# Equations for Final Exam

## 1D Kinematics:

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} \quad \textbf{E1}; \qquad v = \frac{dx}{dt} \quad \textbf{E2}; \qquad s_{avg} = \frac{total\ distance}{total\ time} \quad \textbf{E3}$$

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} \quad \textbf{E4}; \qquad a = \frac{dv}{dt} \quad \textbf{E5}$$

$$v = v_0 + at \quad \textbf{E6}; \qquad x = x_0 + v_0 t + \frac{1}{2}at^2 \quad \textbf{E7}; \qquad v^2 = v_0^2 + 2a(x - x_0) \quad \textbf{E8}$$

## 2D Kinematics:

$$\vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{x_2 - x_1}{t_2 - t_1}\hat{\imath} + \frac{y_2 - y_1}{t_2 - t_1}\hat{\jmath} + \frac{z_2 - z_1}{t_2 - t_1}\hat{k} = v_{avg,x}\hat{\imath} + v_{avg,y}\hat{\jmath} + v_{avg,z}\hat{k} \quad \textbf{E9};$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath} + \frac{dz}{dt}\hat{k} = v_x\hat{\imath} + v_y\hat{\jmath} + v_z\hat{k} \quad \textbf{E10}$$

$$\vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{v_{2x} - v_{1x}}{t_2 - t_1}\hat{\imath} + \frac{v_{2y} - v_{1y}}{t_2 - t_1}\hat{\jmath} + \frac{v_{2z} - v_{1z}}{t_2 - t_1}\hat{k} = a_{avg,x}\hat{\imath} + a_{avg,y}\hat{\jmath} + a_{avg,z}\hat{k} \quad \textbf{E11}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{\imath} + \frac{dv_y}{dt}\hat{\jmath} + \frac{dv_z}{dt}\hat{k} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k} \quad \textbf{E12}$$

## Projectile Motion

**Horizontal:**

$v_{0x} = v_0 \cos\theta_0$ **E13**; $x = x_0 + v_{0x}t$ **E14**

**Vertical:**

$v_{0y} = v_0 \sin\theta_0$ **E15**; $v_y = v_{0y} - gt$ **E16**; $y = y_0 + v_{0y}t - \frac{1}{2}gt^2$ **E17**

$v_y^2 = v_{0y}^2 - 2g(y - y_0)$ **E18**

## Newton's Law

**Newton's first Law** states that if there is no applied force on an object will move in a straight line at constant speed. **Constant Velocity**

**Newton's Second Law**: An object acted on by a force, $\vec{F}$, accelerates according to $\vec{F} = m\vec{a}$
When there is more than one force acting on an object, the net effect is found using the **Principle** of **Superposition** of **Forces to** calculate the $\boldsymbol{net\ force}$:

$$\vec{F}_{net} = \sum_i \vec{F}_i = M\vec{a}$$

**Newton's Third Law:** When two bodies interact, the forces on the bodies from each other are always equal in magnitude and opposite in direction

**Friction:** $f_{s,max} = \mu_s F_N > f_s$, $f_k = \mu_k F_N$

Centripetal: acceleration $a = \frac{v^2}{R}$ **[E19]**; force, $F_c = \frac{mv^2}{R}$ **[E20]**

## Work and Energy

$K = \frac{1}{2}mv^2$ **E21**; Work $W = \vec{F} \cdot \vec{d}$ **E22**;
Work-Energy Theorem $W = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$ **E23**
Work by gravity, $W_g = -mg\Delta y$ **E24**; spring, $U_{el} = \frac{1}{2}kx^2$ **E25**; Hooke's Law, $F_x = -kx$ **E26**
Mechanical Energy: $E_{mec} = K + U$ **E27**; $E_{mec,i} = K_i + U_i = K_f + U_f = E_{mec,f}$ **E28**
Work by external force, $W_{ext} = \Delta K + \Delta U = \Delta E_{mec}$ **E29**;
$W_{applied} = Fd = \Delta E_{mec} + \Delta E_{th}$, $\Delta E_{th} = f_k d$ **E30.**
$\vec{r}_{com} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots + m_N\vec{r}_N}{m_1 + m_2 + \cdots + m_N} = \frac{\sum_{i=1}^{N} m_i\vec{r}_i}{M}$ $[\boldsymbol{E31}]$; $\vec{v}_{com} = \frac{d\vec{r}_{com}}{dt} = \frac{\sum_{i=1}^{N} m_i\vec{v}_i}{M}$ $[\boldsymbol{E32}]$; $\vec{a}_{com} = \frac{d\vec{v}_{com}}{dt} = \frac{\sum_{i=1}^{N} m_i\vec{a}_i}{M}$ $[\boldsymbol{E33}]$; $\vec{F}_{net} = M\vec{a}_{com}$ $[\boldsymbol{E34}]$
**Momentum**, $\vec{p} = m\vec{v}$ **E35**; $\vec{F}_{net} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt} = \frac{d\vec{p}}{dt}$ **E36**; Impulse, $\vec{J} = \int_{t_i}^{t_f} \vec{F}dt$ **E37**;
$J = F_{avg}\Delta t$ **E38**; $\vec{J} = \vec{p}_f - \vec{p}_i = \Delta\vec{p}$ **E39**; $\vec{P}_i = \vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$ **E40**. **Elastic Collision (stationary target)**: $v_{1f} = \frac{(m_1 - m_2)v_{1i}}{m_1 + m_2}$ **E41**; $v_{2f} = \frac{2m_1 v_{1i}}{m_1 + m_2}$ **E42.**

**Rotational Variables:**
$\omega_{avg} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$ $\boldsymbol{E43}$; $\omega = \frac{d\theta}{dt}$ **E44** ; $\alpha_{avg} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}$ $\boldsymbol{E45}$; $\alpha = \frac{d\omega}{dt}$ $\boldsymbol{E46}$;
Constant angular acceleration: $\omega = \omega_0 + \alpha t$ $\boldsymbol{E47}$; $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$ **E48**; $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$ $\boldsymbol{E49}$; $\theta = \theta_0 + \frac{1}{2}(\omega_0 + \omega)t.$ $\boldsymbol{E50}$; $\omega_{avg} = \frac{1}{2}(\omega_0 + \omega)$ **E51**;
Relating linear and angular variable: $s = r\theta$ **E52**; $v = r\omega$ **E53**; $a = r\alpha$ **E54**;

**Center-of-Mass (COM):**
$\vec{r}_{com} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots + m_N\vec{r}_N}{m_1 + m_2 + \cdots + m_N} = \frac{\sum_{i=1}^{N} m_i\vec{r}_i}{M}$ **E55**; $\vec{v}_{com} = \frac{d\vec{r}_{com}}{dt} = \frac{\sum_{i=1}^{N} m_i\vec{v}_i}{M}$ **E56**; $\vec{a}_{com} = \frac{d\vec{v}_{com}}{dt} = \frac{\sum_{i=1}^{N} m_i\frac{d\vec{v}_i}{dt}}{M}$ **E57**; 2nd Law for System, $\vec{F}_{net} = M\vec{a}_{com}$ **E58**; Total Mass $M = \sum_{i=1}^{N} m_i$.

**Moment of Inertia and Rotational Kinetic Energy** see table 10.5.1
Moment of Inertia $I = \sum_{i=1}^{N} m_i r_{i,\perp}^2$ **E59**; $K_{rot} = \frac{1}{2}I\omega^2$ $\boldsymbol{E60}$; Conservation of Mechanical Energy with Rotation: $E_{mec} = K_{linear} + K_{rot} + U$, $E_{mec,i} = E_{mec,f}$ **E61**; Magnitude of Torque on a rigid body due to a force, magnitude $\tau = F_t r = Fr_\perp$ **E62**; Newton's second law for rotation $\tau_{net} = I\alpha$ **E63**; work by torque $W_{rot} = \tau\Delta\theta$ $\boldsymbol{E64}$; Combined Rotation and Translation $K = \frac{1}{2}Mv_{com}^2 + \frac{1}{2}I_{com}\omega^2$ **E65**; **No-slip** rolling $s = r\Delta\theta$; $v_{com} = r\omega$; $a_{com} = r\alpha$ **E66**.

**Point-particle rotation:**
$\vec{\tau} = \vec{r} \times \vec{F}$ **E67**; angular momentum of point particle about origin O $\vec{\ell} = \vec{r} \times \vec{p}$ **E68**; angular momentum of a rigid bout about a rotational axis $\vec{L} = I\vec{\omega}$ **E69**; 2nd Law for rotation in terms of angular momentum $\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$ **E70**; 2nd Law for rotation for point particle $\vec{\tau}_{net} = \frac{d\vec{\ell}}{dt} = m\vec{r} \times \frac{d\vec{a}}{dt}$ **E71**; Conservation of angular momentum, $\vec{L}_{total,initial} = \vec{L}_{total,final}$, $\vec{L}_{total}$ is the total angular momentum of all rigid bodies and point particles of an isolated system **E72**