Random Walk

Binomial distribution:
Consider a trial with two outcomes, with the outcome 1 occuring with probability p, and outcome 2 with probability q = 1 – p. If there are n trials, the probability that outcome 1 occurs k times (and outcome 2 occurs n – k times) is

$$P(k;n,p) = \binom{n}{k} p^k (1-p)^{n-k}$$

With

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

Example: Consider a 1D random walk with right step probability equal to p = 0.5, and left step probability equal to p = 1 – q = 0.5. Calculate the probability that after n = 10000 steps, there will be k = 5000 right steps and n – k = 5000 left steps. This is

$$P(5000;10000{,}0.5) = \binom{10000}{5000} 0.5^{5000} (0.5)^{10000-5000} = \frac{10000!}{5000!\,5000!} (0.5)^{10000}$$

Since 10000! is a very large number, we must use equation 4.2 in your turn 4a

$$lnN! = NlnN - N + \frac{1}{2} ln(2\pi N)$$

Hence

$$ln10000! = 10000ln10000 - 10000 + \frac{1}{2} ln(2\pi \times 10000) = 82108.928$$

$$ln10000! = e^{82108.928}$$

But as done in class

$$e = 10^{0.4343}$$

And

$$ln10000! = e^{82108.928} = 10^{82108.928\times 0.4343} = 10^{35659.9}$$

Also

$$ln5000! = 5000ln5000 - 5000 + \frac{1}{2} ln(2\pi \times 5000) = 37591.14$$

And

$$ln5000! = e^{37591.14} = 10^{37591.14\times 0.4343} = 10^{16325.8}$$

Also

$$(0.5)^{10000} = (10^{-0.30103})^{10000} = 10^{-3010.3}$$

This gives

$$P(5000;10000{,}0.5) = \frac{10000!}{5000!\,5000!} (0.5)^{10000} = \frac{10^{35659.9}}{10^{16325.8} \times 10^{16325.8}} \times 10^{-3010.3}$$

$$P(5000;10000{,}0.5) = 10^{3008.23} \times 10^{-3010.3} = 10^{-2.067} \sim 0.0086$$

Hint: For a random walk of +L right step and – L left step. If after n = 10000 steps, x = - 10L, this means that it will have taken 4995 right steps and 5005 left steps. If x = 4000L, then there must be 7000 right steps and 3000 left steps.