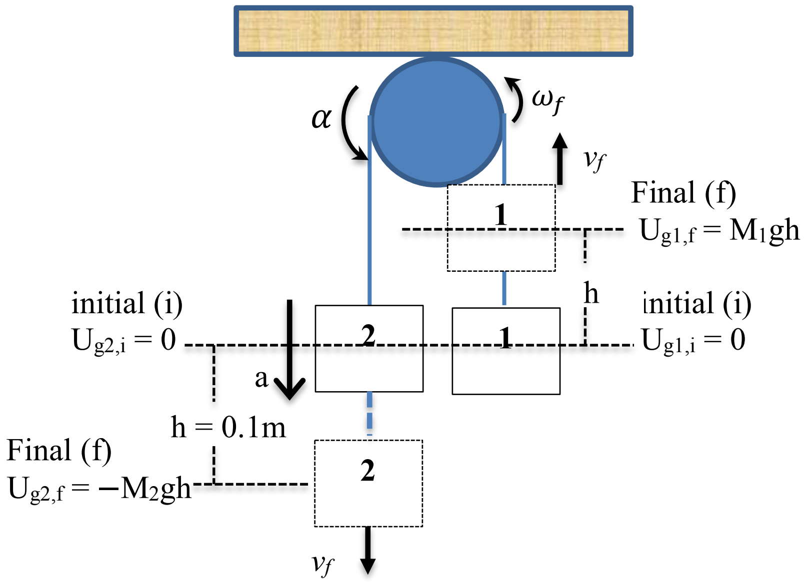
Using [2], $f_s = \frac{I}{R^2}a = \frac{0.0625\ \text{kg•m2}}{(0.25m)^2} \times 3.33\frac{m}{s^2} = 3.33N$ left, or $\vec{f}_s = -3.33N\hat{\imath}$.

$\alpha = \frac{a}{R} = \frac{3.33\frac{m}{s^2}}{0.25m} = 13.33\frac{rad}{s^2}$ (**2 points**)

C) By direct calculation, show that the friction between the floor and the wheel is sufficient to **prevent slipping**, so that the **friction** is **static**.

$f_{s,max} = Mg\mu_s = 8.62 > 3.33N = f_s$, and there will be no slipping (**1 point**)

20. (**10 points**) **Box 1** ($M_1 = 5kg$) and **Box 2** ($M_2 = 8kg$) are connected by a string that is passed over a pulley of radius, R = 10 cm, and moment of inertia, $I = 0.015 kg \cdot m^2$. When the system is **released** from **rest**, Box 2 accelerates downward without the rope slipping, and the pulley rotate CCW.



Definition:
<u>Potential Energy (PE)</u>
$U_{g2,i}$ initial PE of 2
$U_{g2,f}$ Final PE of 2
$U_{g1,i}$ initial PE of 1
$U_{g1,f}$ Final PE of 2

<u>Kinetic Energy (KE)</u>
No initial KE
Final KE below
$$\frac{1}{2}M_1 v_f^2 + \frac{1}{2}M_2 v_f^2 + \frac{1}{2}I\omega_f^2$$
v_f Final speed of box 1 and 2
ω_f final angular speed of pulley

A) Use **Conservation** of **Mechanical Energy** to find the speed of Box 2 after it has fallen 0.1m. **Hint:** write $I\omega^2 = \frac{I}{R^2}(R\omega)^2$, and use the **no-slip condition.**

The physics is summarized above

Initial (i)				Final (f)		
0	$+$	0	$=$	$\frac{1}{2}M_1 v_f^2 + \frac{1}{2}M_2 v_f^2 + \frac{1}{2}I\omega_f^2$	$+ \; M_1 gh$	$- \; M_2 gh$
No KE		No PE		Linear and Rotational KE	1 gains PE	2 loses PE

(**3 points**)

Using the hint: $I\omega_f^2 = \frac{I}{R^2}\left(R\omega_f\right)^2$ and the no-slip condition, $v_f = R\omega_f$ gives $I\omega_f^2 = \frac{I}{R^2}v_f^2$, and the energy equation above becomes (**1 point**)

$$0 = \frac{1}{2}M_1 v_f^2 + \frac{1}{2}M_2 v_f^2 + \frac{1}{2}\frac{I}{R^2}v_f^2 + M_1 gh - M_2 gh$$

$0 = \frac{1}{2}\left(M_1 + M_2 v_f^2 + \frac{I}{R^2}\right)v_f^2 + M_1 gh - M_2 gh \rightarrow v_f^2 = \frac{-M_1 gh + M_2 gh}{\frac{1}{2}\left(M_1 + M_2 + \frac{I}{R^2}\right)}$, (**3 points**)

Substituting all the data gives, $v_f = 0.636 m/s$. (**1 point**)

B) Use the answer of part **A)** and kinematics equations to find the acceleration, a, of Box 1, and the angular acceleration, α, of the pulley.

$v_f^2 = v_i^2 + 2ah \rightarrow (0.636 m/s)_f^2 = 0 + 2a \times 0.1m \rightarrow a = 2m \cdot s^{-2}$(**1 point**)

$\alpha = \frac{a}{R} = \frac{2m \cdot s^{-2}}{0.1m} = 20\frac{rad}{s^2}$ (**1 point**)

Useful Equations

Kinematics $x = x_0 + v_{0x}t + (1/2)a_xt^2$, $v_x = v_{0x} + a_xt$, $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$, $v_x = dx/dt$; for free-fall problem substitute y for x, and a = $-g$. $a_x = dv_x/dt$; $\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$; $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$; **average speed** s_{avg} = (total distance)/(total time); **average velocity (x-com)** $v_{avg,x} = (x_2 - x_1)/(t_2 - t_1)$, **average acceleration (x-com)** $a_{avg,x} = (v_{2x} - v_{1x})/(t_2 - t_1)$. **Newton's Laws** $\vec{F}_{net} = \sum \vec{F}_i = 0$ (Object in equilibrium); $\vec{F}_{net} = m\vec{a}$ (Nonzero net force); **Weight:** $F_g = mg$, $g = 9.8 m/s^2$; **Centripetal acceleration** $a_{rad} = \frac{v^2}{r}$;

Friction $f_s \le \mu_s F_N$, $f_k = \mu_k F_N$. **Hooke's Law** $F_x = -kx$. **Work and Energy** $W = \vec{F} \bullet \vec{d} = (F\cos\theta)d = F_{\|}d$; $W^{net} = \Delta K = (1/2)mv_f^2 - (1/2)mv_i^2$ (valid if W^{net} is the **net** or **total work done** on the object); $W^{grav} = -mg(y_f - y_i)$ (gravitational work), $W^{el} = -\left((1/2)kx_f^2 - (1/2)kx_i^2\right)$ (elastic work)

Conservation of Mechanical Energy (only **conservative forces** are present) $E_{mech} = U + K$
$W^{net} = -\Delta U = -(U_2 - U_1) = \Delta K = K_2 - K_1, U_1 + K_1 = U_2 + K_2, U_{grav} = mgy, U_{el} = (1/2)kx^2$
Also $\Delta E_{mech} = \Delta U + \Delta K = (U_f - U_i) + (K_f - K_i) = 0 \to \Delta K = -\Delta U$

CONSERVATION of **ENERGY**:
Non-Conservative Forces: with **no friction** $W_{ext} = \Delta E_{mech}$ (W_{ext} work done by **external**), with $\Delta E_{mech} = \Delta U + \Delta K = (U_f - U_i) + (K_f - K_i)$; **with friction** $W_{external} = \Delta E_{mech} + \Delta E_{th}$, $\Delta E_{th} = f_k d$, d is **magnitude** of **displacement**.

Work due to variable force 1D: $W = \int_{x_i}^{x_f} F_x\,dx \equiv$ area under F_x vs. x, from $x = x_i$ to x_f

Momentum: $\vec{P} = m\vec{v}$, **Impulse:** $\vec{J} = \int_{t_1}^{t_2} \vec{F}dt = \vec{F}_{av}(t_2 - t_1)$, **Impulse-Momentum** $\vec{J} = \Delta\vec{P} = \vec{P}_2 - \vec{P}_1$ **Newton's Law in Terms of Momentum** $\vec{F}_{net} = d\vec{p}/dt$. For $\vec{F}_{net} = 0$, $d\vec{p}/dt = 0$ gives **momentum conservation**: $\vec{P} \equiv$ constant. **Rotational Kinematics Equations**: $\omega_{avg} = (\theta_2 - \theta_1)/(t_2 - t_1)$, $\alpha_{avg} = (\omega_2 - \omega_1)/(t_2 - t_1)$ For α_z =constant, $\omega = \omega_0 + \alpha t$, $\theta = \theta_0 + \omega_0 t + (1/2)\alpha t^2$,. $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$ **Linear and angular variables:** $s = r\theta$, $v = r\omega$, $a_{\tan} = R\alpha$ (tangential) , $a_{rad} = v^2/r = \omega^2 r$ (radial) **Moment of Inertia and Rotational Kinetic Energy** $I = \sum_{i=1}^{N} m_i r_i^2$, $K_{rot} = (1/2)I\omega^2$. **Center of Mass** (COM) $\vec{r}_{com} = \sum m_i \vec{r}_i / M$, $M = \sum m_i$; $\vec{v}_{com} = \sum m_i \vec{v}_i / M$; $\vec{a}_{com} = \sum m_i \vec{a}_i / M$. **Newton's Second Law for System**: $\vec{F}_{net} = M\vec{a}_{com}$, where $\vec{F}_{net}$ is the **net external force** acting on the system of N particles. **Torque and Newton's Laws of Rotating Body:** rigid body $\tau = Fr_\perp$, $\vec{\tau}_{net} = \sum \vec{\tau}_i^{ext} = I\alpha$, $r_\perp$ -moment arm about axis; point $\vec{\tau} = \vec{r} \times \vec{F}$ about origin O.

Combined Rotation and Translation of a Rigid Body $K = (1/2)Mv_{com}^2 + (1/2)I_{com}\omega^2$, $\vec{F}_{net} = M\vec{a}_{com}$, $\vec{\tau}_{net} = I_{com}\vec{\alpha}$. **Rolling without slipping** $s = R\theta$, $v_{com} = R\omega$, $a_{com} = R\alpha$.