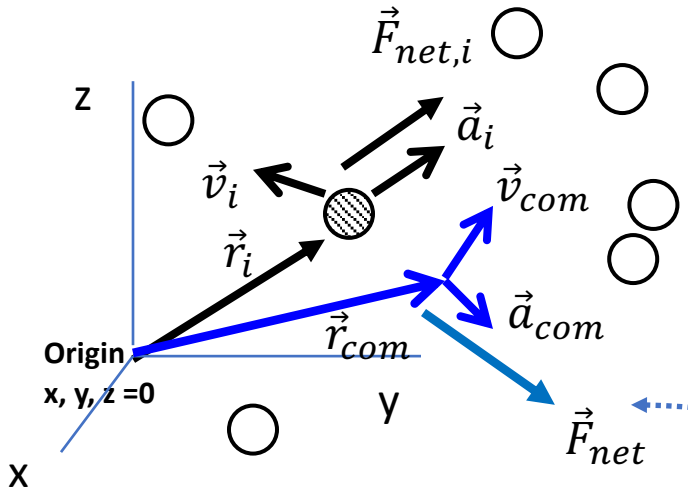
System of N Particles: Newton's Law for System

$$\vec{r}_{com} = \frac{\sum_{i=1}^{N} m_i \vec{r}_i}{M} \qquad [E1], \qquad M = \sum_{i=1}^{N} m_i$$

$$\vec{v}_{com} = \frac{d\vec{r}_{com}}{dt} = \frac{\sum_{i=1}^{N} m_i \vec{v}_i}{M} \qquad [E3]$$

$$\vec{a}_{com} = \frac{d\vec{v}_{com}}{dt} = \frac{\sum_{i=1}^{N} m_i \vec{a}_i}{M} \qquad [E5]$$



Newton's second law on i[th] particle

Net force on i[th] particle $\vec{F}_{net,i} = m_i \vec{a}_i$

Substituting into [E5]

$$\vec{a}_{com} = \frac{\sum_{i=1}^{N} \vec{F}_{net,i}}{M}$$

Define the net force on the system of N particles

$$\vec{F}_{net} = \sum_{i=1}^{N} \vec{F}_{net,i}$$

$$\vec{F}_{net} = M\vec{a}_{com} \qquad [E6]$$

Newton's second law for a system of particles.

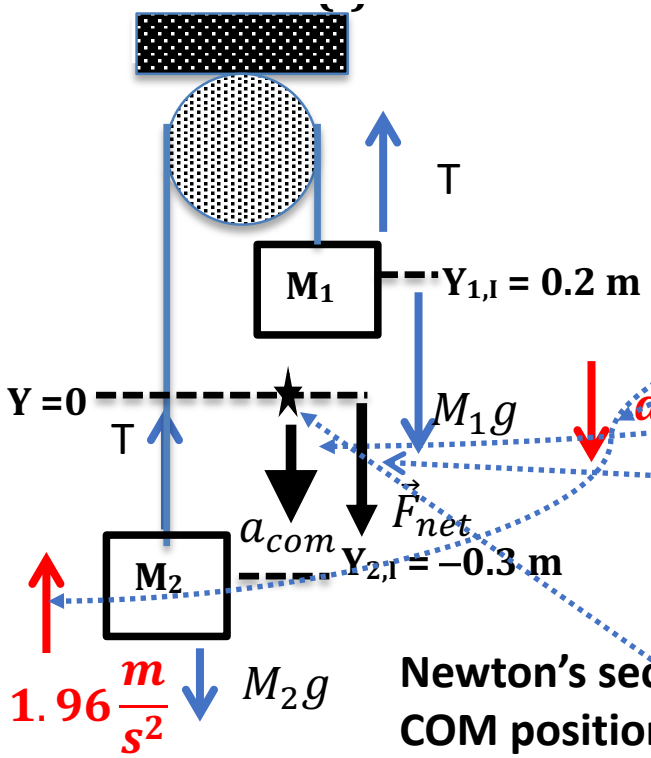
Physical Interpretation: For the first 8 chapters we have treated objects (Books, Boxes, etc.) as though they are point particles of total mass M. E6 justifies this by stating that when a net force $\vec{F}_{net}$ is applied on an object of mass M that is **not** a **point particle**, the effect is the same as though that force is applied on an object of mass M at the com position of mass, $\vec{r}_{com}$ (see E1).

Example on Newton's second law for system

A **M_1 = 3 kg** block and a **M_2 = 2 kg** block are connected by a string threated through a **frictionless** pulley system. **Initially (I)** the blocks are held at **rest**. It is then released. Analyze the com acceleration, $\vec{a}_{com}$

Calculate the **initial** Y **center-of-mass** (**COM**) position. X-comp is not important

$$Y_{com,I} = \frac{M_1 Y_{i,I} + M_2 Y_{2,I}}{M_1 + M_2} = \frac{3kg \times 0.2m + 2kg \times (-0.3m)}{3kg + 2kg} = 0$$



Draw FBD and find **acceleration (in red)**

M_1: $T - M_1 g = -M_1 a$ [1]

M_2: $T - M_2 g = M_2 a$ [2]

[2] minus [1]

$$a = \frac{M_1 g - M_2 g}{M_1 + M_2} = 1.96 \frac{m}{s^2}$$

Use [1], $T = M_1 g - M_1 a = 23.52N$

$$a_{com} = \frac{M_1 a_1 + M_2 a_2}{M_1 + M_2} = -0.392 \frac{m}{s^2}$$

Find y component of $\vec{F}_{net}$

$$F_{net,y} = -M_1 g + T + T - M_2 g = -1.96N$$

$$F_{net,y} = M a_{com}$$

$$-1.96N = (3kg + 2kg)\left(-0.392 \frac{m}{s^2}\right) = -1.96N$$

System obeys Newton's second law

Newton's second Law for system:

COM position, $\vec{r}_{com}$, accelerates according to Newton's Law, $\vec{F}_{net} = M\vec{a}_{com}$