PHYS2332-Modern Physics II
Winter 2020 Assignment #2

Due on Friday January 31, 2020. Note useful equations on page 2.

Do problem 38 of Chapter 9.

Do problem 39 of Chapter 9. Again to receive any marks you must show works. You will receive no marks for copying down the answers from the back of the book.

Question 3 **QUANTUM COUNTING: A)** In assignment 2, problem 28 of chapter 9, we calculate the Fermi energy of aluminum, $E_F = 11.63eV$. If the bulk concentration is $N/V = 6\times10^{28}m^{-3}$, what concentration of electrons has energy in the range $E = 5eV$ to $E = 15eV$. **B)** On page 330 for ^4He, the textbook found that for helium concentration $\left(\frac{N}{V}\right) = 2.11\times10^{28}\frac{^4He}{m^3}, T_C = 3.06K$. Determine the concentration of BE condensate at $T = 0K$, 1.5K, and 4.0K.

Question 4 Consider a **neutron star** of 1.5 solar mass ($M_{Sun} = 2\times10^{30}kg$), A) Assume that the star is made of neutrons ($m_n = 1.66\times10^{-27}kg$), to calculate the equilibrium radius of the star, R_{star}. B) Use the result in A to calculate the fermi energy, E_F, and temperature, T_F. Does the result indicate that the neutrons of the star form a degenerate gas? C) Calculate the pressure due to the degenerate neutron gas in atmospheric pressure ($1atm = 1.01325\times10^5 Pa$). **NOTE:** Assume spherical shape, $V = (4\pi/3)R^3$.

Question 5 The critical temperature $T_C = \frac{h^2}{2mk_C}\left(\frac{N}{2\pi V(2.315)}\right)^{2/3}$ of Bose condensation was discussed in class and also in text (see equation 9.65, page 3.30). As discussed in the text, using liquid helium density $N/V = 2.11\times10^{28}m^{-3}$ we have $T_B = 3.06K$. Note that the derivation is for normal helium, ^4He, with total spin S = 0, with spin degeneracy zero. For **two** ^3He (2 protons + 1 neutron) with spin S =0 or 1, assume S = 1 with degeneracy 2S + 1 = 3. What is the new value of T_C? Hint: Use the equation in the appendix. Further ^3He is a fermion of spin S = ½ , but at really low temperature, **two** ^3He form a "bonded" pair that is effectively a boson of spin $S = 1$, which can form a BE condensate.

Question 6 Gold has an atomic mass 197 amu, a density $19.3\times10^3 kg/m^3$, a Fermi energy 5.54eV, and a **conductivity** of $4.9\times10^7\Omega^{-1}\bullet m^{-1}$. Estimate the mean free path, ℓ, and the average collision time between collisions, τ, under the assumption that each gold atom contributes one conduction electron.

Appendix

Degenerate Fermion Gas: **Fermi energy,** $E_F = \frac{h^2}{8m}\left(\frac{6}{(2S+1)\pi}\frac{N}{V}\right)^{2/3}$, where $S = \frac{1}{2}, \frac{3}{2}, \ldots$ is the spin of the fermions. For elcctrons, protons, and neutrons, $S = \frac{1}{2}$, and we obtain the same relation 9.42. **Internal Energy (average total energy)**, $U = \frac{3}{5}NE_F$. **Pressure** (P) $PV = \frac{2}{3}U$, 1atm = $1.01325 \times 10^5 Pa$. **Fermi Temperature:** $T_F = E_F / k_B$. **Fermi Speed** $u_F = \sqrt{2E_F / m}$

Drude Model: conductivity $\sigma = ne^2\ell / mu_F$, ℓ mean free path, mean collision time $\tau = \ell / u_F$, resitivity $\rho = 1/\sigma$, resistance $R = L\rho / A$.

White Dwarfs and Neutron Star: Non-relativistic approximate theory

Equilibrium Star Radius, $R_{star} = \frac{N^{5/3}h^2}{4GM_{star}^2 m_{fermion}}\left(\frac{9}{4\pi^2}\right)^{2/3}$, M_{star} is the mass of the star, $m_{fermion}$ mass of the identical fermions that produces the degenerate gas pressure, $G = 6.67 \times 10^{-11} m^3 \bullet kg^{-1} \bullet s^{-2}$ is Newton's gravitational constant.

Bose-Einstein Condensate:

Critical Temperature: $T_c = \frac{h^2}{2mk_B}\left(\frac{1}{2\pi(2.315)}\frac{1}{2S+1}\frac{N}{V}\right)^{2/3}$, where $S = 0,1,2\ldots$ is the spin of the bosons, and for He4, $S = 0$, and we obtain the same relation 9.65. If N is the total number of boson, and N_0 is the number of particles in the Bose-Einstein condensate state: $N_0 = N\left(1 - \left(\frac{T}{T_c}\right)^{3/2}\right), T \le T_C$, $N_0 = 0, T > T_C$.

QUANTUM COUNTING: Density of state, $g(E) = (2S+1)\frac{2\pi V}{h^3}(2m)^{3/2}E^{1/2}$. For degenerate gas $g(E) = \frac{3N}{2}E_F^{-3/2}E^{1/2}$, $N = \int_0^{E_F} g(E)dE$. Number of fermions in the energy range E_1 to E_2 we integrate $N_{E_1 to E_2} = \int_{E_1}^{E_2} g(E)dE$. This assumes that $E_2 < E_F$. In the case where $E_1 < E_F$ but $E_2 > E_F$, $N_{E_1 to E_2} = \int_{E_1}^{E_F} g(E)dE$.