PHYS2332-Modern Physics II
Winter 2018, Assignment #2

Assigned on Tuesday 23 January. Due on Wednesday 31 January, 5 pm.

Do Problem 8, 16 and 18 of chapter 8.

Question 1 For the Bohr hydrogen atom, the **time-independent Schrödinger equation** in spherical coordinate is given by equation 7.3 with Coulomb potential energy,

$V=-\frac{e^2}{4\pi\varepsilon_0}\frac{1}{r}$, and energy Eigenvalue, $E_n=-\frac{m}{2\hbar^2}\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2\frac{1}{n^2}=-\frac{13.6eV}{n^2}, n=1,2,3\ldots$,

where m is the electron's mass (effectively equal to μ), which by **direct substitution** give

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi_{nlm_\ell}}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi_{nlm_\ell}}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi_{nlm_\ell}}{\partial\phi^2}$$

$$+\frac{2m}{\hbar^2}\left(E_n+\frac{e^2}{4\pi\varepsilon_0}\frac{1}{r}\right)\psi_{nlm_\ell}=0$$

A) Using equation 7.17 and table 7.1 and 7.2, explicitly write out $\psi_{21-1}(r,\theta,\phi)$.

B) Directly substitute the relation of part A) into the above **partial differential equation** (PDE) and perform the partial differentiation to verify that $\psi_{21-1}(r,\theta,\phi)$ is the solution of the **time-independent Schrödinger equation**.

Question 2 Unsöld Theorem states that the electron's cloud distribution of an atom with filled or half-filled subshell is isotropic (spherical), consequently the atom in that state has zero **total orbital angular momentum**, L = 0. This is illustrated by the mathematical relation $\sum_{m_\ell=-\ell}^{m_\ell=\ell}\left|Y_{\ell,m_\ell}\right|^2=\frac{2\ell+1}{4\pi}$.

A) Write down the ground state configuration of Nitrogen (N), Z = 7.

B) Verify that the electron cloud's distribution of **Nitrogen's half-filled 2p subshell** is isotropic by showing that $\sum_{m_\ell=-\ell}^{m_\ell=\ell}\left|Y_{\ell,m_\ell}\right|^2=\frac{2\ell+1}{4\pi}$ is true for the half-filled 2p subshell (ℓ).

Question 3 Atomic State of Titanium (Ti, Z = 22) The ground state of Ti has $S = 1$ and $L = 3$.

(A) Write down the ground state electronic configuration of Ti.

(B) What are the possible values of J? Write down the **spectroscopic notation** for these values of S, L and J.

(C) Which on of the states in part (B) represent the ground state of Ti. Briefly justify your answer.