

PHYS3511-Biological Physics
Fall 2018, Assignment #5
Due Wednesday October 7, 2018

Read Chapter 5 before attempting the solution

Exercise 1) Chapter 5, Problem 5.2

Exercise 2) Chapter 5, Problem 5.5

Exercise 3) An E. Coli of radius about $r = 0.7 \mu\text{m}$ is observed to swim at $v = 80 \mu\text{m} \cdot \text{s}^{-1}$. Estimate the amount of ATP that it must produce per second, in order for the E. Coli to swim at this rate. Compare this with the number of ATP needed to produce one E. Coli per second. DATA: ATP hydrolysis produce $\Delta G = -30 \text{kJ} \cdot \text{mol}^{-1}$.

Exercise 4) From class note, I showed the 1st law of Thermodynamics:

$dE = TdS - PdV + \mu dN$, which gives $dS = \frac{1}{T}dE + \frac{P}{T}dV - \frac{\mu}{T}dN$. Hence we know that the Entropy $S(E, V, N)$ is a function of E, V and N. This gives the differential formula

$dS = \left(\frac{\partial S}{\partial E}\right)_{V,N} dE + \left(\frac{\partial S}{\partial V}\right)_{E,N} dV - \left(\frac{\partial S}{\partial N}\right)_{E,V} dN$. Comparing coefficients we obtain:

$$\left(\frac{\partial S}{\partial E}\right)_{V,N} = \frac{1}{T}; \left(\frac{\partial S}{\partial V}\right)_{E,N} = \frac{P}{T}; \left(\frac{\partial S}{\partial N}\right)_{E,V} = -\frac{\mu}{T}$$

For an ideal gas of non-interacting particles the Entropy is given by the Sackur-Tetrode equation:

$$S = Nk_B \left[\ln \left\{ \left(\frac{V}{N}\right) \left(\frac{4\pi m E}{3Nh^2}\right)^{3/2} \right\} + \frac{5}{2} \right].$$

The above can be used to find the ideal gas equation, $PV = Nk_B T$, and the equipartition theorem, $E = \frac{3}{2} Nk_B T$.

Now we consider the entropy of a **dilute attractive gas** with **Entropy**:

$$S = Nk_B \left[\ln \left\{ \left(\frac{V}{N}\right) \left(\frac{4\pi m}{3Nh^2}\right)^{3/2} \left(E + a \frac{N^2}{V}\right)^{3/2} \right\} + \frac{5}{2} \right].$$

A) Use $\left(\frac{\partial S}{\partial E}\right)_{V,N} = \frac{1}{T}$ to find an analogous equipartition equation.

B) and $\left(\frac{\partial S}{\partial V}\right)_{E,N} = \frac{P}{T}$ to find an analogous equation to the ideal gas equation

BONUS: For the van der waals gas the Entropy is

$$S = Nk_B \left[\ln \left\{ \left(\frac{V - Nb}{N}\right) \left(\frac{4\pi m}{3Nh^2}\right)^{3/2} \left(E + a \frac{N^2}{V}\right)^{3/2} \right\} + \frac{5}{2} \right]$$

Find the van der Waals equation of state (this equation is easily found online)

NOTE: van der Waals equation of state is $\left(P + a \left(\frac{N}{V}\right)^2\right) (V - Nb) = Nk_B T$. The equation of state of an attractive gas is found by setting $b = 0$ in the **van der Waals equation of state**.

way? How many reducing equivalents of NADH and NADPH are consumed or created along the way? In order to answer this question completely, you may need to consider a few other pathways as well. For example, it is likely that the displayed glycolysis pathway does not actually start with glucose. To get glucose 6-phosphate from glucose requires 1 ATP. One of the steps in the serine synthesis pathway is coupled to a conversion of L-glutamate to 2-ketoglutarate. You will have to look up the "glutamate biosynthesis III" pathway to determine the cost of regenerating L-glutamate from 2-ketoglutarate. Assuming that each NADH or NADPH is equivalent to 2 ATP, which is a reasonable conversion factor for bacteria, what is the net energy cost to synthesize one molecule of serine in units of ATP and units of $k_B T$?

• **5.2 The sugar budget revisited**

In Chapter 3, we worked out the rate of sugar uptake to provide the construction materials for a dividing bacterium. However, as shown in this chapter, sugar molecules also provide the *energy* needed to perform macromolecular synthesis. Amend the estimate of Chapter 3 to include the fact that sugar supplies construction materials and the energy needed to assemble them. You might find it useful to look at the macromolecular energy costs revealed in Table 5.2. How many sugars are needed to provide the energy and construction materials for making a new cell? Make an estimate for the average rate of sugar uptake for a dividing bacterium in light of this amendment to our earlier estimates.

• **5.3 A feeling for the numbers: covalent bonds**

- (a) Based on a typical bond energy of $150 k_B T$ and a typical bond length of 0.15 nm, use dimensional analysis to estimate the frequency of vibration of covalent bonds.
- (b) Assume that the Lennard-Jones potential given by

$$V(r) = \frac{a}{r^{12}} - \frac{b}{r^6}$$

describes a covalent bond (though real covalent bonds are more appropriately described by alternatives such as the Morse potential that are not so convenient mathematically).

• **5.5 A feeling for the numbers: comparing multiplicities**

Boltzmann's equation for the entropy (Equation 5.30) tells us that the entropy difference between a gas and a liquid is given by

$$S_{\text{gas}} - S_{\text{liquid}} = k_B \ln \frac{W_{\text{gas}}}{W_{\text{liquid}}}. \quad (5.64)$$

From the macroscopic definition of entropy as $dS = dQ/T$, we can make an estimate of the ratios of multiplicities by noting that boiling of water takes place at fixed T at 373 K.

- (a) Consider a cubic centimeter of water and use the result that the heat needed to boil water (the latent heat of vaporization) is given by $Q_{\text{vaporization}} = 40.66 \text{ kJ/mol}$ (at 100°C) to estimate the ratio of multiplicities of water and water vapor for this number of molecules. Write your result as 10 to some power. If we think of multiplicities in terms of an ideal gas at fixed T , then

$$\frac{W_1}{W_2} = \left(\frac{V_1}{V_2}\right)^N. \quad (5.65)$$

What volume change would one need to account for the liquid/vapor multiplicity ratio? Does this make sense?

- (b) In the chapter, we discussed the Stirling approximation and the fact that our results are incredibly tolerant of error. Let us pursue that in more detail. We have found that the typical types of multiplicities for a system like a gas are of the order of $W \approx e^{10^{25}}$. Now, let us say we are off by a factor of 10^{1000} in our estimate of the multiplicities, namely, $W = 10^{1000} e^{10^{25}}$. Show that the difference in our evaluation of the entropy is utterly negligible whether we use the first or second of these results for the multiplicity. This is the error tolerance that permits us to use the Stirling approximation so casually! (This problem was adapted from Ralph Baierlein, *Thermal Physics*, Cambridge University Press, 1999.)

• **5.6 Stirling approximation revisited**

The Stirling approximation is useful in a variety of different settings. The goal of the present problem is to work through a more sophisticated treatment of this approximation than

Table 5.2: Biosynthetic cost in ATP equivalents to synthesize the macromolecules of a single *E. coli* cell.

Class	Biosynthetic cost (aerobic) – ATP equivalent
Protein	4.5×10^9
DNA	3.5×10^8
RNA	1.6×10^9
Phospholipid	3.2×10^9
Lipopolysaccharide	3.8×10^8
Peptidoglycan	1.7×10^8
Glycogen	3.1×10^7