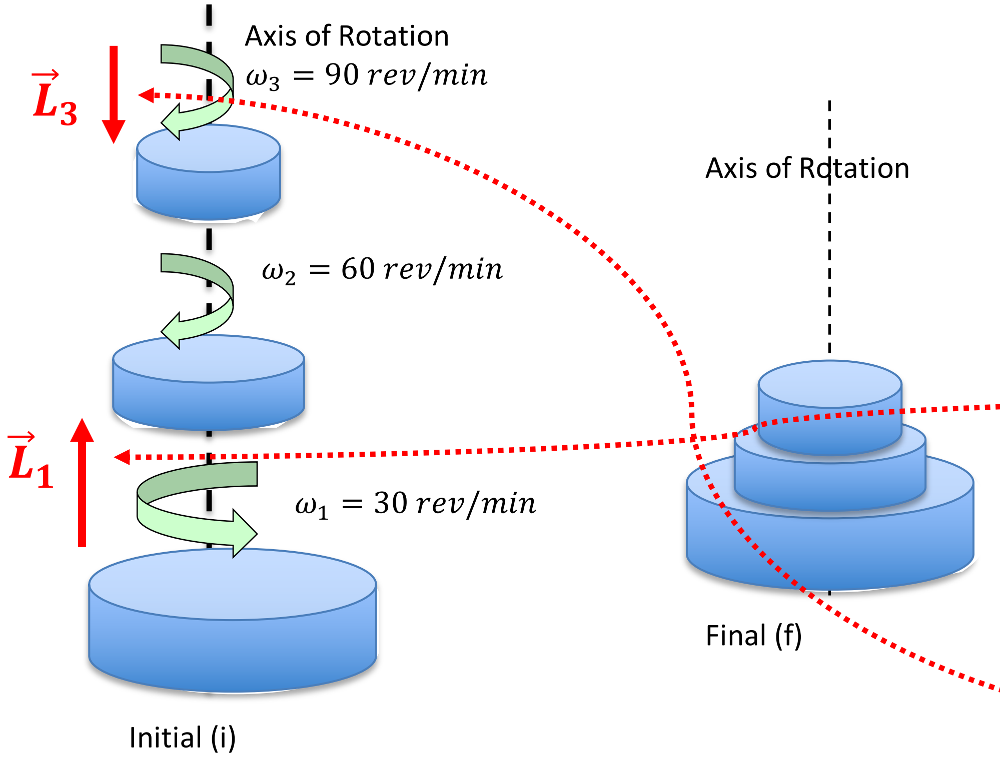
Problem 6: Part A

In Figure, disk 1 is hollow with M_1 = 3kg , radius R_1 = 0.5 kg, Disk 3 is solid with M_3 = 0.8 kg, radius R_3 = 0.15 m. Data on disk 2 is unknown . Initially, the disks are rotation as shown. They undergo a collision to the final state (right), where the disks stick together, with an angular velocity of $\omega_f = 2.453 rad \cdot s^{-1}$, in the same direction of ω_1.

A) Calculate the **initial** (left) angular momentum of each of the disks 1 and 2, and indicate the directions beside each disk. Hence find the moment of inertia and angular momentum of disk 2. **B)** Calculate the final angular momentum of all three disks on the right. Indicate the direction of the final angular momentum, $\vec{L}_f$. **C)** Find the final angular velocity (magnitude and direction) if the I_2 you calculated is **tripled**.



Solution of Part A

Assume **up** (+z and **ccw** about axis of rotation) to be **positive**.

Disk 1: $I_1 = M_1 R_1^2 = 5kg \times (0.5m)^2 = 1.25 kg \cdot m^2$

$\omega_1 = 30 \frac{rev}{min} = \pi \frac{rad}{s}$ $\quad L_1 = I_1 \omega_1 = 3.927 \frac{kg \cdot m^2}{s}$

Using the **right-hand rule (rhr)** direction of $\vec{L}_1$ is **up**, as in figure.

$$\vec{L}_1 = I_1 \omega_1 = 3.927 \frac{kg \cdot m^2}{s} \hat{k}$$

Disk 3: $I_3 = \frac{1}{2} M_3 R_3^2 = \frac{1}{2} 0.8kg \times (0.15m)^2 = 9 \times 10^{-3} kg \cdot m^2$

$\omega_3 = 90 \frac{rev}{min} = 3\pi \frac{rad}{s}$ $\quad L_3 = I_3 \omega_3 = -0.084823 \frac{kg \cdot m^2}{s}$

Using the **right-hand rule (rhr)** direction of $\vec{L}_3$ is **down**, as in figure.

$$\vec{L}_3 = I_3 \omega_3 = -0.084823 \frac{kg \cdot m^2}{s} \hat{k}$$

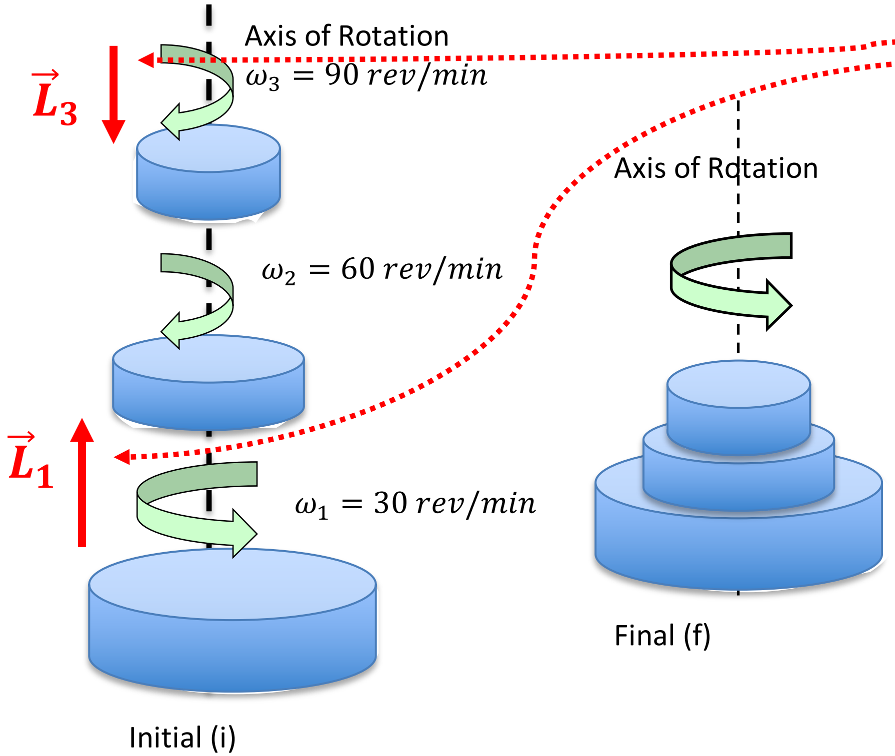
Problem 6: Part A (Continued 2)

In Figure, disk 1 is hollow with M_1 = 3kg , radius R_1 = 0.5 kg, Disk 3 is solid with M_3 = 0.8 kg, radius R_3 = 0.15 m. Data on disk 2 is unknown . Initially, the disks are rotation as shown. They undergo a collision to the final state (right), where the disks stick together, with an angular velocity of $\omega_f = 2.453 rad \cdot s^{-1}$, in the same direction of ω_1.

A) Calculate the **initial** (left) angular momentum of each of the disks 1 and 2, and indicate the directions beside each disk. Hence find the moment of inertia and angular momentum of disk 2.

<u>Solution of Part A</u>

Assume **up** (+z and **ccw** about axis of rotation) to be **positive**.



$$\vec{L}_1 = I_1\omega_1 = 3.927\frac{kg\cdot m^2}{s}\hat{k} \qquad \vec{L}_3 = -I_3\omega_3 = -0.084823\frac{kg\cdot m^2}{s}\hat{k}$$

AFTER COLLISION: Final Moment of inertia

$$I_f = I_1 + I_2 + I_3 = 1.25 kg\cdot m^2 + I_2 + 9\times10^{-3} kg\cdot m^2 = 1.259 kg\cdot m^2 + I_2$$

Final angular momentum $L_f = I_f\omega_f$, $\omega_f = 2.453 rad\cdot s^{-1}$,

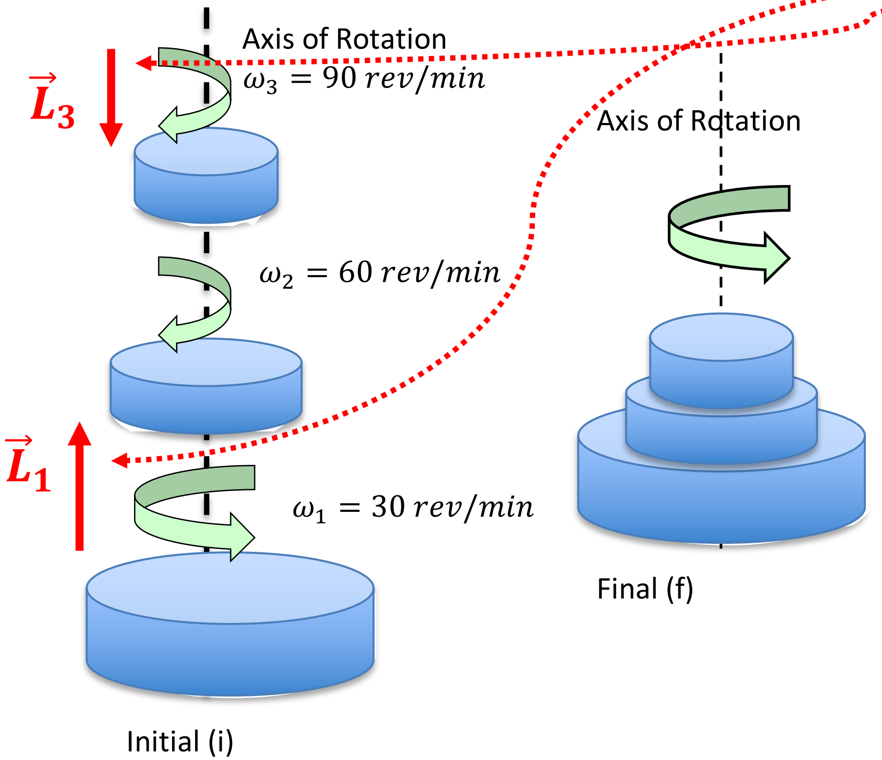
$$L_f = I_f\omega_f = (1.259 kg\cdot m^2 + I_2)\times 2.453 rad\cdot s^{-1}$$

$$L_f = I_f\omega_f = 3.088327\frac{kg\cdot m^2}{s} + I_2\omega_f$$

Problem 6: Part A (Continued 3)

In Figure, disk 1 is hollow with M_1 = 3kg , radius R_1 = 0.5 kg, Disk 3 is solid with M_3 = 0.8 kg, radius R_3 = 0.15 m. Data on disk 2 is unknown . Initially, the disks are rotation as shown. They undergo a collision to the final state (right), where the disks stick together, with an angular velocity of $\omega_f = 2.453 rad \cdot s^{-1}$, in the same direction of ω_1.

A) Calculate the **initial** (left) angular momentum of each of the disks 1 and 2, and indicate the directions beside each disk. Hence find the moment of inertia and angular momentum of disk 2.



<u>Solution of Part A</u>

Assume **up** (+z and **ccw** about axis of rotation) to be **positive**.

$\vec{L}_1 = I_1\omega_1 = 3.927 \frac{kg\cdot m^2}{s}\hat{k}$ $\quad \vec{L}_3 = I_3\omega_3 = -0.084823 \frac{kg\cdot m^2}{s}\hat{k}$

AFTER COLLISION: Final Angular Momentum

$$L_f = I_f\omega_f = 3.088327 \frac{kg\cdot m^2}{s} + I_2\omega_f$$

CONSEVATION OF ANGULAR MOMENTUM

Initial $L_i = I_1\omega_1 - I_2\omega_2 - I_3\omega_3 = I_f\omega_f = L_f$ Final

The term $-I_2\omega_2$ is negative sign due to CW rotation about the axis

$$3.927 \frac{kg\cdot m^2}{s} - I_2\omega_2 - 0.084823 \frac{kg\cdot m^2}{s} = 3.088327 kg\cdot m^2\cdot s^{-1} + I_2\omega_f$$

$$0.75385 \frac{kg\cdot m^2}{s} = I_2(\omega_f + \omega_2) \rightarrow I_2 = \frac{0.75385 \frac{kg\cdot m^2}{s}}{2.453 rad\cdot s^{-1} + 2\pi rad\cdot s^{-1}}$$

$$\boxed{I_2 = 0.0863 kg\cdot m^2}$$

Problem 6: Part B

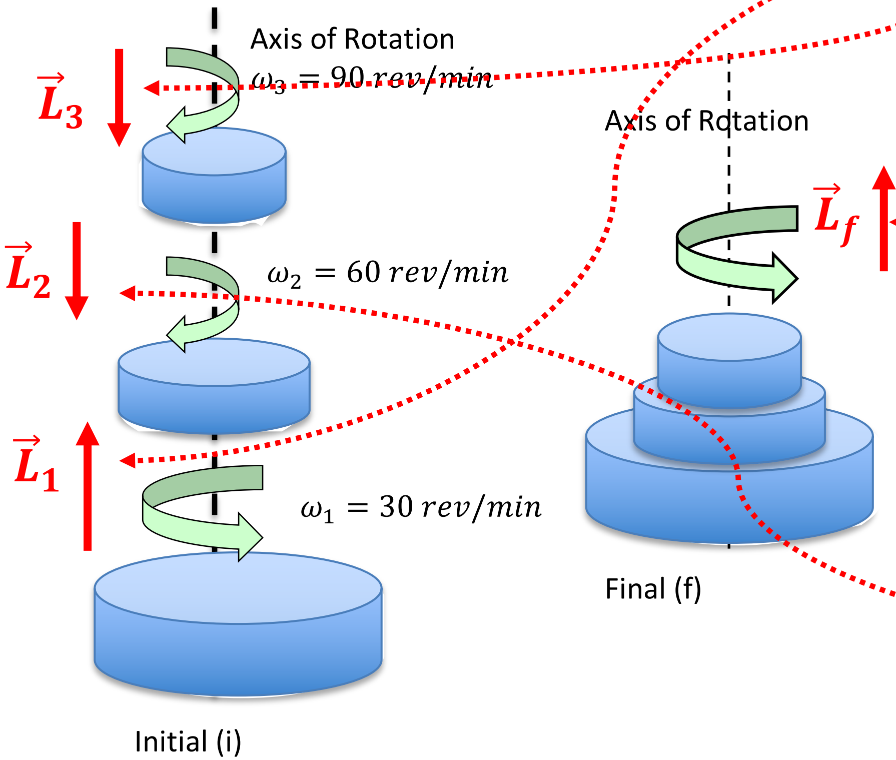
In Figure, disk 1 is hollow with M_1 = 3kg , radius R_1 = 0.5 kg, Disk 3 is solid with M_3 = 0.8 kg, radius R_3 = 0.15 m. Data on disk 2 is unknown . Initially, the disks are rotation as shown. They undergo a collision to the final state (right), where the disks stick together, with an angular velocity of $\omega_f = 2.453 rad \cdot s^{-1}$, in the same direction of ω_1.

B) Calculate the final angular momentum of all three disks on the right. Indicate the direction of the final angular momentum, $\vec{L}_f$.

Solution of **Part B**: Below are results from **part A**

Assume **up** (+z and **ccw** about axis of rotation) is **positive**.

$$\vec{L}_1 = I_1\omega_1 = 3.927\frac{kg\cdot m^2}{s}\hat{k} \qquad \vec{L}_3 = I_3\omega_3 = -0.084823\frac{kg\cdot m^2}{s}\hat{k}$$



$$L_f = I_f\omega_f = 3.088327\frac{kg\cdot m^2}{s} + I_2\omega_f$$

$$I_2 = 0.0863 kg\cdot m^2$$

$$L_f = I_f\omega_f = 3.088327\frac{kg\cdot m^2}{s} + (0.0863 kg\cdot m^2)(2.453 rad\cdot s^{-1})$$

$$\vec{L}_f = 3.11\frac{kg\cdot m^2}{s}\hat{k}$$

$$\omega_2 = 60\frac{rev}{min}\times\frac{2\pi\, rad\cdot rev^{-1}}{60s\cdot min^{-1}} = 2\pi\frac{rad}{s}$$

$$L_2 = -I_2\omega_2 = -(0.0863 kg\cdot m^2)(2\pi\, rad\cdot s^{-1}) = -0.542\frac{kg\cdot m^2}{s}$$

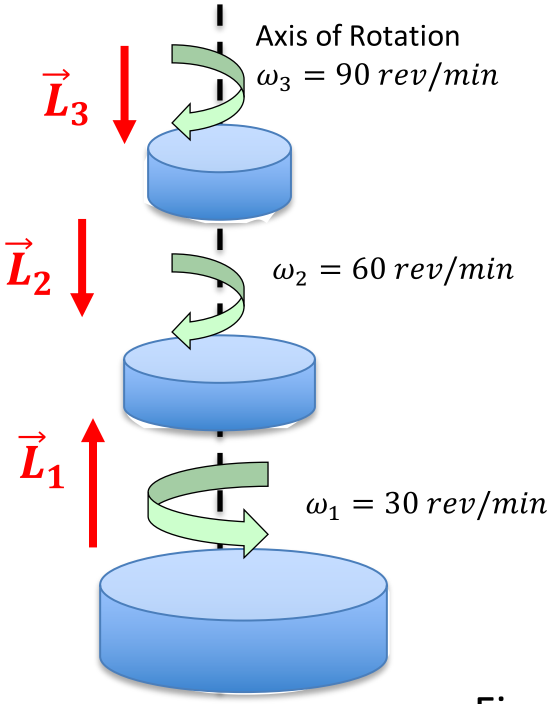
$$\vec{L}_2 = -0.542\frac{kg\cdot m^2}{s}\hat{k}$$

Problem 6: Part C

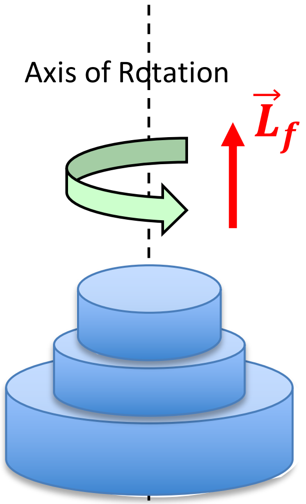
In Figure, disk 1 is hollow with M_1 = 3kg , radius R_1 = 0.5 kg, Disk 3 is solid with M_3 = 0.8 kg, radius R_3 = 0.15 m. Data on disk 2 is unknown . Initially, the disks are rotation as shown. They undergo a collision to the final state (right), where the disks stick together, with an angular velocity of $\boldsymbol{\omega_f} = ?$. In part A we showed tat the mo $I_2 = 0.0863 kg \cdot m^2$ moment of inertia of disk 2 is $I_2 = 0.0863 kg \cdot m^2$.

C) Find the final angular velocity (magnitude and direction) if the I_2 you calculated is **tripled**.

Assume **up** (+z and **ccw** about axis of rotation) is **positive**.



Initial (i)



Final (f)

Solution **C:**

Disk 2: $I_2 = 3 \times 0.0863 kg \cdot m^2 = I_2 = 0.259 kg \cdot m^2$

Disk 1: $I_1 = M_1 R_1^2 = 1.25 kg \cdot m^2$ **Disk 3**: $I_3 = \frac{1}{2} M_3 R_3^2 = 9 \times 10^{-3} kg \cdot m^2$

CONSEVATION OF ANGULAR MOMENTUM

Initial $L_i = I_1\omega_1 - I_2\omega_2 - I_3\omega_3 = I_f\omega_f = L_f$ Final

$$L_i = 1.25 kg \cdot m^2 \times \pi \frac{rad}{s} - 0.259 kg \cdot m^2 \times 2\pi \frac{rad}{s} \text{-} -9 \times 10^{-3} kg \cdot m^2 \times 3\pi \frac{rad}{s}$$

$$L_i = 2.215 kg \cdot m^2 \cdot s^{-1}$$

Final moment of inertia $I_f = I_1 + I_2 + I_3 = 1.5179 kg \cdot m^2$

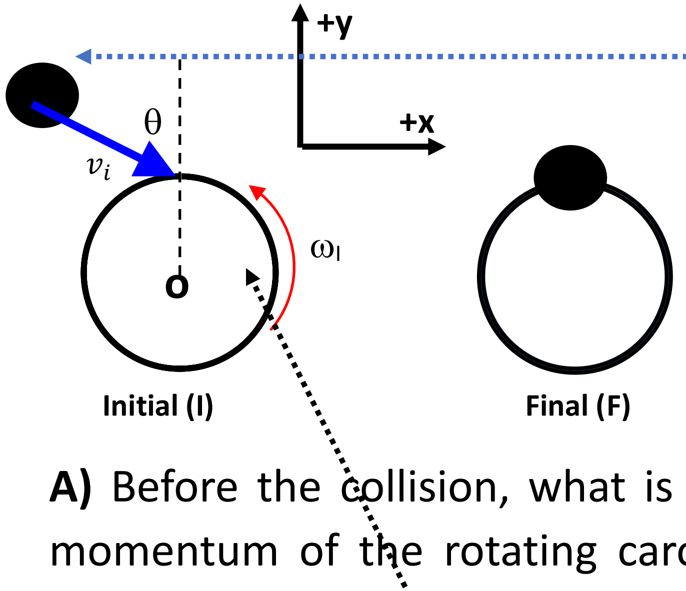
Final Angular Momentum $L_f = I_f\omega_f = 2.215 kg \cdot m^2 \cdot s^{-1} = L_i$ **Initial**

$$\omega_f = \frac{L_i}{I_f} = \frac{2.215 kg \cdot m^2 \cdot s^{-1}}{1.5179 kg \cdot m^2} \rightarrow \omega_f = 1.403 rad \cdot s^{-1}$$

Final angular momentum is $L_f = I_f\omega_f = 2.215 kg \cdot m^2 \cdot s^{-1}$, up (CCW) $\vec{L}_f = 2.215 \frac{kg \cdot m^2}{s} \hat{k}$

Problem 7: Part A and B

In figure, a carousel has a radius of 3.0 m and a moment of inertia of $I_C = 2000 kg \cdot m^2$, for rotation about axis perpendicular to the its center. The carousel is rotating unpowered and without friction with an angular velocity of 1.1 rad/s. A 120-kg super-athlete runs with a velocity of $v_i = 10\frac{m}{s}$, in a direction at $\theta = 30^{\circ}$with the vertical, at the rim of the carousel, as shown below. He grabs hold of a pole on the rim.



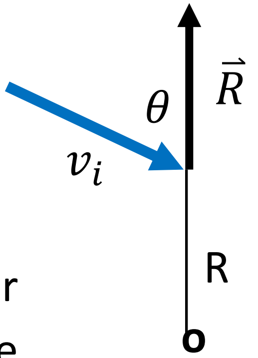
Direction perpendicular to x-y plane
⊙ indicates +z out of the page
⊗ indicates −z into page
USE THIS CONVENTION TO INDICATE THE DIRECTION OF ANGULAR MOMENTUM

A) Before the collision, what is the magnitude of the angular momentum of the rotating carousel, $\vec{L}_C$, with respect to the center of the **carousel**? What is the **direction** of $\vec{L}_c$? Directions (+x, +y, +z, -x, -y, -z) are as indicated in the above figure.

$L_C = I_C\omega_I = 2200 kg \cdot m^2 \cdot s^{-1}$ with direction ⊙, +z

$$\vec{L}_C = (2200 kg \cdot m^2 \cdot s^{-1})\,\hat{k}$$

B) Before the collision, what is the magnitude of the angular moment of the running **120-kg man**, $\vec{L}_M$, with respect to the center of the carousel? What is the **direction** of $\vec{L}_M$?



Angular Momentum about Origin **o**

$$\vec{L}_M = \vec{R} \times \vec{p}$$

$$L_M = Rmv_i \sin\theta = 3m \times 120kg \times 10\frac{m}{s} \sin 30^{\circ}$$

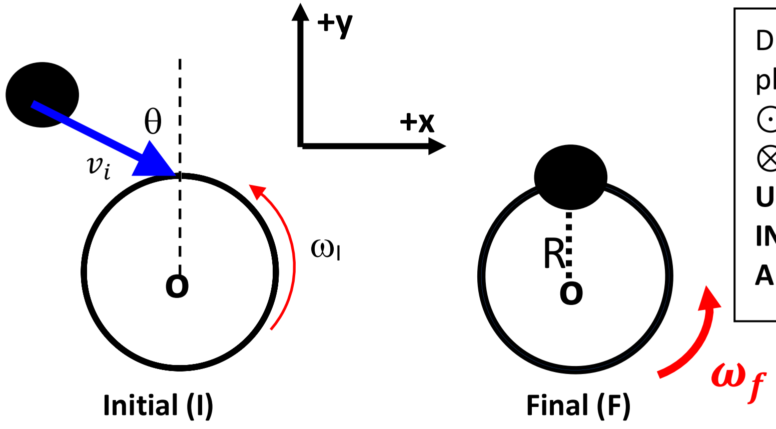
$$L_M = 1800 kg \cdot m^2 \cdot s^{-1}$$

Direction? $\vec{L}_M = -(1800 kg \cdot m^2 \cdot s^{-1})\,\hat{k}$

⊗, $-z$

Problem 7: Part C

In figure, a carousel has a radius of 3.0 m and a moment of inertia of $I_C = 2000kg \cdot m^2$, for rotation about axis perpendicular to the its center. The carousel is rotating unpowered and without friction with an angular velocity of 1.1 rad/s. A 120-kg super-athlete runs with a velocity of $v_i = 10\frac{m}{s}$, in a direction at $\theta = 30^{\circ}$with the vertical, at the rim of the carousel, as shown below. He grabs hold of a pole on the rim.



Direction perpendicular to x-y plane
⊙ indicates +z out of the page
⊗ indicates –z into page
USE THIS CONVENTION TO INDICATE THE DIRECTION OF ANGULAR MOMENTUM

A) Answer: $\vec{L}_C = (2200kg \cdot m^2 \cdot s^{-1})\,\hat{k}$

B) Answer: $\vec{L}_M = -(1800kg \cdot m^2 \cdot s^{-1})\,\hat{k}$

C) After the collision when the man is on the carousel, what is the magnitude of the final angular velocity of the carousel (with the man on it), ω_F? What is the **direction** of the final angular velocity $\vec{\omega}_F$? **Note:** $I_{total} = I_C + mR^2$

$$I_{total} = I_C + mR^2$$

$$I_{total} = \underbrace{2000kg \cdot m^2}_{\text{Carousel}} + \underbrace{120kg(3m)^2}_{\text{Man}} = 3080kg \cdot m^2$$

$$\underbrace{L_c + L_M}_{\text{Initial}} = \underbrace{I_{total}\omega_f}_{\text{Final}}$$

$$2200kg \cdot m^2 \cdot s^{-1} - 1800kg \cdot m^2 \cdot s^{-1} = 400kg \cdot m^2 = I_{total}\,\omega_f$$

Final Angular Velocity

$$\omega_f = \frac{400kg \cdot m^2}{3080kg \cdot m^2} = 0.136rad \cdot s^{-1}$$

Direction? ⊙, +z or CCW

$$\vec{\omega}_f = (0.136\,rad \cdot s^{-1})\,\hat{k}$$