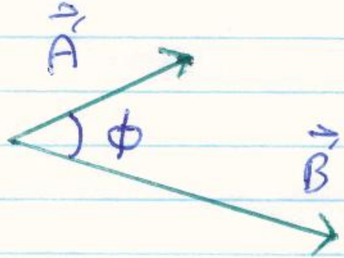
SCALAR PRODUCT (DOT PRODUCT)

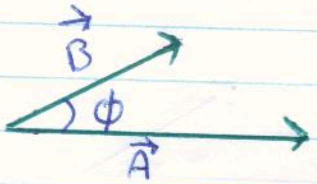
Consider TWO VECTORS $\vec{A}$ & $\vec{B}$, which makes an angle ϕ with each other.



Their SCALAR (DOT) PRODUCT is defined as

$$\boxed{\vec{A} \cdot \vec{B} = A B \cos \phi = |\vec{A}||\vec{B}| \cos \phi}$$

MODIFY to include



$0^\circ < \phi < 90^\circ$

$\cos\phi > 0,$

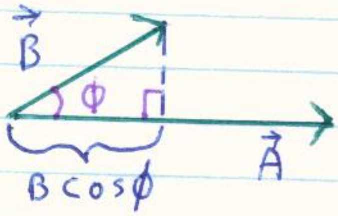
$\vec{A} \cdot \vec{B} > 0$

$90^\circ < \phi < 180^\circ$

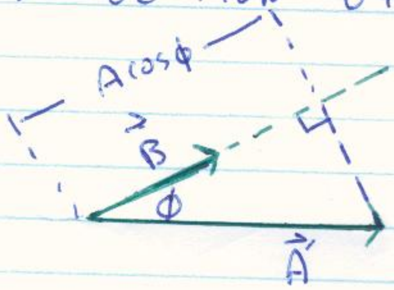
$\cos\phi < 0$

$\vec{A} \cdot \vec{B} < 0$

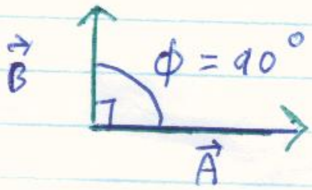
It is said that $\vec{A} \cdot \vec{B}$ is the projection of $\vec{B}$ onto $\vec{A}$. This is because $B\cos\phi$ is the component of $\vec{B}$ in the direction of $\vec{A}$



SIMILARLY $A\cos\phi$ is the COMPONENT of $\vec{A}$ in the direction of $\vec{B}$



Hence if $\vec{A}$ is perpendicular to $\vec{B}$ $(\vec{A} \perp \vec{B})$



$\vec{A} \cdot \vec{B} = 0$

COMPONENT OF $\vec{B}$ ALONG $\vec{A}$ is zero

$\vec{A} \cdot \vec{B} = AB\underbrace{\cos 90^\circ}_{0} = 0$

ALSO SINCE $\cos 90^\circ = 0$

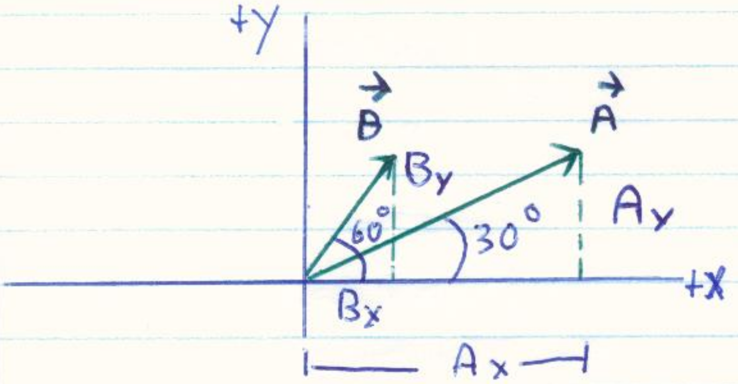
$$\vec{A} \cdot \vec{B} = AB \cos 90^\circ = 0$$

The DOT PRODUCT OF TWO-DIMENSION VECTORS

$$\boxed{\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y}$$

EXAMPLE $\vec{A}$, $A = 4.0$, $\theta = 30^\circ$

$\vec{B}$, $B = 2.0$, $\theta = 60^\circ$



CAN SEE that ϕ between $\vec{A}$ & $\vec{B}$ is 30°! VERIFY BY DOT PRODUCT.

$A_x = 4.0 \cos 30^\circ = 4.0 \times \frac{\sqrt{3}}{2} = 2.0\sqrt{3}$

$A_y = 4.0 \sin 30^\circ = 4.0 \times \frac{1}{2} = 2.0$

$B_x = 2.0 \cos 60^\circ = 2.0 \times \frac{1}{2} = 1.0$

$B_y = 2.0 \sin 60^\circ = 2.0 \times \frac{\sqrt{3}}{2} = \sqrt{3}$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y$$

$$= (2\sqrt{3})(1.0) + (2.0)(\sqrt{3})$$

$$= 4.0\sqrt{3}$$

RECALL NOW

$$\vec{A} \cdot \vec{B} = AB \cos\phi$$

Divide through by AB

$$\cos\phi = \frac{\vec{A} \cdot \vec{B}}{AB} = \frac{4\sqrt{3}}{4 \times 2} = \frac{\sqrt{3}}{2}$$

$$\cos\phi = \sqrt{3}/2$$

$$\phi = \arccos\left(\sqrt{\tfrac{3}{2}}\right) = 30^\circ$$