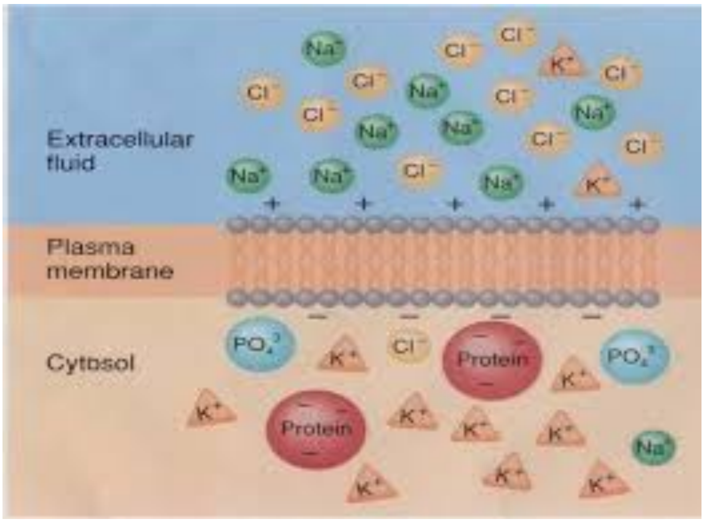
Membrane Resting Potential and the Sodium Anomaly

Below is an image of a typical membrane, where the **intercellular** region (in) inside the cell is filled with **cytosol** (water plus organelles, ions and proteins), and the **extracellular** region (out) is filled with extracellular fluid (mainly ionic solutions).



Resting Membrane Potential: The interior (in) is usually more electrically more negative than the exterior (out): $\Delta V = V_{in} - V_{out} \approx -60mV$.

Sodium Anomaly: In physics, it is well known that **positive ions** move to more **negative potential** region, and **negative ions** move to more **positive potential**. It is then not surprising that there are more **potassium** ions, **K^+**, outside (extracellular) than inside (intracellular), and there are more **chloride** ions, **Cl^-**, inside than outside. But it is surprising that there are more **sodium** ions, **Na^+**, outside (positive potential) than inside (negative potential). This is called the **sodium anomaly** in Phillip Nelson's textbook. Data below shows the ion concentration of a typical **living** (*in vivo*) **cell.**

Data of mamalian cell Membrane Rest Potential $\Delta V = -86mV$

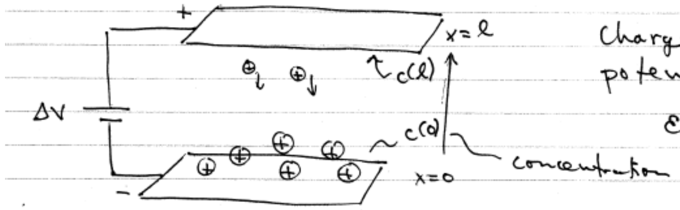
ion	*z* (valence)	Intracellular $c_{i,in}$, mM	Extracellular $c_{i,out}$, mM	V_i^{Nersst}, mV
K^+	+1	140	4	**?**
Na^+	+1	4	140	?
Cl^-	-1	4	120	**?**

Nernst Potential:

In class and in the textbook we calculate the Nernst's potential $V_i^{Nernst} = \frac{k_B T}{ze} \ln \frac{c_{i,out}}{c_{i,in}}$.

For above data: K^+, $V_{K^+}^{Nernst} = -90mV$; Na^+, $V_{Na^+}^{Nernst} = 89mV$; Cl^-, $V_{Cl^-}^{Nernst} = -86mV$. Note that they are **different** than the membrane potential $\Delta V = -86mV$. On the next page, I will illustrate why this is a significant point.

Consider the distribution of charge in the presence of a voltage (battery):



Charge has a potential energy

$\mathcal{E} = q \cdot V(x)$

NB: q has sign, here it is positive

The energy difference between charges on the top and bottom plate is:

$$\Delta\mathcal{E} = \mathcal{E}(\ell) - \mathcal{E}(0) = q\big(V(\ell) - V(0)\big) = q\Delta V$$

- From Boltzmann: $c(\ell) = c(0)\, e^{-\Delta\mathcal{E}/k_BT} = c(0)\, e^{-q\Delta V/k_BT}$

or $$\boxed{\ln\left(\frac{c(\ell)}{c(0)}\right) = -\frac{q\Delta V}{k_BT}}$$ Nernst relation

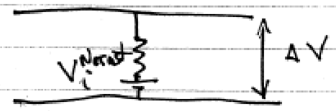
The physical interpretation of the above is that in the presence of an **applied voltage** ΔV , the equilibrium charge distribution will be as the diagram above with more **positive ions** on the **negative bottom plate**. Another interpretation is that if there is charge distribution as in the above figure, then there will be a voltage $\Delta V = -\frac{k_BT}{q}\ln\frac{C_{top}}{C_{bottom}}$. Since the top is more positive than negative we identify the **top** as **extracellular** (out) and the **bottom** as **intracellular** (in), so for cells we have $\Delta V = -\frac{k_BT}{q}\ln\frac{C_{out}}{C_{in}} = -V_i^{Nernst}$. Basically the potential ΔV , depicted above is **negative** of the Nernst potential equation V_i^{Nernst} .

Interpretation of Nernst Potential: From Ohm's law we know that an applied voltage will induce a current, $\Delta V = IR$. In the diagram above yhe Nernst potential V_i^{Nernst} calculated for the above distribution is the applied voltage needed to prevent a flow of charge – i.e. a current.

What happens in living cells? In the above data (previous page) we see that in general the Nernst potential is different from the membrane potential, $V_{K^+}^{Nernst} < \Delta V$ and $V_{Na^+}^{Nernst} > \Delta V$, but $V_{Cl^-}^{Nernst} = \Delta V$. This means that there must be a **leak current**, as will be illustrated in the next example.

• Recall Ohm's Law: $I = \frac{\Delta V}{R}$ or $I = G \cdot \Delta V$ ← conductance

• Flow on ions:



• if $\Delta V = V_{nernst}$ → no flow

• Net drop is $(\Delta V - V_{nernst}^{i})$ for ion species i

so for ionic species i, the current flux is

$$\boxed{j_i = \frac{I_i}{\underset{\uparrow \text{ area}}{A}} = g_i\left(\Delta V - V_i^{Nernst}\right)}$$

where $g_i \equiv$ membrane conductance of species, i.
(i = Na^+, Cl^-, K^+ etc)

IMPORTANT POINTS:

1. In the above definition the flow of ions is **positive** if it is from the inside (in) to outside (out), but in **class notes** we assume that **positive flow** is from **outside** (out) to **inside** (in), so the equation should be for **our course and assignment**: $I_i = Ag_i\left(V_i^{Nernst} - \Delta V\right) = \left(V_i^{Nernst} - \Delta V\right)/R_i$, where A is the effective cross section area, and $R_i = Ag_i$ is the resistance of the ion i (Na^+, K^+, Cl^-).
2. Note that the conductance is $G = 1/R$ and the conductance per unit area is $g = G/A$. Hence $g = AG = A/R$, which is how we derived the above equation.
3. For **negative ions** such as Cl^-, the ion flow direction is reversed as defined above.
4. In general, $\Delta V \neq V_i^{Nernst}$. The data above, $V_{Cl^-}^{Nernst} = \Delta V$ is **unusal**.