Practice Modern Physics II, W2018, Set 3

Question 1: Symmetric (Boson) and Anti-symmetric (Fermions) Wavefunction

A) Consider a system of **two fermions**. Which of the following wavefunctions can describe the system? $\psi_{n1,n2}(\vec{r}_1,\vec{r}_2)=\frac{1}{\sqrt{2}}\{\psi_{n1}(\vec{r}_1)\psi_{n2}(\vec{r}_2)+\psi_{n1}(\vec{r}_2)\psi_{n2}(\vec{r}_1)\}$ or $\psi_{n1,n2}(\vec{r}_1,\vec{r}_2)=\frac{1}{\sqrt{2}}\{\psi_{n1}(\vec{r}_1)\psi_{n2}(\vec{r}_2)-\psi_{n1}(\vec{r}_2)\psi_{n2}(\vec{r}_1)\}$. Justify your answer.

B) **Two identical particles in a box** (in one dimension 1D) is described by the **total wavefunction**, $\psi(x_1,x_2)=\frac{1}{\sqrt{2}}\left\{\frac{2}{L}\sin\left(\frac{2\pi x_1}{L}\right)\sin\left(\frac{4\pi x_2}{L}\right)-\frac{2}{L}\sin\left(\frac{4\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right)\right\}$, where L is the length of the box and x_1 and x_2 are the positions of particle 1 and 2, respectively. What are the **quantum states** of the two particles? Are the two particles **bosons** or **fermions**? **Explain your answer**. What is the **total energy** of the two identical particles? The mass of a particle is $m=1.67\times10^{-27}kg$, and L = 1 nm. **Hint**: A particle in a box of length, L, is described by the wavefunction $\psi_n=\sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right)$, where $n=1,2,3\ldots$ is the **quantum state** of the particle. For a particle in the n state, its energy is $E_n=\frac{n^2\pi^2\hbar^2}{2mL^2}$.

C) Repeat part D) for the wavefunction,

$$\psi(x_1,x_2)=\frac{1}{\sqrt{2}}\left\{\frac{2}{L}\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{3\pi x_2}{L}\right)+\frac{2}{L}\sin\left(\frac{3\pi x_1}{L}\right)\sin\left(\frac{\pi x_2}{L}\right)\right\}.$$

D) repeat for the wavefunction $\psi(x_1,x_2)=\frac{2}{L}\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{\pi x_2}{L}\right)$.

E) Consider **two electrons** in a box of width L = 1.50 nm. The quantized energy of **each electron** is $E_n=\frac{n^2\pi^2\hbar^2}{2mL^2}$, $n=1,2,3\ldots.$, where $m=9.1\times10^{-31}kg$ is the mass of the electron. Calculate the **ground state energy** (lowest energy) and **first excited energy** (second lowest energy) of the **two electrons** system. Express your answer in eV ($1eV=1.6\times10^{-19}J$).

Question 2 Magic Numbers and Hassium

As discussed in the textbook, certain values of protons, Z and neutrons N are called "magic' since they are believed to correspond to nuclear stability. These **magic numbers** are 2, 8, 20, 28, 50, 82, and 126. However, there are other magic numbers associated only with protons or neutrons. For example, Hassium, $^{270}_{108}Hs$ is one of the heaviest isotopes ever observed, and is stable for about 10s. Its surprising stability is attributed by some to be due to the fact that 108 is a magic number for protons, and 162 is a magic number for neutron.

A) Despite this explain in no more than **two sentences**, the physical reasons why $^{270}_{108}Hs$ should **not be stable.**

B) Use the nuclear liquid drop model (see equation sheet) to calculate the binding energy of $^{48}_{28}Ni$. Does the results support the magic number hypothesis? Justify in **two sentences.**

C) $^{270}_{108}Hs$ has atomic mass of 270.13429u. Use this to calculate the binding energy, and compare (no more than **two sentences**) with the results of part B).

<u>Question</u> 3 β^+ and α decay, and electron capture.

A) Consider the nuclear reaction, $^{14}_{8}O \rightarrow {}^{14}_{7}N + \beta^+ + v_e$. Calculate the **disintegration energy** of the decay using $Q = \left[M\left({}^{14}_{8}O\right) - M\left({}^{14}_{7}N\right) - 2m_e\right]c^2$. In **no more than two sentences**, explain why it is necessary to included the mass of two electrons in the equation for Q. In **one sentence**, justify, based on your answer, why the decay will or will not occur? What is v_e? In no more than **two sentences**, explain how that the law of quantum mechanics requires its presence in the reaction

$1u \rightarrow mc^2 = 931.5MeV$, $m_e c^2 = 511keV = 0.511MeV$,

B) Repeat part A), but for electron capture $^{14}_{8}O + \beta^- \rightarrow {}^{14}_{7}N + v_e$. Why does the relation for Q of the electron capture not include $-2m_e$?

C) Calculate the **disintegration energy** of the decay $^{16}_{8}O \rightarrow {}^{A}_{Z}C + \alpha$. Determine the value of A and Z for $^{A}_{Z}C$. In **one sentence**, justify, based on your answer, why the decay will or will not occur?

<u>Question 4</u> Elementary Particle Physics

A) State **one (or two) reason** why the following two reactions **are not allowed**:

$\pi^- \rightarrow e^- + \gamma$ and $\pi^+ \rightarrow e^- + e^+ + \mu^+$

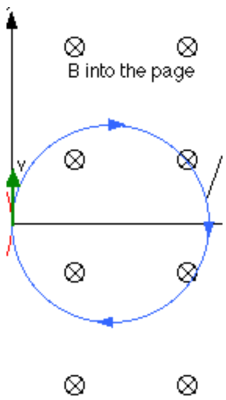
B) Complete the following reactions: $\mu^- + p \rightarrow n + ?$; $n + p \rightarrow \Sigma^0 + n + ?$. Briefly justify your answers?

C) Determine the quark composition of Λ , π^- and π^+. See equation sheet for information on Λ.

D) Quarks are spin ½ particles. The Ξ^0 baryon has a quark composition ssu. Is this composition consistent with the Pauli exclusion principle? Why or why not?

Question 5) Cyclotron The figure below shows a particle undergoing a **uniform circular motion** in a cyclotron of radius $r = 5.5m$ by a magnetic field of $B = 1.3T$. It was then determined that the particle has a kinetic energy of $K = 2000MeV$.

A) Using the appropriate equation, find the **relativistic momentum**, *p*, of the particle. **HINT:** Assume that the particle has a charge of magnitude $|q| = e$, and the small cross ⊗ means that the magnetic field $\vec{B}$ points into the page, while the charge particle follows a clockwise path.



B) Starting from $K = E - mc^2$ and $E = \sqrt{p^2c^2 + (mc^2)^2}$, show that the rest mass energy can be expressed as $mc^2 = (p^2c^2 - K^2)/2K$.

C) Use the results of part A and B to find the rest mass energy of the particle in MeV. Inspect the nuclear physics data on the equation sheet to infer the identity of this particle. **NOTE**: Select the particle in the table 14.4 with the closest mass, and charge.