PHYS2332-Modern Physics II
Winter 2018
Assignment #1, REVIEW of the Method of Quantum Mechanics (QM)

Assigned on Friday January 12, 2018. Due on Friday 19, 2018.

There are useful equations on page 2.

Question 1 Consider the 1D harmonic oscillator (HOS) **eigen-solution** of equs 6.57 to 6.59 (page 223) $\psi_n = H_n(x)\exp-(\alpha x^2/2)$, which corresponds to the **energy eigenvalue**, $E_n = \hbar\omega(n+1/2)$, $\omega = \sqrt{k/m}$.

A) Show that the arbitrary **time-independent** function state $\psi_a = \left(\sqrt{\frac{2}{5}} - i\sqrt{\frac{2}{5}}\right)\psi_0 + i\sqrt{\frac{1}{5}}\psi_2$ is **not** a **solution** of the **time-independent Schrodinger equation**.

B) Show that the arbitrary **time-dependent** function state $\Psi_a = \left(\sqrt{\frac{2}{5}} - i\sqrt{\frac{2}{5}}\right)\psi_0 \exp\left(-\frac{iE_0 t}{\hbar}\right) + i\left(\sqrt{\frac{1}{5}}\right)\psi_2 \exp\left(-\frac{iE_2 t}{\hbar}\right)$ is a **solution** of the **time-dependent Schrodinger equation**.

C) In no more than 3 sentences comment on the physical significant of the results of part A and B. If a measurement were made on a 1D HOS that is in the state described by Ψ_a, what would be the possible values of the energy? What would be the probability for each value of the energy?

D) If the measurement find HOS in n = 2 state, then find the **most probable** position. Repeat in the case where HOS is in state n = 0.

Question 2 Consider the particle in the infinite square-well potential of section 6.3, and the eigen-solution, ψ_n, of equation 6.34 and 6.35. Focus on the ground state (n = 1), $\psi_1 = (2/L)^{1/2}\sin(\pi x/L)$. **A)** Calculate the probability that the particle occupy a position in the range x = L/12 to x = L/6. **B)** Calculate the average momentum of the particle. **C)** Find the most **probable positions** of the particle. **D)** What is the probability that the particle occupy a position at x = 0 and x = 2L.

Question 3 **Schrodinger cat** in **two boxes**. Read the article in the link: https://www.sciencenewsforstudents.org/article/famous-physics-cat-now-alive-dead-and-two-boxes-once

In **no more** than **100 words**, discuss the similarity in this experiment with the Schrodinger cat state discussed in class.

Question 4 Problem 4, Chapter 8.

Appendix

1D probability distribution:

$|\psi(x)|^2 dx = \psi^*(x)\psi(x)dx$ is the probability that a particle occupies a position in the range x to $x+dx$, valid for small dx. **Most probable position**, $dP(x)/dx = 0$, with $P(x) = |\psi(x)|^2$. Probability that the particle occupy a position in the range $x_1 \le x \le x_2$ is $P_{x_1 \le x \le x_2} = \int_{x_1}^{x_2} P(x)dx = \int_{x_1}^{x_2} |\psi(x)|^2 dx$.

Normalization: It follow from the above that since the total probability must be one, the normalization condition is $\int_{x_1}^{x_2} P(x)dx = \int_{x_1}^{x_2} |\psi(x)|^2 dx = 1$.

Expectation (Average) values: Let $f(x)$ represent a physical quantity, the average $\langle f \rangle = \int_{-\infty}^{\infty} dx \psi^* f \psi$.

For the momentum, use equation 6.21 $\hat{p} = \frac{\hbar}{i}\frac{\partial}{\partial x}$, the **average momentum** is

$\langle p \rangle = \int_{-\infty}^{\infty} dx \psi^* \hat{p} \psi = \frac{\hbar}{i}\int_{-\infty}^{\infty} dx \psi^* \frac{\partial \psi}{\partial x}$.