Practice Modern Physics II, W2018, Set 2

Question 1: 2D Maxwell-Boltzman Distribution

A) In 2D the speed distribution becomes $f_{2D}(v)=\beta m v \exp\left(-\beta \frac{mv^2}{2}\right)$ with $v^2=v_x^2+v_y^2$, material. **Briefly explain** the mathematical meaning of $f_{2D}dv$, and explain why the **normalization** of f_{2D} requires $\int_0^{\infty} f_{2D}\,dv=1$. **Verify** by direct integration that $\int_0^{\infty} f_{2D}\,dv=1$.

B) Find the average speed in 2D, $\bar{v}$ of a neutron at T = 300K. Data: Mass of neutron in equation sheet. HINT: $\int_0^{\infty} x^2 \exp\left(-ax^2\right)dx=(\pi)^{1/2}/\left(4a^{3/2}\right)$.

C) Using $\frac{df_{2D}}{dv}=0$ find the most probable speed of a neutron at T = 300K, and compare with your answer in part B)

Question 2 Beryllium atoms as a Degenerate Fermi Gas

In Question 3 of assignment 6, we consider helium-3, ${}_2^3He$, whose nucleus is made of two protons and one neutron, with two electrons orbiting its nucleus. We showed that its total spin can be $S_{tot}=1/2, 3/2$, which shows that it is a fermion. We calculate the its Fermi energy: $\varepsilon_F=6.87\times10^{-23}J \rightarrow \varepsilon_F=4.3\times10^{-4}eV$, and Fermi temperature $T_F=4.97K$. There are **two important points**:

Point 1: At atmospheric pressure, helium-3 is a **gas** at room temperature (~300K), but becomes a **liquid** at temperatures lower than 3.19 K. It is not a solid at very high pressure > 10 atm. This is an important point, since in order for system of **identical fermions** (electrons, helium-3, …) to behave like a **degenerate gas**, the fermions must be free to move, just like free electrons in metals. As mentioned before, the Fermi temperature of electrons in metals is usually very high > 5000K, so at room temperature they can behave like **quantum degenerate gas**. Helium-3 are fermions that remains a fluid (liquid or gas), which is free to move till T = 0, as far as we know. They do become a **quantum degenerate gas** at very low temperature.

Point 2 You may as what is the difference between a **quantum degenerate ideal gas**, and **Maxwell-Boltzman (MB) ideal** gas. For the **MB ideal gas**: the internal energy is $U = Nk_BT$, where N is the total number of particles, and the pressure, P, is given by $PV = Nk_BT$ or $P = nk_BT$ with $n = N/V$ being the number density. For a system of **quantum degenerate ideal gas**: $U=\frac{3}{5}N\varepsilon_F$ (9.46), and $P=\frac{2}{5}n\varepsilon_F$, valid for sufficiently low temperature (which in some case is room temperature ~ 300K). As discussed in class the reason that the quantum Fermi gas has such high energy/pressure at sufficiently low temperature is that two identical fermions cannot occupy the same low energy quantum states – this is the Pauli Exclusion Principle.

For this question consider Beryllium, Z = 4, with nucleus of 4 protons and 5 neutrons, ${}^{9}_{4}Be$ (most common isotope). At atmospheric pressure, its boiling point is 2742K, and its melting point is 1560K. Hence it is a **liquid** from 1560 K to 2742K, where its mass density remains relatively constant at 1.69 g/cm^3. The atomic mass of Beryllium is $m_{Be} = 9.01u$.

A) Calculate the possible total spin of ${}^{9}_{4}Be$.

B) Assume that ${}^{9}_{4}Be$ has total spin S_{tot} = ½, and that it is a liquid. Calculate its Fermi energy and temperature. Based on your result, and the data, do you think that ${}^{9}_{4}Be$ will become a quantum degenerate gas at low temperatures. Why?

C) At the Fermi temperature calculated in part B), estimate the **pressure** and **energy**, U, of **one mole** $N = 6.023\times10^{23}$ of "gas" of ${}^{9}_{4}Be$ if it is a MB gas, and it is a **quantum degenerate gas**. For this question, assume erroneously that ${}^{9}_{4}Be$ remain a gas even at such low temperature.

Question 3 Semiconductors:

A) Using the band theory of solids explain the differences in the electrical conductivity behavior of conductors, insulators, and semiconductors. For full marks draw energy band diagram to illustrate your answer.

B) Explain the differences between an n-type and a p-type semiconductor.

C) Germanium has atomic number Z = 32. Germanium is doped with Aluminium (Z = 13), would the resulting semiconductor be an n-type or an p-type. Briefly explain your answer.

D) Repeat part C with phosphorus (Z = 15) as the dopant.

Question 4 Magnetism and Superconductivity

Titanium (Z = 22), ${}^{48}_{22}Ti$, atomic mass 47.947 u, mass density 4.507 g/cm^3.

A) Write the electronic configuration of titanium, and determine the number of unpaired d electrons. Do you think Ti is ferromagnetic or paramagnetic. Why? **HINT:** use table 8.1 on page 275

B) In fact it is Titanium is paramagnetic. Calculate the **saturation** (maximum) **magnetization**, M_{max}. Hence calculate the **maximum magnetic field** that can be induced by the d spins, $B_{max} = \mu_0 M_{max}$, where $\mu_0 = 4\pi \times 10^{-7} T \bullet m / A$. **Hint:** see question 7 of assignment 5.

C) Use Table 10.5 to determine the transition temperature T_c, and the critical field at zero temperature $B_c(0)$ of Titanium. For Titanium, use equation 10.44, 10.46 and 10.47 to calculate the zero temperature gap $E_g(0)$, and the energy gap at T = 0.33 K, and the critical magnetic field at T = 0.33 K, $B_c\left(0.33K\right)$.

D) A superconductor is a **perfect diamagnet** (also called the Meissner effect) such that the total magnetic field inside a superconductor is zero $B_{inside} = 0$. Give a brief physical explanation of how the Meissner effect occurs. Can the Meissner effect be explain by the spin magnetic field calculated in part B.

E) Using the result of part A, and the fact that Zinc, ${}^{64}_{30}Zn$ has atomic mass 63.93u to estimate the critical temperature of ${}^{64}_{30}Zn$. Compare your data with table 10.5.

F) Study problem 51 chapter 10 f assignment 5