Conservation of Energy: Summary, example, and an exercise

Read section 8.4 titled “WORK DONE ON A SYSTEM BY AN EXTERNAL FORCE”. Pay attention to equation 8.25 and 8.26:

$$W = \Delta K + \Delta U = \Delta E_{mec},$$

where W is the work done by non-conservative external force (including work by friction), and ΔE_{mec} is the change in mechanical energy, ΔU is the change in potential energy arising from gravity and/or spring (at least in this course), and ΔK is the change in the kinetic energy of the system. Equation 8.33 rewrite the equation as:

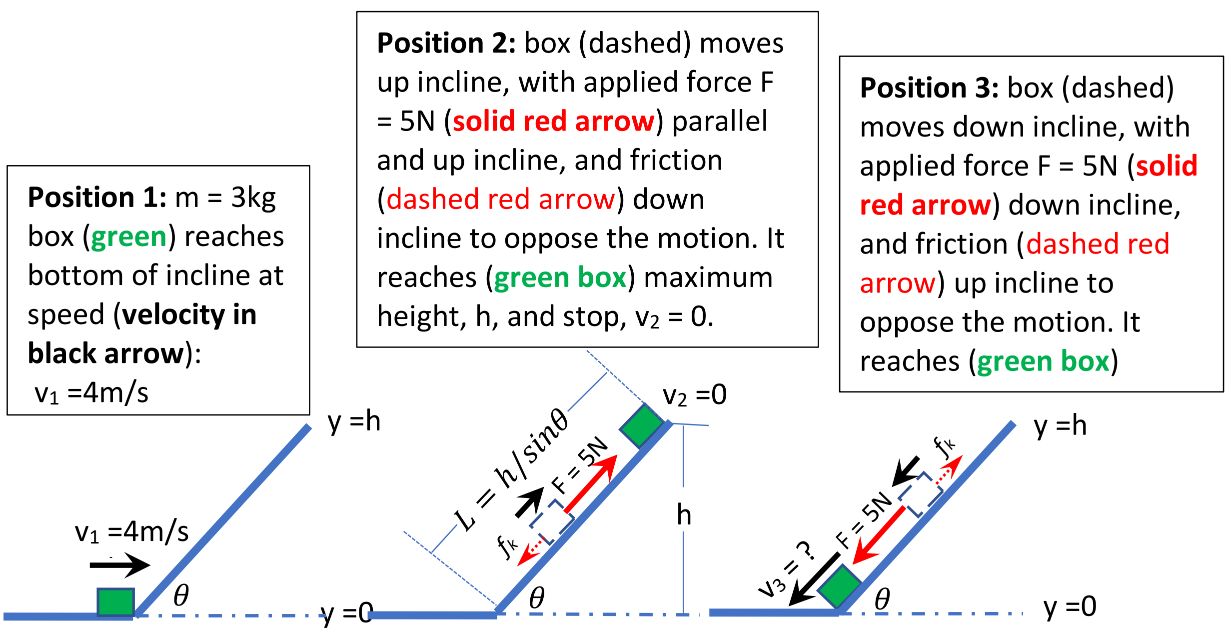
$$W = \Delta E_{mec} + \Delta E_{th},$$

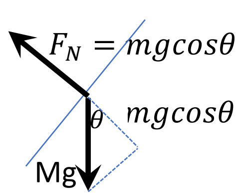
where $\Delta E_{th} = f_k d$ is the thermal energy passed into (dissipated) the environment. In his case the work done by external force W does not include friction.

Example

The figure below shows a green box of mass 3 kg moving on a level frictionless incline at speed of v_1 = 3 m/s. It reaches an $\theta = 53.1^0$incline with friction coefficients of $\mu_k = 0.1$and $\mu_s = 0.15$. As it starts to slide up the incline, an engine on the box applies a force F = 5N up and parallel to the incline on the box. The box moves up till it reaches a maximum height. As it slides down the incline the direction of the F = 5N force is now applied down parallel to the incline. Find the maximum height, h, and the speed of the box, v_3, when it slid back to the bottom.

Solution As always begin by drawing a diagram below that shows all relevant physics. For conservation of energy problem, it is crucial to clearly define an initial and final positions. These are position 1, 2 and 3 as shown below.





Kinetic Friction:

$$f_k = F_N\mu_k = mgcos\theta\mu_k$$

$$f_k = 3kg \times (9.8m \cdot s^{-2})cos53.1 \times 0.1 = 1.764N$$

Finding Maximum height: **Initial position 1**, $y_1 = 0$, $v_1 = 4m/s$; **final position 2**, $y_2 = h$, $v_2 = 0$. Use equation 8:28, and the fact that the box will move a distance L along the incline

$$W = \Delta E_{mec} + \Delta E_{th} \rightarrow FL = \Delta K + \Delta U + f_k L,$$

with the change in **kinetic energy** $\Delta K = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = -\frac{1}{2}mv_1^2$, and **potential** energy $\Delta U = U_2 - U_1 = mgy_2 - mgy_1 = mgh$, and using $h = Lsin\theta = Lsin53.1 = 0.8L$ gives

$$FL = -\frac{1}{2}mv_1^2 + \mathrm{mgh} + f_k L = -\frac{1}{2}mv_1^2 + 0.8\mathrm{mgL} + f_k L,$$

$$L = \frac{0.5mv_1^2}{0.8mg + f_k - F} = \frac{0.5 \times 3kg \times (4m \cdot s^{-1})^2}{0.8 \times 3kg(9.8m \cdot s^{-2}) + 1.764N - 5N} = 1.183m.$$

The maximum height is $h = 0.8L = 0.8 \times 1.183m = 0.947m.$

Speed when box falls to bottom, v_3: **Initial position** 2, $y_2 = h$, $v_2 = 0$; **final position 3**, $y_3 = 0$, v_3 =?. Use equation 8:28, and the fact that the box will move a distance L along the incline

$$W = \Delta E_{mec} + \Delta E_{th} \rightarrow FL = \Delta K + \Delta U + f_k L,$$

with the change in **kinetic energy** $\Delta K = \frac{1}{2}mv_3^2 - \frac{1}{2}mv_2^2 = \frac{1}{2}mv_3^2$, and **potential** energy $\Delta U = U_3 - U_2 = mgy_3 - mgy_2 = -mgh$, and using $h = Lsin\theta = Lsin53.1 = 0.8L$ gives

$$FL = \frac{1}{2}mv_3^2 - \mathrm{mgh} + f_k L = \frac{1}{2}mv_3^2 - 0.8\mathrm{mgL} + f_k L,$$

$$\frac{1}{2}mv_3^2 = FL + 0.8\mathrm{mgL} - f_k L \rightarrow v_3^2 = \frac{2L}{m}(F + 0.8mg + f_k)$$

$$v_3 = \sqrt{\frac{2 \times 1.183m}{3kg}(0.8 \times 3kg(9.8m \cdot s^{-2}) - 1.764N + 5N)} = 4.9\frac{m}{s}$$

The maximum height is $h = 0.8L = 0.8 \times 1.183m = 0.947m.$

EXERCISE:

Verify your calculation above by applying equation, $W = \Delta E_{mec} + \Delta E_{th}$ to **Initial position** 1, $y_1 = 0$, $v_1 = 4m/s$; **final position 3**, $y_3 = 0$, v_3 =? You may use the value of L = 1.183m and h = 0.947 m.