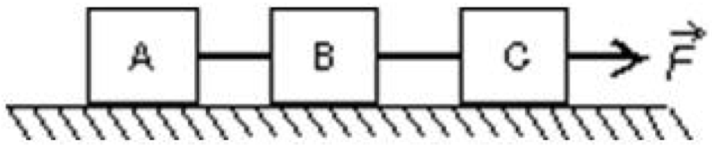
Newton’s Law

Theory and Problems

A Multiple Choice: Part I

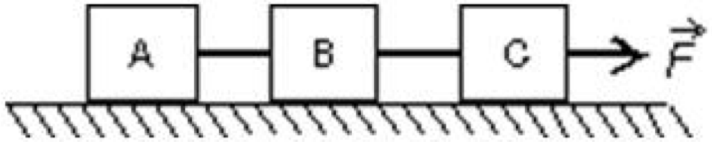
- Three blocks (A, B, C), each having the same mass *M*, are connected by strings as shown. Block C is pulled to the right by a force that causes the entire system to accelerate. Neglecting friction, the **net force** acting on block B is:



- **A)** 0; **B)** $\vec{F}/3$; **C)** $\vec{F}/2$; **D)** $2\vec{F}/3$; **E)** $\vec{F}$
- Solution: Composite System and Newton second law
- $F = 3Ma \rightarrow a = \frac{F}{3M}$
- Net force on any box: $F_{net,1box} = Ma = \frac{F}{3}$, **B**

A Multiple Choice: Part II

- Three blocks (A, B, C), each having the same mass *M*, are connected by strings as shown. Block C is pulled to the right by a force that causes the entire system to accelerate. Neglecting friction, the **Tension in rope** connecting B and C:



- **A)** 0; **B)** $\vec{F}/3$; **C)** $\vec{F}/2$; **D)** $2\vec{F}/3$; **E)** $\vec{F}$
- Solution: Composite System and Newton second law
- $F = 3Ma \rightarrow a = \frac{F}{3M}$
- Net force on any box: $F_{net,1box} = Ma = \frac{F}{3}$
- Horizontal force on C gives $F - T_{BC} = F_{net,1box} = \frac{F}{3} \rightarrow T_{BC} = \frac{2F}{3}$. D

Friction Section 6.1

- In Figure 6.1.1 (a) a top slab sliding over a bottom slab experiences a friction force, $\vec{f}$, opposing its motion.
- Friction is due to the microscopic contacts between the two surfaces. This is seen in Figure 6.1.1 (b).
- The friction depends on the properties of the surface, such as its molecular makeup, and its smoothness/roughness.

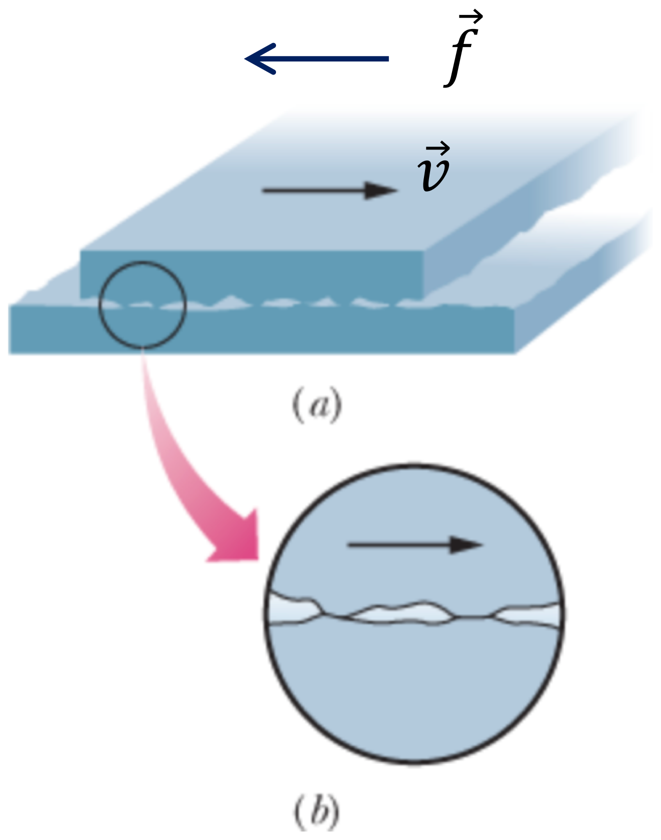


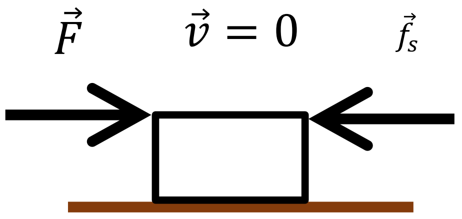
Figure 6.1.1

Static and Kinetic Friction

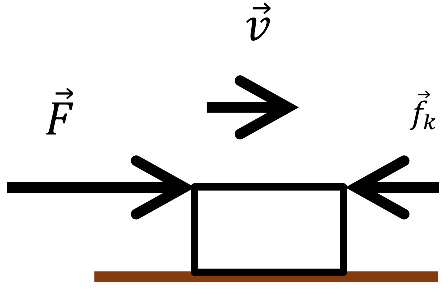
When a Force, $\vec{F}$, is applied to an object at **rest** on a **surface** with **friction**, there is a **force** of **static friction**, $\vec{f_s}$, that cancels the applied force **up** to a **maximum value**:

$$f_{s,max} = \mu_s F_N > f_s$$

where μ_s is the coefficient of static friction



$$\vec{F} = \vec{f}_s$$
$$\vec{F}_{Net} = \vec{F} - \vec{f_s} = 0$$



$$\vec{F} > \vec{f}_k$$
$$\vec{F}_{Net} = \vec{F} - \vec{f_k} > 0$$

If the object is moving with a velocity, $\vec{v}$, on a **surface** with **friction**, there is a **force** of **kinetic friction**, $\vec{f_k}$ in the opposite direction of the velocity, with a magnitude:

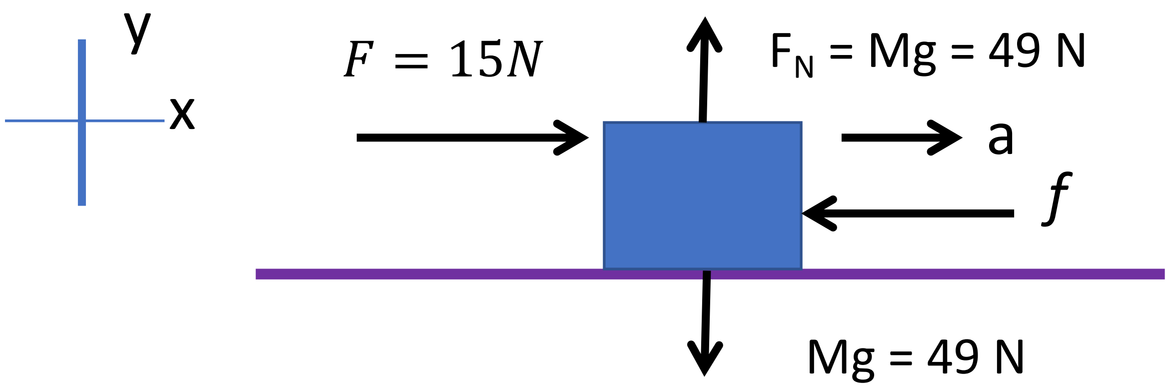
$$f_k = \mu_k F_N$$

where μ_k is the coefficient of kinetic friction.

If an object at rest is acted on by an applied $\mathrm{F} > f_{s,max}$, the object will move and the friction force equals $f_k = \mu_k F_N$

Simple Friction Problem I

- Below is a box of mass M = 5 kg at **rest** on a surface with friction coefficient, $\mu_k = 0.15$ and $\mu_s = 0.3$. A horizontal force F = 15 N is applied. Calculate the force of Friction and acceleration.



Y-comp: $F_{Net,y} = F_N - Mg = 0$

$$F_N = 49N$$

Maximum Friction: $f_{s,max} = \mu_S F_N = 14.7N$

F = 15N > 14.7N. It will Move!

Kinetic Friction: $\underline{f_k = \mu_k F_N = 7.35N}$

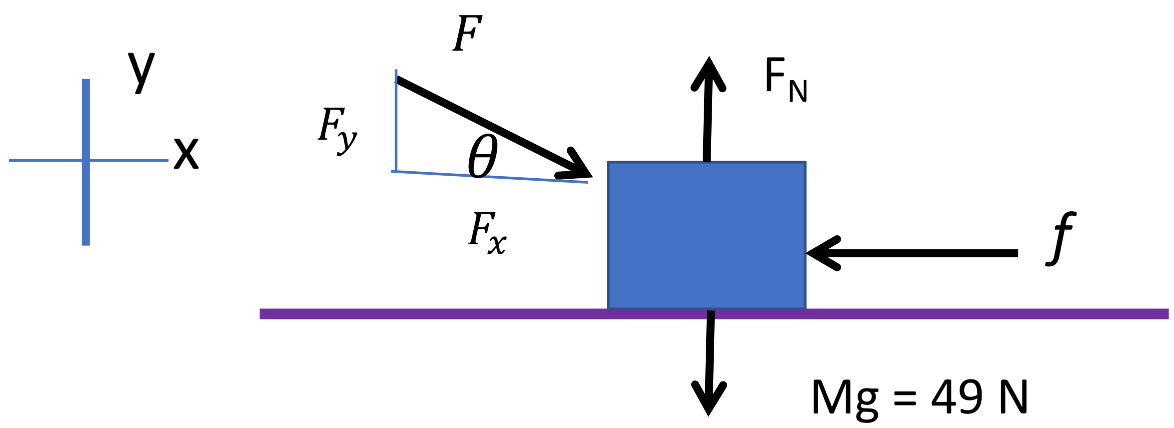
Newton's 2nd Law: $F_{Net,x} = F - f_k = Ma$

$$15N - 7.35N = 5kg \times a$$

$$\underline{a = 1.53m \cdot s^{-2}}$$

Simple Friction Problem II

- Below is a box of mass M = 5 kg at **rest** on a surface with friction coefficient, $\mu_k = 0.15$ and $\mu_s = 0.3$. A force F = 15 N, applied at angle $\theta = 36.9^{\circ}$ is applied. Calculate the force of Friction and acceleration.



Y-comp: $F_{Net,y} = F_N - F\sin\theta - Mg = 0$

$$F_N = 15N \times 0.6 + 49N = 58N$$

Maximum Friction: $f_{s,max} = \mu_S F_N = 17.8N$

$F_x = 15N\cos\theta = 12N < 17.8\text{N}$

F_x cannot overcome static friction

Friction force is static, f_s

$$F_{Net,x} = F_x - f_s = 0$$

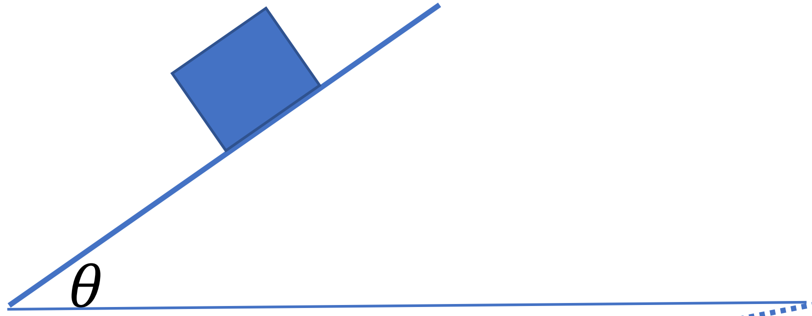
$$f_s = 15N\cos\theta = 12N$$

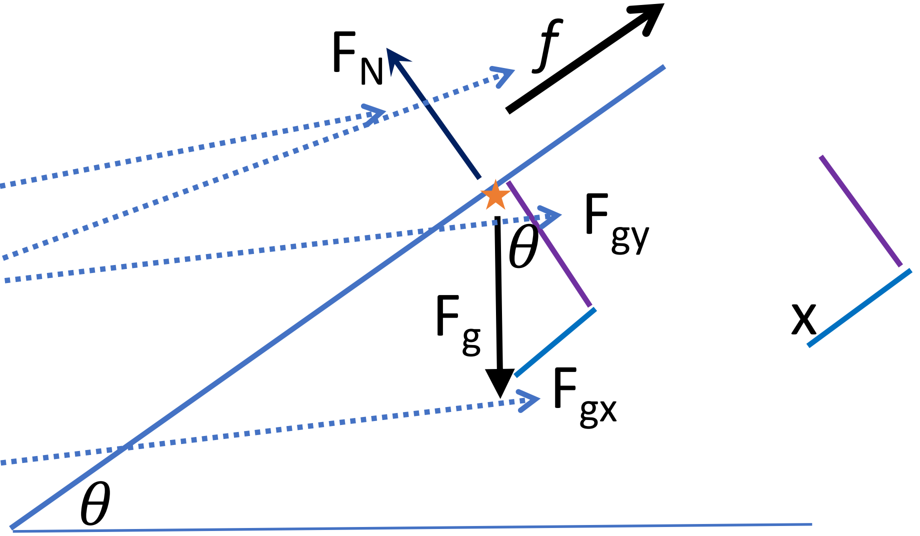
$$\underline{a = 0}$$

Incline with Friction Problem I

- A **box** of **mass m = 8 kg** is **resting** on a $\theta = 36.9^{\circ}$ incline, with $\mu_k = 0.4$ and $\mu_s = 0.8$.
- Find the force of friction and the acceleration.

Direction of friction, $\vec{f}$, is **opposite** direction of motion … or **possible motion**





Draw FBD

Y-com: $F_{Net,y} = F_N + F_{gy} = 0$

$$F_N - mg\cos 36.9 = 0 \rightarrow F_N = 62.72N$$

Maximum Static Friction: $f_{s,max} = F_N \mu_s = 50.176N$

$$F_{gx} = mg\sin 36.9 = 47.04N < f_{s,max}$$

F_{gx} cannot overcome static friction:

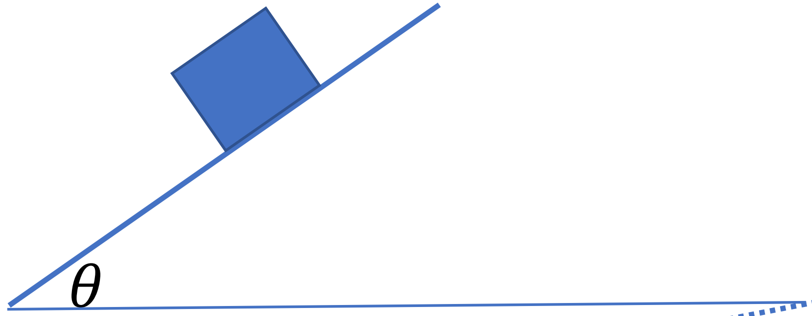
Acceleration is zero!

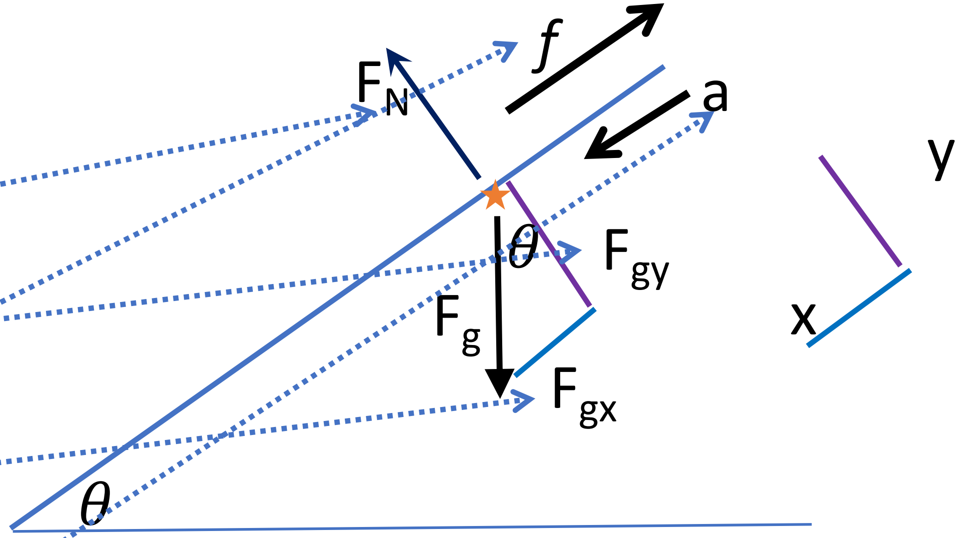
X-comp: $F_{Net,x} = F_{gx} - f_s = 0 \rightarrow f_s = 47.04N$

Incline with Friction Problem II

- A **box** of **mass m = 8 kg** is **resting** on a $\theta = 53.1^{\circ}$ incline, with $\mu_k = 0.4$ and $\mu_s = 0.8$.
- Find the force of friction and the acceleration.

Direction of friction, $\vec{f}$, is **opposite** direction of motion … or **possible motion**





Draw FBD

Y-com: $F_{Net,y} = F_N + F_{gy} = 0$

$$F_N - mg\cos 53.1 = 0 \rightarrow F_N = 47.04N$$

Maximum Static Friction: $f_{s,max} = F_N\mu_s = 37.632N$

$$F_{gx} = mg\sin 53.1 = 62.72N > f_{s,max}$$

F_{gx} overcome static friction

Kinetic Friction, $\underline{f_k = f = F_N\mu_k = 18.8N}$

X-comp: $F_{Net,x} = F_{gx} - f_k = ma \rightarrow a = 5.49m \cdot s^{-2}$

What if the box is moving down?

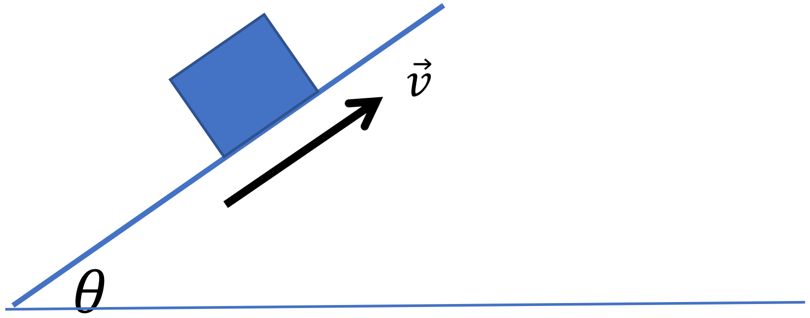
Would the acceleration change?......No!

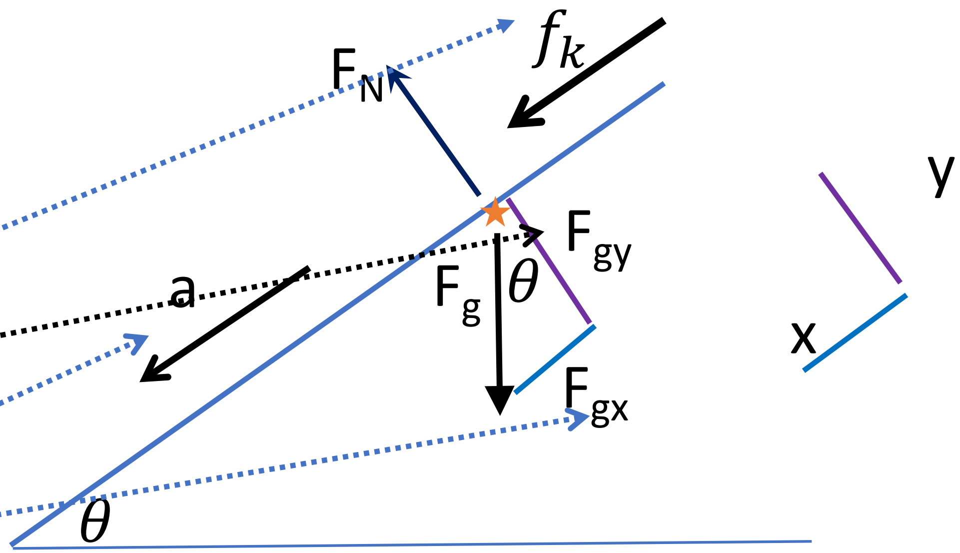
Friction will still be up incline, and $a = 5.49m \cdot s^{-2}$!

Incline with Friction Problem III, **moving up**

- A **box** of **mass m = 8 kg** is **moving up** a $\theta = 53.1^{\circ}$ incline, with $\mu_k = 0.4$ and $\mu_s = 0.8$.
- Find the force of **friction** and the **acceleration**.

Direction of friction, $\vec{f}$, is **opposite** direction of **motion**.





Draw FBD

Y-com: $F_{Net,y} = F_N + F_{gy} = 0$

$$F_N - mg\cos 53.1 = 0 \rightarrow F_N = 47.07N$$

Since already moving, **Kinetic Friction**:

$$f_k = F_N \mu_k = 18.8N$$

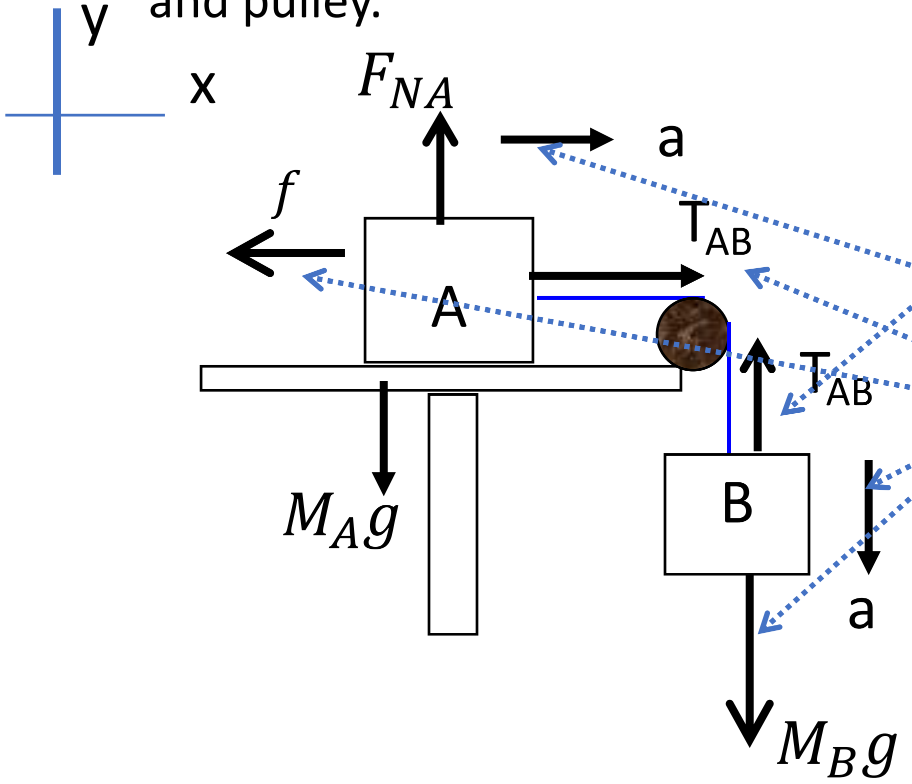
$$F_{gx} = mg\sin 53.1 = 62.7N$$

X-comp: $F_{Net,x} = F_{gx} + f_k = ma$

Acceleration: $a = \frac{62.7N + 18.8N}{8kg} = 10.18\,\frac{m}{s^2}$

2-Body with friction I

In the diagram below block A has a mass of 4.00 kg and block B has mass 8.00 kg. Block A is resting on a table with **frictionless coefficient**, $\mu_s = 0.3$, $\mu_k = 0.2$. Block A is released from rest, calculate the Tensions T_{AB} and Acceleration a. The rope is massless, and there is **no friction** between the rope and pulley.



<u>Draw FBD **B**</u>:

Y-comp: $T_{AB} - M_B g = -M_B a$ [1]

<u>Draw FBD **A**</u>:

Y-comp: $F_{NA} = 39.2N$

Maximum static friction $f_{s,max} = F_{NA}\mu_s = 11.76\ N$. Will it move?

$M_B g$ make A move **right**: $M_B g - f_s = 66.64N > 0$

It will move! Friction is kinetic, $f = f_k = F_{NA}\mu_k = 7.84N$

X-comp: $T_{AB} - f_k = M_A a$,

$$T_{AB} = f_k + M_A a \quad [2]$$

<u>Solution</u>: Subs [2] into [1]

$$f_k + M_A a - M_B g = -M_B a \rightarrow M_B g - f_k = (M_B + M_A)a$$

$$a = \frac{M_B g - f_k}{M_B + M_A} = \frac{8kg \times 9.8m \cdot s^{-2} - 7.84N}{8kg + 4kg} = 5.88m \cdot s^{-2}$$

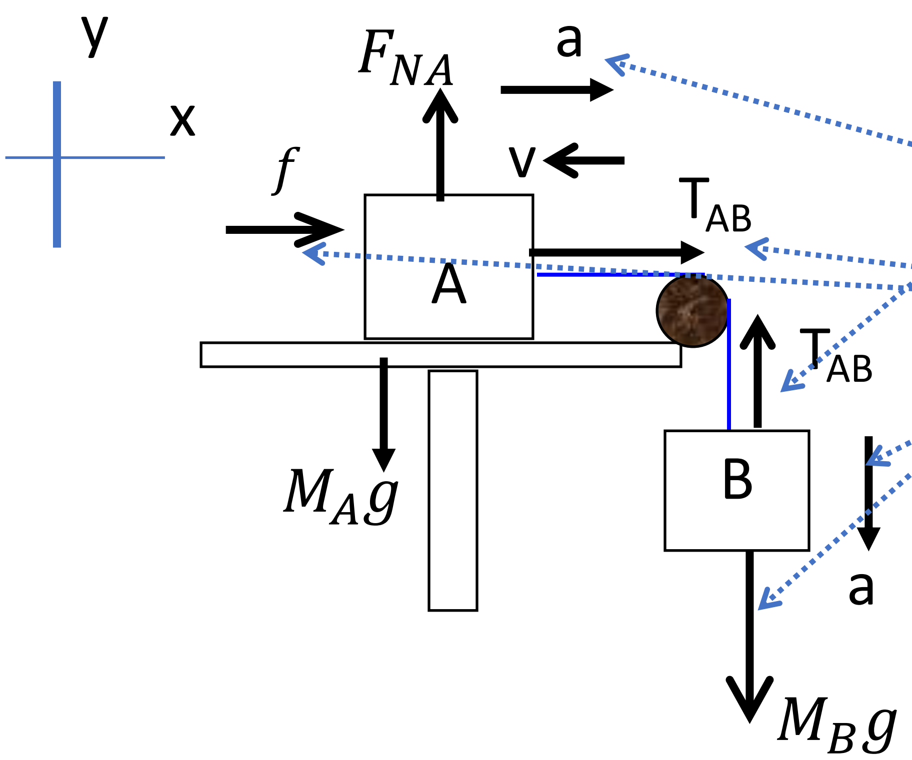
Use [2] $T_{AB} = f_k + M_A a = 7.84N + 4kg \times 5.88m \cdot s^{-2} = 31.36N$

Verify with [1]

$$T_{AB} = 8kg(g - a) = 31.36N$$

2-Body with friction II

In the diagram below block A has a mass of 4.00 kg and block B has mass 8.00 kg. Block A is on a table with **frictionless coefficient**, $\mu_s = 0.3$, $\mu_k = 0.2$, and **moving left**. Calculate the Tensions T_{AB} and Acceleration a. The rope is massless, and there is **no friction** between the rope and pulley. Will the acceleration increase compare to the last problem?



<u>Draw FBD **B**</u>:

Y-comp: $T_{AB} - M_B g = -M_B a$ [1]

<u>Draw FBD **A**</u>:

Y-comp: $F_{NA} = 39.2N$

It already moving! Friction is kinetic, $f = f_k = F_{NA}\mu_k = 7.84N$

X-comp: $T_{AB} + f_k = M_A a$,

$$T_{AB} = -f_k + M_A a \quad [2]$$

<u>Solution</u>: Subs [2] into [1]

$$-f_k + M_A a - M_B g = -M_B a \rightarrow M_B g + f_k = (M_B + M_A)a$$

$$a = \frac{M_B g + f_k}{M_B + M_A} = \frac{8kg \times 9.8m \cdot s^{-2} + 7.84N}{8kg + 4kg} = 7.19m \cdot s^{-2}$$

Use [2] $T_{AB} = -f_k + M_A a = -7.84N + 4kg \times 7.19m \cdot s^{-2} = 20.92N$

Verify with [1]

$$T_{AB} = 8kg(g - a) = 20.88N$$