Useful Equations

Kinematics $x = x_0 + v_{0x}t + (1/2)a_x t^2$, $v_x = v_{0x} + a_x t$, $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$, $v_x = dx/dt$; $a_x = dv_x/dt$; $\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$; $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$; **average speed** s_{avg} = (total distance)/(total time); **average velocity (x-com)** $v_{avg,x} = (x_2 - x_1)/(t_2 - t_1)$, **average acceleration (x-com)** $a_{avg,x} = (v_{2x} - v_{1x})/(t_2 - t_1)$. **Newton's Laws** $\vec{F}_{net} = \sum \vec{F}_i = 0$ (Object in equilibrium); $\vec{F}_{net} = m\vec{a}$ (Nonzero net force); **Weight:** $F_g = mg$, $g = 9.8 m/s^2$; **Centripetal acceleration** $a_{rad} = \frac{v^2}{r}$; **Friction** $f_s \le f_{s,\max} = \mu_s F_N$, $f_k = \mu_k F_N$. **Hooke's Law** $F_x = -kx$. **Work and Energy** $W = \vec{F} \bullet \vec{d} = (F\cos\theta)d = F_{\parallel} d$;

$W^{net} = \Delta K = (1/2)mv_f^2 - (1/2)mv_i^2$ (valid if W^{net} is the **net** or **total work done** on the object); $W^{grav} = -mg(y_f - y_i)$ (gravitational work), $W^{el} = -\left((1/2)kx_f^2 - (1/2)kx_i^2\right)$ (elastic work) **Conservation of Mechanical Energy** (only **conservative forces** are present) $E_{mech} = U + K$ $W^{net} = -\Delta U = -(U_2 - U_1) = \Delta K = K_2 - K_1, U_1 + K_1 = U_2 + K_2$.

Potential Energy:Gravitational $U_{grav} = mgy$; Elastic Spring $U_{el} = (1/2)kx^2$

Also $\Delta E_{mech} = \Delta U + \Delta K = (U_f - U_i) + (K_f - K_i) = 0 \rightarrow \Delta K = -\Delta U$,

CONSERVATION of **ENERGY**:

Non-Conservative Forces: with **no friction** $W_{ext} = \Delta E_{mech}$ (W_{ext} work done by **external**), with $\Delta E_{mech} = \Delta U + \Delta K = (U_f - U_i) + (K_f - K_i)$; and **with friction**

$W_{external} = \Delta E_{mech} + \Delta E_{th}$, $\Delta E_{th} = f_k d < 0$, d is **magnitude** of **displacement**.

Work due to variable force 1D: $W = \int_{x_i}^{x_f} F_x\, dx \equiv$ area under F_x vs. x, from $x = x_i$ to x_f

Momentum: $\vec{P} = m\vec{v}$, **Impulse:** $\vec{J} = \int_{t_1}^{t_2} \vec{F} dt = \vec{F}_{av}(t_2 - t_1)$, **Impulse-Momentum** $\vec{J} = \Delta\vec{P} = \vec{P}_2 - \vec{P}_1$ **Newton's Law in Terms of Momentum** $\vec{F}_{net} = d\vec{p}/dt$. For $\vec{F}_{net} = 0$, $d\vec{p}/dt = 0$ gives **momentum conservation**: $\vec{P} \equiv$ constant. **Rotational Kinematics Equations**: $\omega_{avg} = (\theta_2 - \theta_1)/(t_2 - t_1)$, $\alpha_{avg} = (\omega_2 - \omega_1)/(t_2 - t_1)$ For $\alpha_z =$ constant, $\omega = \omega_0 + \alpha t$, $\theta = \theta_0 + \omega_0 t + (1/2)\alpha t^2$,. $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$ **Linear and angular variables:** $s = r\theta$, $v = r\omega$, $a_{\tan} = R\alpha$ (tangential) , $a_{rad} = v^2/r = \omega^2 r$ (radial) **Moment of Inertia and Rotational Kinetic Energy** $I = \sum_{i=1}^{N} m_i r_i^2$, $K_{rot} = (1/2)I\omega^2$. **Center of Mass** (COM) $\vec{r}_{com} = \sum m_i \vec{r}_i / M$, $M = \sum m_i$; $\vec{v}_{com} = \sum m_i \vec{v}_i / M$; $\vec{a}_{com} = \sum m_i \vec{a}_i / M$. **Torque and Newton's Laws of Rotating Body:** rigid body $\tau = F r_\perp$, $\vec{\tau}_{net} = \sum \vec{\tau}_i^{ext} = I\alpha$, $r_\perp$ - moment arm about axis; point $\vec{\tau} = \vec{r} \times \vec{F}$ about origin O.

Combined Rotation and Translation of a Rigid Body $K = (1/2)Mv_{com}^2 + (1/2)I_{com}\omega^2$, $\vec{F}_{net} = M\vec{a}_{com}$, $\vec{\tau}_{net} = I_{com}\vec{\alpha}$. **Rolling without slipping** $s = R\theta$, $v_{com} = R\omega$, $a_{com} = R\alpha$.