PHYS4115 Quantum MechanicsII Fall 2015
Assignment 4: Identical Particles and Parity

The due date for this assignment is Wednesday November 18, 2015.

1. Do problem 5.13 from edition of Griffith's. Note that the ground state of carbon and nitrogen is discussed in class.

2. Use Hund's rule, Clebsch-gordan table, and the anti-symmetry of electrons, to show the groun state of Titanium (Z = 22) is $^3F_2$. **SUGGESTION:** 1) for L = 4 explain why all the states must be **symmetric**; 2) for L =3 verify that all the states are **anti-symmetric**.

3. Detail on Nitrogen's ground state $^4S_{3/2}$.

   (A) TOTAL SPIN: In class, it was shown that there are two distinct states: **1)** $S = \frac{3}{2}$, $\left|S = \frac{3}{2}, M_S\right\rangle$, where $M_S = \frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}$, for example $\left|\frac{3}{2}, \frac{3}{2}\right\rangle = |\uparrow\uparrow\uparrow\rangle$; **2)** $S = \frac{1}{2}$, for $M_S = \frac{1}{2}$ with two possibilities, $\left|\frac{1}{2}, \frac{1}{2}\right\rangle_A = \omega^1 |\downarrow\uparrow\uparrow\rangle + \omega^2 |\uparrow\downarrow\uparrow\rangle + \omega^3 |\uparrow\uparrow\downarrow\rangle$, and $\left|\frac{1}{2}, \frac{1}{2}\right\rangle_B = \omega^{-1} |\downarrow\uparrow\uparrow\rangle + \omega^{-2} |\uparrow\downarrow\uparrow\rangle + \omega^{-3} |\uparrow\uparrow\downarrow\rangle$, where $\omega = \exp i\frac{2\pi}{3}$. Find the analogue states $\left|\frac{1}{2}, -\frac{1}{2}\right\rangle_A$ and $\left|\frac{1}{2}, -\frac{1}{2}\right\rangle_B$. Show that $\left|\frac{1}{2}, -\frac{1}{2}\right\rangle_A$ and $\left|\frac{1}{2}, -\frac{1}{2}\right\rangle_B$. are not symmetric and not anti-symmetric.Show that they are orthogonal to the 4 $\left|\frac{3}{2}, M_S\right\rangle$ states.

   (B) TOTAL ORBITAL ANGULAR MOMENTUM ($\overrightarrow{L} = \overrightarrow{L}_1 + \overrightarrow{L}_2 + \overrightarrow{L}_3$): In class we use the clebsch-gordon table to add three p electrons ($\ell_1 = 1, \ell_2 = 1, \ell_3 = 1$) to find the $L = 0$ state, $|0, 0\rangle$. Use the Slater's determine to form $|0, 0\rangle$, where $i^{th} (i = 1, 2, 3)$ particle can occupy $Y_1^1(i) = |1, 1\rangle_i$, $Y_1^0(i) = |1, 0\rangle_i$, $Y_1^{-1}(i) = |1, -1\rangle_i$. Show that this is the same as $|0, 0\rangle$ obtained in class, which verifies that $|0, 0\rangle$ is **anti-symmetric**.

4. Parity of Identical particles

   (a) Show that the parity operator $\widehat{P}$ commutes with the L_z operator.

   (b) Show that the parity operator $\widehat{P}$ commutes with the $L_\pm$ operator.

   (c) Use results of a) and b) to show that the parity operator $\widehat{P}$ commutes with the L^2 operator, by using the relation (from Griffith) $L_\pm L_\mp = L^2 - L_z^2 \mp \hbar L_z$.

   (d) Explain the implication of part c on th eparity of the spherical harmonic $Y_\ell^m(\theta, \phi)$.

   (e) This means that $Y_\ell^m(\theta, \phi)$ is either even or odd under the inversion of coordinate(ie the parity operation) $x \to -x$, $y \to -y$, $z \to -z$. It was shown in class that this operation is equivalent to $\theta \to \pi - \theta$ and $\phi \to \pi + \phi$. It was also shown that $\widehat{P} Y_\ell^m(\theta, \phi) = Y_\ell^m(\pi - \theta, \pi + \phi) = (-1)^\ell Y_\ell^m(\theta, \phi)$
   Using the above results to show that the decay

   $$p^0 \to \pi^0 + \pi^0$$

   cannot occur. The p^0 meson has spin 1, while the π^0 meson has spin 0.
   HINT: Use conservation of total angular momentum and the fact that the two π^0 mesons are **indistinguishable**.

5. **Spin-Spin Interaction Energy**.

(a) Consider two **distinguishable** spin 1/2 particles, $\overrightarrow{S}_1\,(s_1 = 1/2)$ and $\overrightarrow{S}_2\,(s_2 = 1/2)$, with the Hamiltonian $H = a\left(\overrightarrow{S}_1 \cdot \overrightarrow{S}_2\right)$, where a is a constant. Find the **exact** solution for this system.

(b) Consider two **distinguishable** spin 1/2 particles described by the Hamiltonian $H = aS_{1z} + bS_1^2 S_{2z}$. Find the exact solutions.

(c) Consider two **indistinguishable** spin 1/2 particles described by the Hamiltonian $H = aS_{1z} + bS_1^2 S_{2z}$. Find the exact solutions.

(d) Consider two **indistinguishable** spin 1/2 particles described by the Hamiltonian $H = a\left(\overrightarrow{S}_1 \cdot \overrightarrow{S}_2\right) + b\left(S_{1z} + S_{2z}\right)$ .Find the exact solutions.