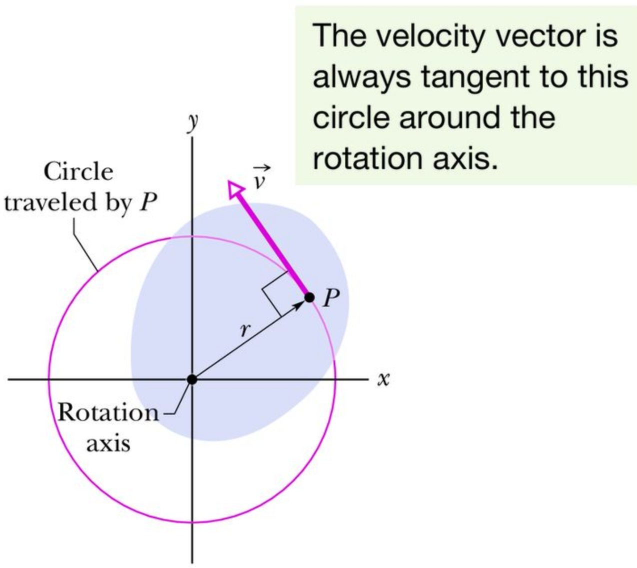
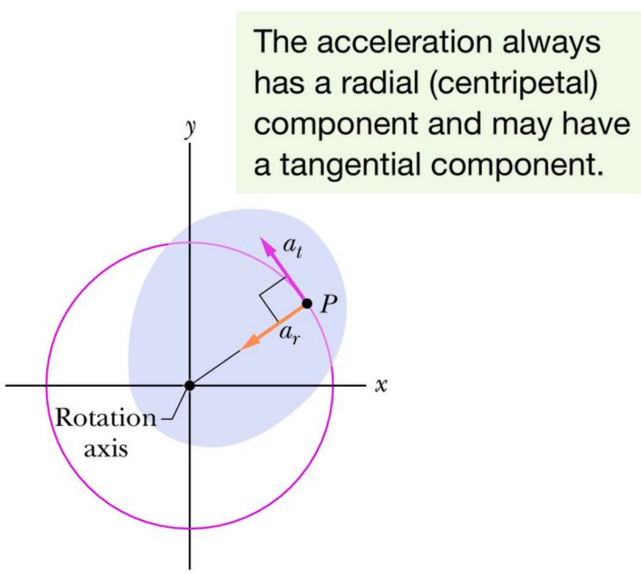
Section 10.3 Relating Linear and Angular Variables

Read this page then do the problem on the following page:

Definitions: 1) s linear arc length, θ angular displacement; 2) v linear speed, ω angular speed; 3) a_t tangential component of linear acceleration, α angular acceleration. Below is a summary of the variables.



(a)



(b)

- A point in a rigid rotating body, at a perpendicular distance *r* from the rotation axis, moves in a circle with radius *r*. If the body rotates through an angle θ, the point moves along an arc with length *s* given by s = θr (radian measure), where θ is in radians: **arc length** $s = r\theta$ **radius times angle. (E1)**
- The linear velocity $\vec{v}$ of the point is tangent to the circle; the point's linear speed *v* is given by v=ωr (radian measure), where ω is the angular speed (in radians per second) of the body, and thus also the point: **linear speed** v=ωr **angular speed times radius (E2)**
- The linear acceleration $\vec{a}$ of the point has both tangential and radial components. The **tangential component** is $a_t = \alpha r$ (radian measure), where α is the magnitude of the angular acceleration (in radians per second-squared) of the body: **Tangential Linear acceleration** $a_t = \alpha r$ **angular speed times radius (E3)**
- The **radial component** of is $a_r = \frac{v^2}{r}$, which is the centripetal acceleration of section 4.5, equation 4-34, covered earlier. Using v=ωr from before we have $a_r = \omega^2 r$.
- If the point moves in uniform circular motion, the period *T* of the motion for the point and the body is
  Period (in seconds) T=2πrv=2πω. **(E4)**
  The period is the time it takes for the body to rotate one revolution (2π radians) or for a point on the body to returns to its original position.

Questions that will help you prepare for assignment 10

Question 1

In the figure on the left, a disk of radius 0.7 m **rotates ccw** at 1.2 rad•s^{-1}. Calculate the **linear speed** of point **P**. Draw the **direction** of the **linear velocity** at point **P**. How long does it takes for point P to perform 2 revolutions. Calculate the linear distance traveled by point P after 2 revolutions.

HINT: Use equation E1, E2, E3, and E4. **See Solution on the last 3 pages**

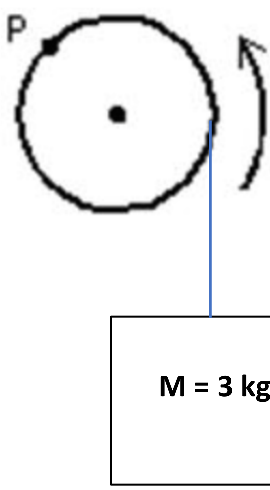
Question 2

In the figure of question 1, the disk starts from rest and accelerate at 0.8 rad•s^{-2}. How long does it take to rotate one revolution. Calculate the linear and angular speed after one revolution. Calculate the linear and angular acceleration after one revolution. Calculate the distance traveled by point p after one revolution.

HINT: Use equation E1, E2, E3, and E4

See Solution on the last 3 pages

Question 3



In the **left figure**, a **pulley** of radius 0.7 m **rotates ccw** at angular speed of 1.2 rad•s^{-1}. As the pulley rotates it hauls up a 3kg box, attached to a **rope** that does not **slip** as the pulley **rotates**.

A) Calculate the **velocity** of the box.

B) The **constant angular acceleration** is 0.5 rad•s^{-2} **cw**. Calculate the **acceleration** of **box**, and its **velocity,** and the **distance** it moves, **after 0.4 s**.

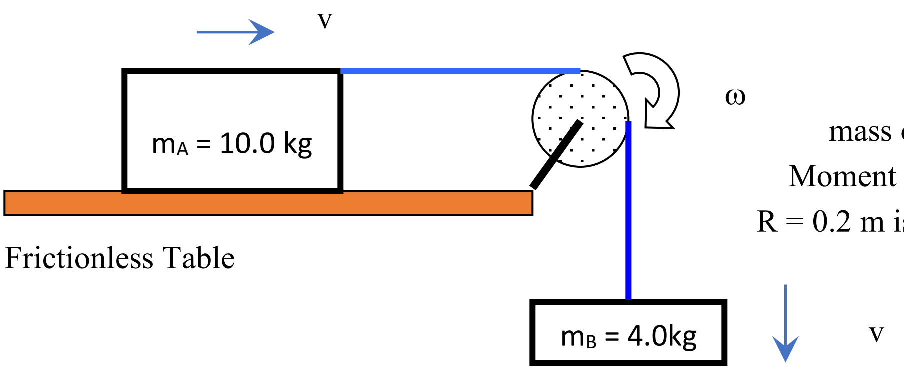
C) (**1 point**) Calculate the **linear acceleration** of the box

D) Calculate the net force on the box and the tension in the rope as the box moves up at constant acceleration. **HINT**: draw free-body diagram (FBD)

See Solution on the last 3 pages

Question 4

In figure below, box A and B are connected by a rope-cylindrical pulley system. Box A is released from **rest**. When Box A is released, box B begin to fall, and the **friction** between the rope and pulley **rotates** the **cylindrical pulley**, **without slipping**. The pulley rotates **clockwise** with an angular velocity, ω. The data are shown in the figure. The ideal (no mass) rope is in **blue**.



mass of pulley is $M_P = 3.0$ kg

Moment of inertia $I = \frac{1}{2} M_p R^2$

R = 0.2 m is the radius of cylinder

See Solution on the last 3 pages

Question 5 This is question is based on question 4

In figure of question 4 assume that box A and the table has friction $\mu_k = 0.11$; $\mu_s = 0.15$.

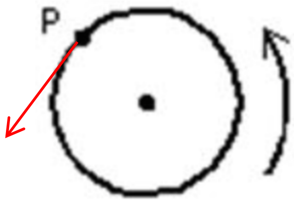
A) Use conservation of energy equation $\Delta E_{mec} + \Delta E_{th} = 0$, with $\Delta E_{mec} = \Delta K + \Delta U$ and $\Delta E_{th} = f_k d$, with $f_k = m_A g \mu_k$, and d = 0.4m, to find the **speed** of the boxes after B has fallen 0.4m. **ANSWER:** 1.27 m/s

B) Now in addition to the friction assume that once the boxes are released from rest, there is a horizontal force of 15 N pushing box A to the right. Use conservation of energy equation $W = \Delta E_{mec} + \Delta E_{th}$, with $\Delta E_{mec} = \Delta K + \Delta U$ and $\Delta E_{th} = f_k d$, with $f_k = m_A g \mu_k$, and d = 0.4m, and external work W = Fd, with F =15N to find the **speed** of the boxes after B has fallen 0.4m. **ANSWER:** 1.57 m/s

ANSWERS and Solutions:

QUESTION 1, Solution

SOLUTION:



$v = \omega r = 1.2 rad \bullet s^{-1} \times 0.7m = 0.84 m \bullet s^{-1}$, see **red arrow** for velocity of point P. **Time** for **2 revolutions** is $\frac{2rev \times 2\pi r}{v} = \frac{2 \times 2\pi \times 0.7m}{0.84m \cdot s^{-1}} = 10.47s$, or $\frac{2rev \times 2\pi}{\omega} = \frac{2 \times 2\pi}{1.2s^{-1}} = 10.47s$. **Linear distance of point P** after **2 revolutions** is $2 \times 2\pi r = 4\pi \times 0.7m = 8.8m$.

QUESTION 2, Answers and Solution

ANSWER: 3.96 s; 3.17 rad•s^{-1}, 2.2 rad•s^{-1}; 0.8 rad•s^{-2}, 0.56 m•s^{-2}
SOLUTION:
Since **one revolution** is $\theta = 2\pi$, we use $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$, with $\theta_0 = 0$, and since disk **starts** from **rest** $\omega_0 = 0$, to give $\theta = \frac{1}{2}\alpha t^2 \rightarrow t = \sqrt{2 \times 2\pi \div 0.8rad \cdot s^{-2}} = 3.96s$; To find the **angular speed** after this time use $\omega = \omega_0 + \alpha t = 0 + (0.8rad \cdot s^{-2}) \times 3.96s = 3.17rad \cdot s^{-1}$. The **linear speed** is $v = \omega r = (3.17rad \cdot s^{-1})(0.7m) = 2.2m \cdot s^{1}$. We are given that the **angular acceleration** is $\alpha = 0.8rad \cdot s^{-2}$, and the **linear acceleration** is $a = \alpha r = 0.8rad \cdot s^{-2} \times 0.7m = 0.56m \cdot s^{2}$. Distance of P after 1 revolution is $2\pi r = 4.4m$

QUESTION 3, Answers and Solution

A) $v_0 = \omega_0 r = 1.2rad \cdot s^{-1} \times 0.7m = 0.84m \cdot s^{-1}$
B) $a = \alpha r = -0.5rad \cdot s^{-2} \times 0.7m = -0.35m \cdot s^{-2}$, negative sign indicate acceleration is **down** (**rotation** is **cw**)
To find distance moved by the box after 4 seconds use $y = y_0 + v_0 t + \frac{1}{2}at^2$, with $y_0 = 0$, $v_0 = 0.84m \cdot s^{-1}$ (**positive** since it was moving **initially up**), and $a = -0.35m \cdot s^{-2}$ (**negative** since **acceleration** is **down**), time is 0.4s

$$y = y_0 + v_0 t + \frac{1}{2}at^2 = (0.84m \cdot s^{-1}) \times 0.4s + \frac{1}{2}(-0.35m \cdot s^{-2})(0.4s)^2 = 0.308m.$$

C) acceleration $-0.35m \cdot s^{-2}$, see part B

D) **ANSWER:** From C, since $a = -0.35m \cdot s^{-2}$; $F_{net,y} = Ma = 3kg \times -0.35m \cdot s^{-2} = -1.05N$, negative means force is down, tension T = 28.35N.

<u>QUESTION 4, Answers and Solution</u>

A) Use conservation of mechanical energy to find the **speed** of box A, after Box B has fallen 0.4 m. **HINT**: Use conservation of mechanical energy equation

$$initial\ (1) \qquad K_1 + U_1 = K_2 + U_2 \qquad Final\ (2)$$

<u>**Partial Solution**</u> The kinetic energy includes linear (first two terms) motion $\frac{1}{2}m_A v^2 + \frac{1}{2}m_A v^2 + \frac{1}{2}I_p\omega^2$, and rotational motion (last term) of rotating cylindrical pulley, the potential energy is gravitational U = mgy.

Use **no-slip condition**, $v = \omega R$, and $I = \frac{1}{2}M_p R^2$ to show that $\frac{1}{2}I_p\omega^2 = \frac{1}{4}M_p v^2$.

This will give the **answer (speed)** $v = 1.42\frac{m}{s}$

B) Calculate the angular speed, ω, after box B has fallen 0.4 m.

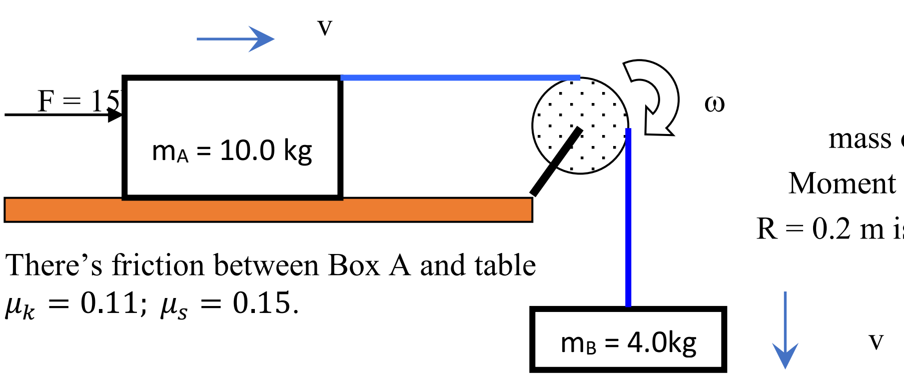
ANSWER: Use **no-slip condition** to find $\omega = 7.1\frac{rad}{s}$

C) Using the answer of part A, calculate the linear acceleration of box A. Calculate the angular acceleration, α, of the cylinder.

ANSWER: Use Kinematic Equations (linear and rotartional) $a = 2.52m \cdot s^{-2}\ (down)$; $\alpha = 12.6\frac{rad}{s^2}\ (cw)$

<u>Question 5: Solution</u>

In figure below, box A and B are connected by a rope-cylindrical pulley system. Box A is released from **rest**. When Box A is released, box B begin to fall, and the **friction** between the rope and pulley **rotates** the **cylindrical pulley**, **without slipping**. The pulley rotates **clockwise** with an angular velocity, ω. The data are shown in the figure. The ideal (no mass) rope is in **blue**.



mass of pulley is M_P = 3.0 kg

Moment of inertia $I = \frac{1}{2}M_p R^2$

R = 0.2 m is the radius of cylinder

There's friction between Box A and table $\mu_k = 0.11$; $\mu_s = 0.15$.

Diagram above depicts part B. If F = 0 then it depicts part A)

A) Use conservation of energy equation $\Delta E_{mec} + \Delta E_{th} = 0$, with $\Delta E_{mec} = \Delta K + \Delta U$ and $\Delta E_{th} = f_k d$, with $f_k = m_A g \mu_k$, and d = 0.4m, to find the **speed** of the boxes after B has fallen 0.4m.

B) Now in addition to the friction assume that once the boxes are released from rest, there is a horizontal force of 15 N pushing box A to the right. Use conservation of energy equation $W = \Delta E_{mec} + \Delta E_{th}$, with $\Delta E_{mec} = \Delta K + \Delta U$ and $\Delta E_{th} = f_k d$, with $f_k = m_A g \mu_k$, and d = 0.4m, and external work W = Fd, with F =15N to find the **speed** of the boxes after B has fallen 0.4m.

A) $\Delta E_{th} = f_k d = m_A g \mu_k d = 10kg \times 9.8\frac{m}{s^2} \times 0.11 \times 0.4m = 4.312J$

$\frac{1}{2}(m_A + m_B)v^2 - 0 + 0 - m_B g \times 0.4m + 4.312J$, with $\frac{1}{2}(m_A + m_B) = 7kg$

$$v = 1.274\frac{m}{s}$$

B) $W = \Delta E_{mec} + \Delta E_{th} \rightarrow Fd = \Delta K + \Delta U \ + \Delta E_{th}$, with same values of in part A)

$$Fd = 6J = \frac{1}{2}(m_A + m_B)v^2 - 0 + 0 - m_B g \times 0.4m + 4.312J$$

$$v = 1.57\frac{m}{s}$$