Kinematics in 2D and 3D

Chapter 4

Position Vector in Three Dimensions (3D)

- $\hat{\imath} = (1,0,0)$ along **x-axis**
- $\hat{\jmath} = (0,1,0)$ along **y-axis**
- $\hat{k} = (0,0,1)$ along **z-axis**
- **Position Vector**
- $\vec{r}(t) = (x(t), y(t), z(t)\ )$

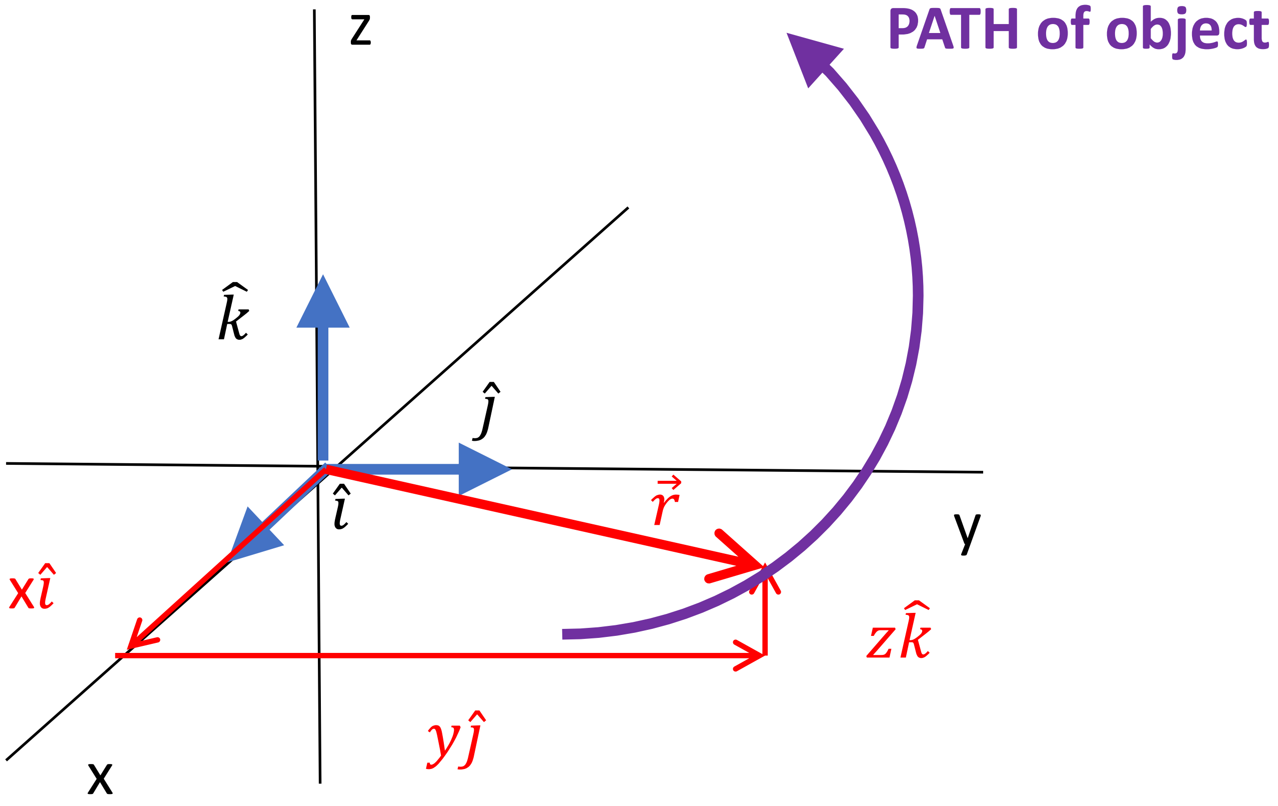


Figure 4.1.1 from textbook

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}. \tag{4.1.1}$$

The *vector components* are $x\hat{i}$, $y\hat{j}$, and $z\hat{k}$, and the coefficients x, y, and z in front of the unit vectors are the *scalar components*.
In the following figure, a particle is located at coordinates (−3 m, 2 m, 5 m).

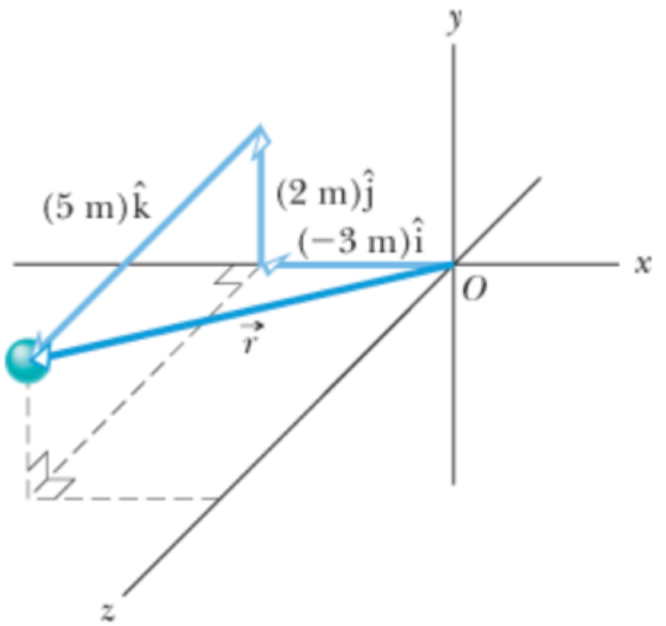
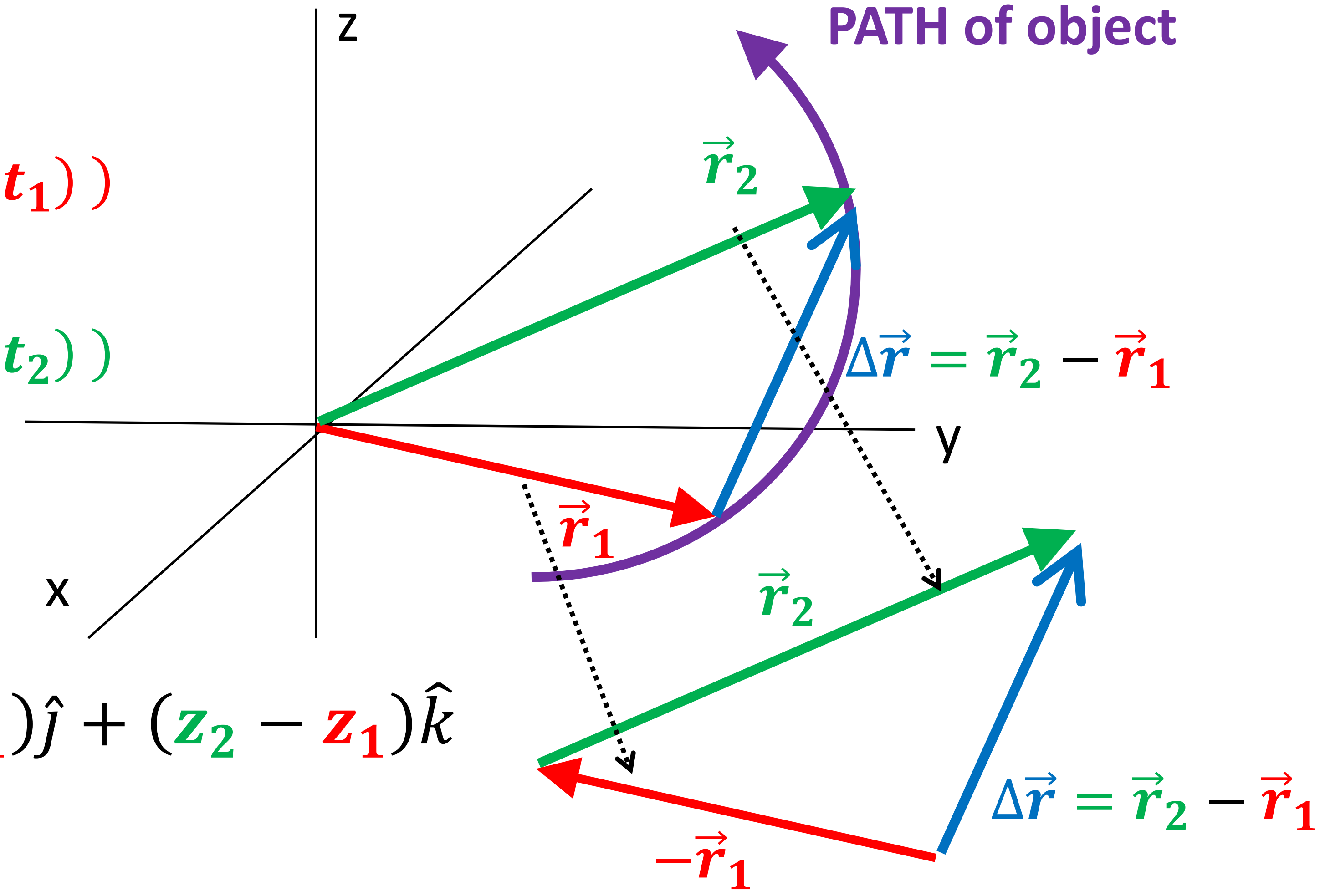


Figure 4.1.1

Displacement in Three Dimensions (3D)

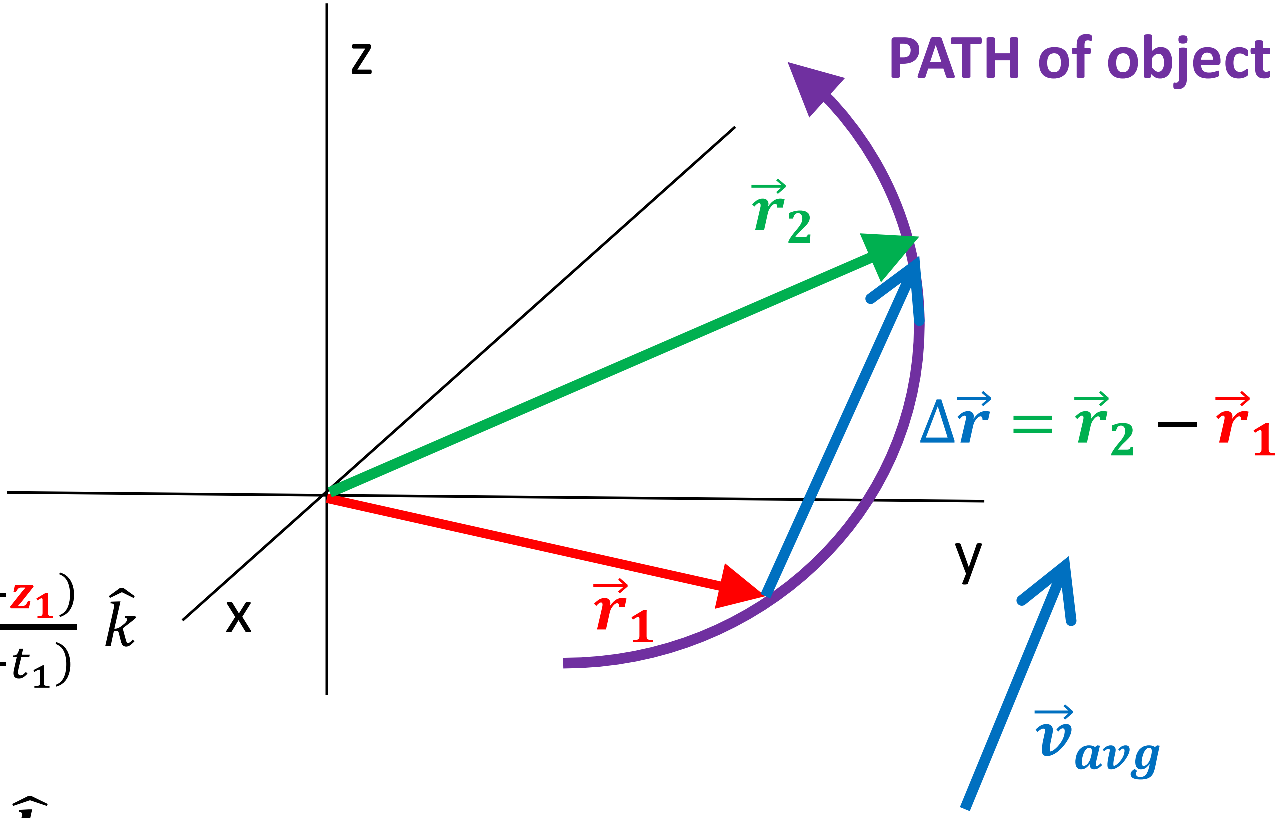
- **Position Vectors**
- Initial time t_1
  - $\vec{\boldsymbol{r}}_1(\boldsymbol{t}_1) = (\boldsymbol{x}_1(\boldsymbol{t}_1), \boldsymbol{y}_1(\boldsymbol{t}_1), \boldsymbol{z}_1(\boldsymbol{t}_1))$
- Final time t_2
  - $\vec{\boldsymbol{r}}_2(\boldsymbol{t}_2) = (\boldsymbol{x}_2(\boldsymbol{t}_2), \boldsymbol{y}_2(\boldsymbol{t}_2), \boldsymbol{z}_2(\boldsymbol{t}_2))$



- <u>**Displacement**</u>
- $\Delta\vec{\boldsymbol{r}} = \vec{\boldsymbol{r}}_2 - \vec{\boldsymbol{r}}_1$
- $\Delta\vec{\boldsymbol{r}} = (\boldsymbol{x}_2 - \boldsymbol{x}_1)\hat{\boldsymbol{\imath}} + (\boldsymbol{y}_2 - \boldsymbol{y}_1)\hat{\jmath} + (\boldsymbol{z}_2 - \boldsymbol{z}_1)\hat{k}$

Average Velocity in Three Dimensions (3D)

- **Position Vectors**
- Initial time t_1
  - $\vec{r}_1(t_1) = (x_1(t_1), y_1(t_1), z_1(t_1))$
- Final time t_2
  - $\vec{r}_2(t_2) = (x_2(t_2), y_2(t_2), z_2(t_2))$
- <u>**Average Velocity**</u>
- $\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t}$
- $\vec{v}_{avg} = \frac{(x_2 - x_1)}{(t_2 - t_1)}\hat{\imath} + \frac{(y_2 - y_1)}{(t_2 - t_1)}\hat{\jmath} + \frac{(z_2 - z_1)}{(t_2 - t_1)}\hat{k}$
- $\vec{v}_{avg} = \frac{\Delta x}{\Delta t}\hat{\imath} + \frac{\Delta y}{\Delta t}\hat{\jmath} + \frac{\Delta z}{\Delta t}\hat{k}$
- $= v_{avg,x}\hat{\imath} + v_{avg,y}\hat{\jmath} + v_{avg,z}\hat{k}$



Instantaneous Velocity (velocity) in Three Dimensions (3D)

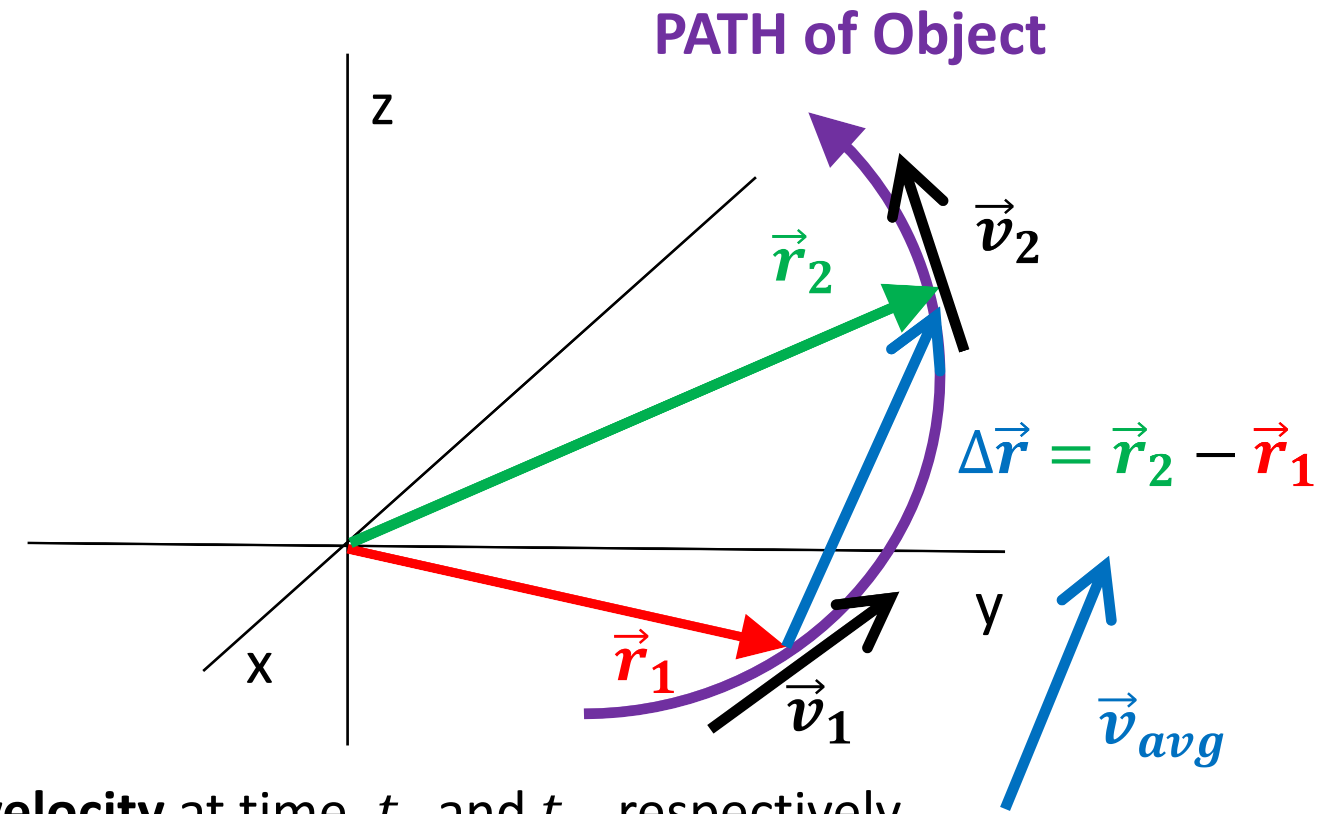
Average Velocity

- $\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t}$
- $\vec{v}_{avg} = \frac{\Delta x}{\Delta t}\hat{\imath} \quad + \quad \frac{\Delta y}{\Delta t}\hat{\jmath} \quad + \quad \frac{\Delta z}{\Delta t}\hat{k}$
- $\qquad = v_{avg,x}\hat{\imath} + v_{avg,y}\hat{\jmath} + v_{avg,z}\hat{k}$

Velocity

In the calculus limit, $\Delta t \to 0$

- $\vec{v} = \lim_{\Delta t \to 0}\frac{\Delta\vec{r}}{\Delta t} = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath} + \frac{dz}{dt}\hat{k}$
- $\qquad = v_x\hat{\imath} + v_y\hat{\jmath} + v_z\hat{k}$
- $\vec{v}_1 = \vec{v}_1(t_1)$ and $\vec{v}_2 = \vec{v}_2(t_2)$ is the **velocity** at time, t_1 and t_2, respectively.
- In general, the **velocity** is a **function** of **time**, $\vec{v} = \vec{v}(t)$



Average Acceleration, $\vec{a}_{avg}$, Acceleration, $\vec{a}$, in Three Dimensions (3D)

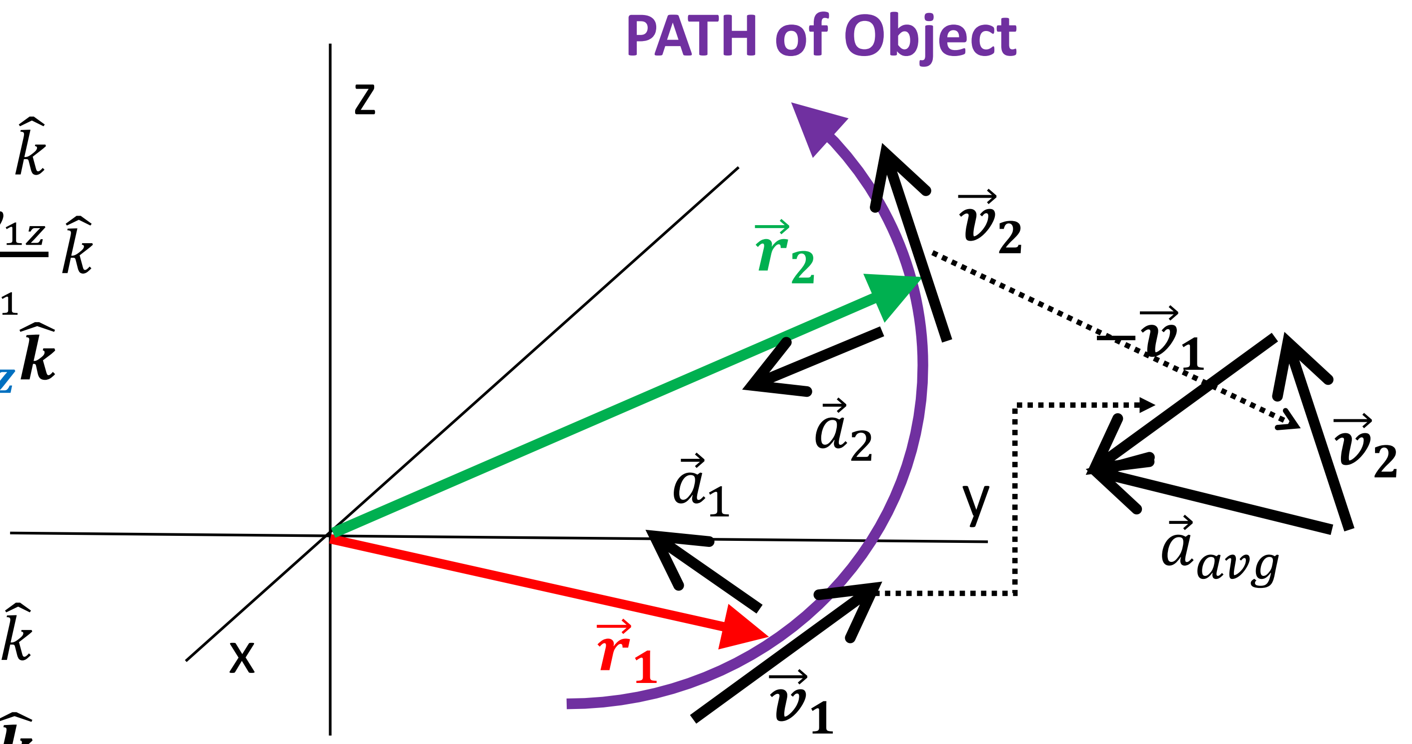
<u>Average Acceleration</u>

- $\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\Delta v_x}{\Delta t}\hat{i} + \frac{\Delta v_y}{\Delta t}\hat{j} + \frac{\Delta v_z}{\Delta t}\hat{k}$
- $= \frac{v_{2x}-v_{1x}}{t_2-t_1}\hat{i} + \frac{v_{2y}-v_{1y}}{t_2-t_1}\hat{j} + \frac{v_{2z}-v_{1z}}{t_2-t_1}\hat{k}$
- $= a_{avg,x}\hat{i} + a_{avg,y}\hat{j} + a_{avg,z}\hat{k}$

<u>Acceleration</u>

In the calculus limit, $\Delta t \to 0$

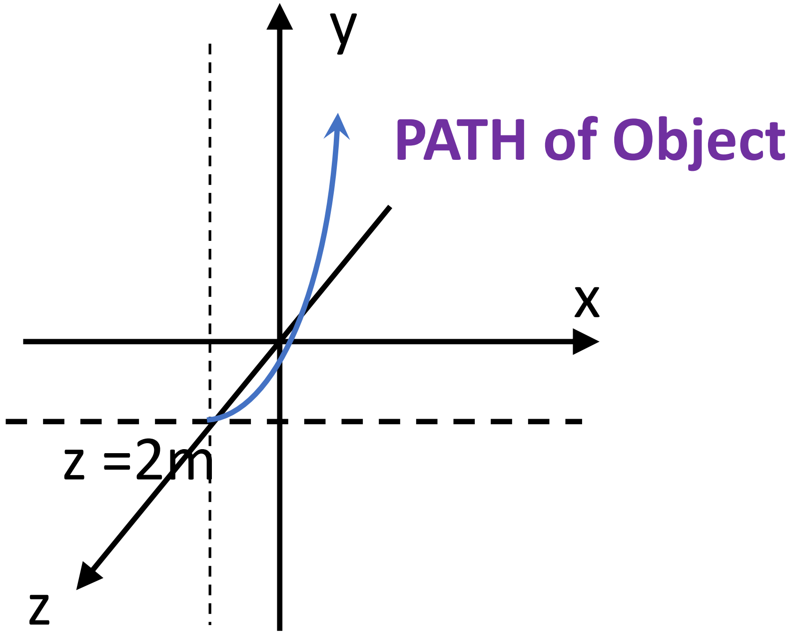
- $\vec{a} = \lim_{\Delta t \to 0}\frac{\Delta\vec{v}}{\Delta t} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k}$
- $= a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$
- $\vec{a}_1 = \vec{a}_1(t_1)$ and $\vec{a}_2 = \vec{a}_2(t_2)$ is the **velocity** at time, t_1 and t_2, **respectively**.
- In general, the **velocity** is a **function** of **time**, $\vec{a} = \vec{a}(t)$.



Example of Kinematics in 2D

An object follows a path $\vec{r} = \left(1\frac{m}{s^2}\right)t^2\hat{\imath} + \left(1\frac{m}{s^4}\right)t^4\hat{\jmath} + (2m)\hat{k}$

- A) Draw the path of the object from t > 0.
- B) Find average velocity from t = 1s to 2 s.
- C) Find average acceleration from t = 1s to 2s. Plot.
- D) Find acceleration at t = 1s and at 2s.



<u>Solution of part A</u>

It's clear, $\mathrm{x} = \left(1\frac{m}{s^2}\right)t^2, \mathrm{y} = \left(1\frac{m}{s^4}\right)t^4, \mathrm{z} = 2\mathrm{m}$

$$\mathrm{y} = \left(1\frac{m}{s^4}\right)t^4 = \left[\left(1\frac{m^{1/2}}{s^2}\right)t^2\right]^2 \rightarrow y = x^2$$

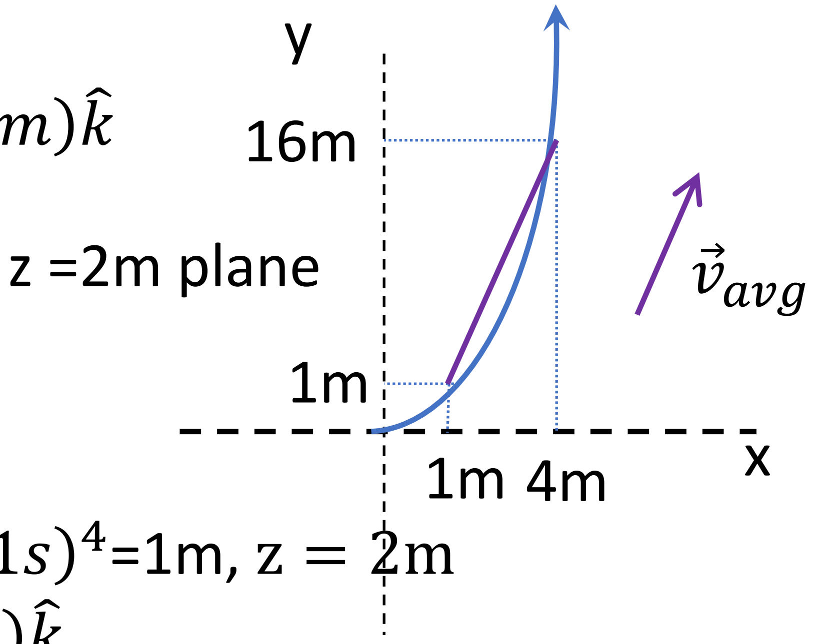
Path is a **parabola** in the plane defined by $z = 2m$

The relation $y = x^2$ omits unit. If **units** were included $\rightarrow y = (1m^{-1})x^2$

Example of Kinematics in 2D

An object follows a path $\vec{r} = \left(1\frac{m}{s^2}\right)t^2\hat{\imath} + \left(1\frac{m}{s^4}\right)t^4\hat{\jmath} + (2m)\hat{k}$

- B) Find the average velocity from t = 1s to 2 s.



Solution of part B

At $t = 1s$, $\mathrm{x}(1s) = \left(1\frac{m}{s^2}\right)(1s)^2 = 1m$, $\mathrm{y}(1s) = \left(1\frac{m}{s^4}\right)(1s)^4$=1m, $\mathrm{z} = 2\mathrm{m}$

$$\vec{r}(1s) = (1m)\hat{\imath} + (1m)\hat{\jmath} + (2m)\hat{k}$$

At $t = 2s$, $\mathrm{x}(2s) = \left(1\frac{m}{s^2}\right)(2s)^2 = 4m$, $\mathrm{y}(2s) = \left(1\frac{m}{s^4}\right)(2s)^4 = 16m$, $\mathrm{z} = 2\mathrm{m}$

$$\vec{r}(2s) = (4m)\hat{\imath} + (16m)\hat{\jmath} + (2m)\hat{k}$$

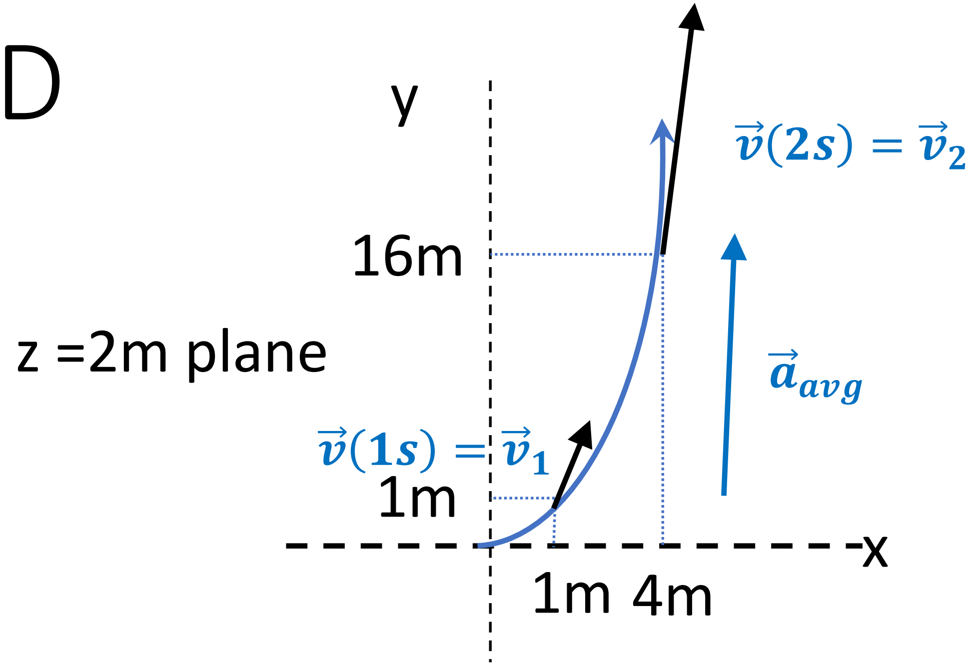
$$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{(\boldsymbol{x_2} - \boldsymbol{x_1})}{(t_2 - t_1)}\hat{\boldsymbol{\imath}} + \frac{(\boldsymbol{y_2} - \boldsymbol{y_1})}{(t_2 - t_1)}\hat{\jmath} + \frac{(\boldsymbol{z_2} - \boldsymbol{z_1})}{(t_2 - t_1)}\hat{k}$$

$$\vec{v}_{avg} = \frac{(4m - \boldsymbol{1m})}{(2s - 1s)}\hat{\boldsymbol{\imath}} + \frac{(16m - \boldsymbol{1m})}{(2s - 1s)}\hat{\jmath} + \frac{(2m - \boldsymbol{2m})}{(2s - 1s)}\hat{k} \rightarrow \vec{v}_{avg} = \left(3\frac{m}{s}\right)\hat{\imath} + \left(15\frac{m}{s}\right)\hat{\jmath}$$

Example of Kinematics in 3D

An object follows a path $\vec{r} = \left(1\frac{m}{s^2}\right)t^2\hat{\imath} + \left(1\frac{m}{s^4}\right)t^4\hat{\jmath} + (2m)\hat{k}$

- C) Find average acceleration from t = 1s to 2s. Plot



<u>**Solution of part C**</u>

First find **velocity** as a function of time t by **differentiation** wrt time, t.

- $\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath} + v_z\hat{k} = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath} + \frac{dz}{dt}\hat{k}$
- $v_x = \frac{dx}{dt} = \frac{d}{dt}\left(\left(1\frac{m}{s^2}\right)t^2\right) = 2\times\left(1\frac{m}{s^2}\right)t^{2-1} = \left(2\frac{m}{s^2}\right)t$
- $v_y = \frac{dy}{dt} = \frac{d}{dt}\left(\left(1\frac{m}{s^4}\right)t^4\right) = 4\times\left(1\frac{m}{s^4}\right)t^{4-1} = \left(4\frac{m}{s^4}\right)t^3$
- $v_z = \frac{dz}{dt} = \frac{d}{dt}(2m) = 0 \rightarrow \qquad \vec{v} = \left(2\frac{m}{s^2}\right)t\hat{\imath} + \left(4\frac{m}{s^4}\right)t^3\hat{\jmath}$

At $t = 1s$, $\vec{v}(1s) = \left(2\frac{m}{s^2}\right)(1s)\hat{\imath} + \left(4\frac{m}{s^4}\right)(1s)^3\hat{\jmath} = \left(2\frac{m}{s}\right)\hat{\imath} + \left(4\frac{m}{s}\right)\hat{\jmath}$

At $t = 2s$, $\vec{v}(2s) = \left(2\frac{m}{s^2}\right)(2s)\hat{\imath} + \left(4\frac{m}{s^4}\right)(2s)^3\hat{\jmath} = \left(4\frac{m}{s}\right)\hat{\imath} + \left(32\frac{m}{s}\right)\hat{\jmath}$

$$\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{4\frac{m}{s} - 2\frac{m}{s}}{2s - 1s}\hat{\imath} + \frac{32\frac{m}{s} - 4\frac{m}{s}}{2s - 1s}\hat{\jmath} = \left(2\frac{m}{s^2}\right)\hat{\imath} + \left(28\frac{m}{s^2}\right)\hat{\jmath}$$

<u>D) Find acceleration at t = 1s and at 2s.</u>

$$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} = \frac{dv_x}{dt}\hat{\imath} + \frac{dv_y}{dt}\hat{\jmath}$$

$$\vec{a} = \frac{d}{dt}\left(\left(2\frac{m}{s^2}\right)t\right)\hat{\imath} + \frac{d}{dt}\left(\left(4\frac{m}{s^4}\right)t^3\right)\hat{\jmath}$$

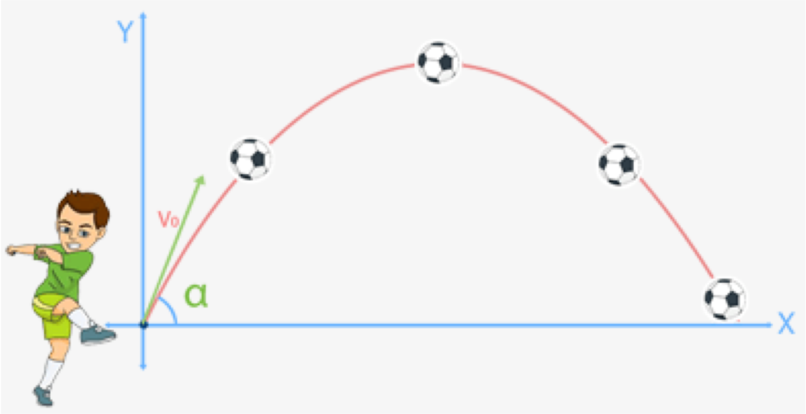
$$\vec{a} = \left(2\frac{m}{s^2}\right)\hat{\imath} + \left(12\frac{m}{s^4}\right)t^2\hat{\jmath}$$

$$\vec{a}(1s) = \left(2\frac{m}{s^2}\right)\hat{\imath} + \left(12\frac{m}{s^2}\right)\hat{\jmath}$$

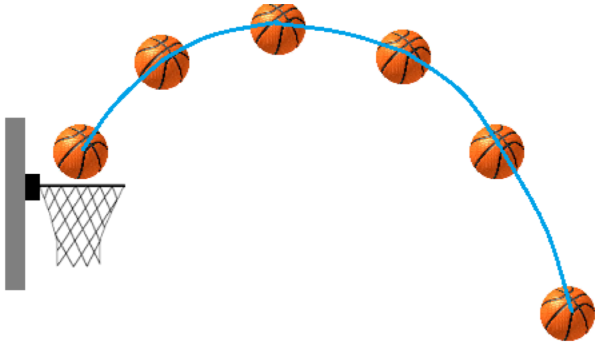
$$\vec{a}(2s) = \left(2\frac{m}{s^2}\right)\hat{\imath} + \left(192\frac{m}{s^2}\right)\hat{\jmath}$$

Projectile Motion

- An object launched near the earth's surface with an initial velocity, $\vec{v}_0$
- The Object then it follows a path (i.e. a projectile) that dictated by the **force** of **gravity**

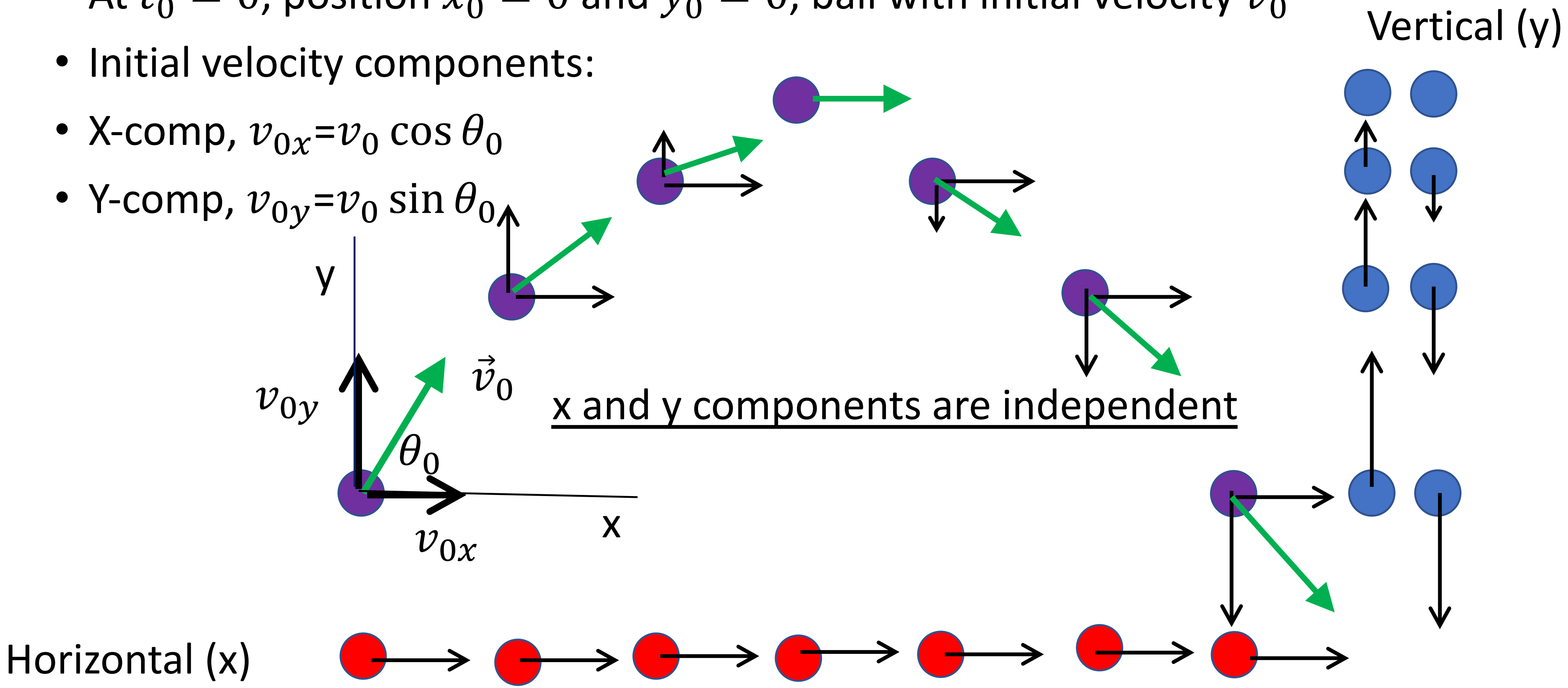


$$a_y = -g\hat{j}$$



Projectile Motion: **Motion diagram** illustrates that the x- and y- components are **independent**

- At $t_0 = 0$, position $x_0 = 0$ and $y_0 = 0$, ball with initial velocity $\vec{v}_0$
- Initial velocity components:
- X-comp, v_{0x}=$v_0 \cos\theta_0$
- Y-comp, v_{0y}=$v_0 \sin\theta_0$

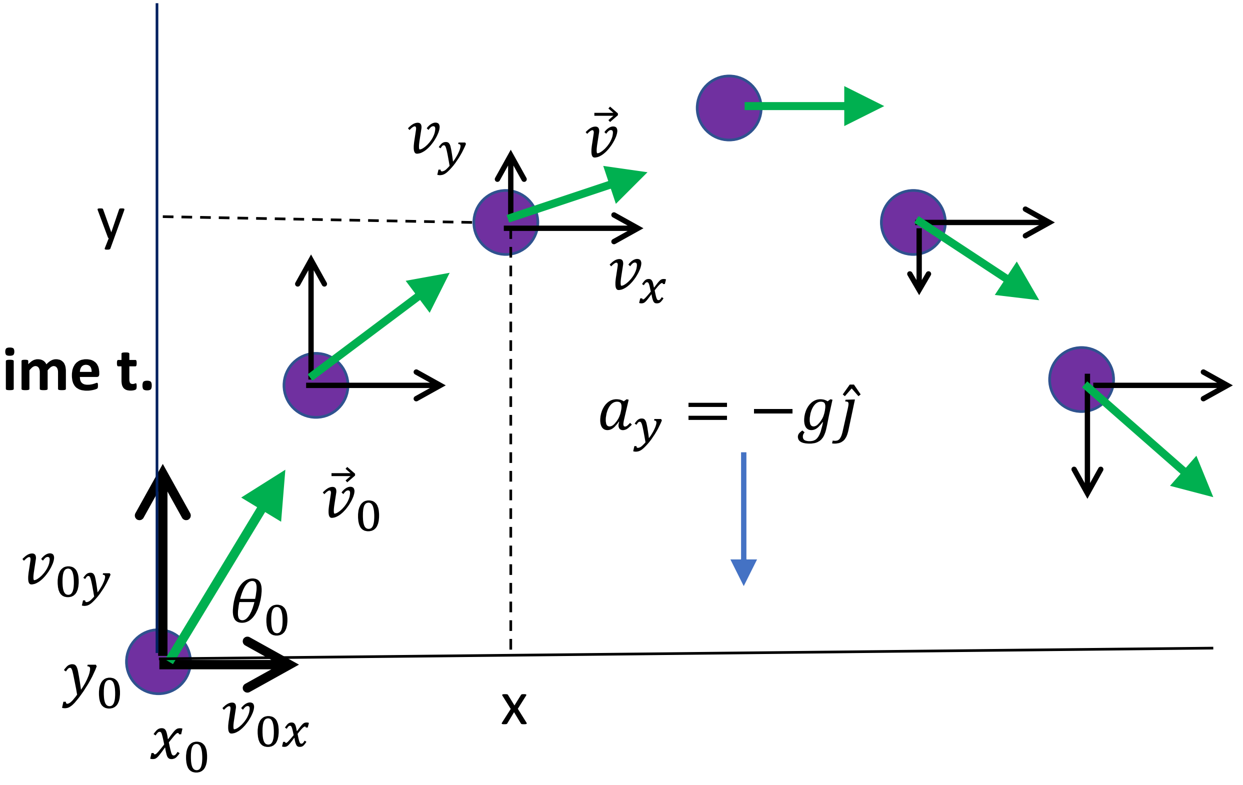


Projectile Motion: Kinematics Equations

- At $t_0 = 0$, position $x_0 = 0$ and $y_0 = 0$, ball with initial velocity $\vec{v}_0$
- Initial velocity components:
- X-comp, v_{0x}=$v_0 \cos\theta_0$
- Y-comp, v_{0y}=$v_0 \sin\theta_0$

Equations to find **position**, $x(t), y(t)$

and **velocity**, $\vec{v}(t) = v_x(t)\hat{\imath} + v_y(t)\hat{\jmath}$ at **time t.**



<u>Horizontal (x)</u>

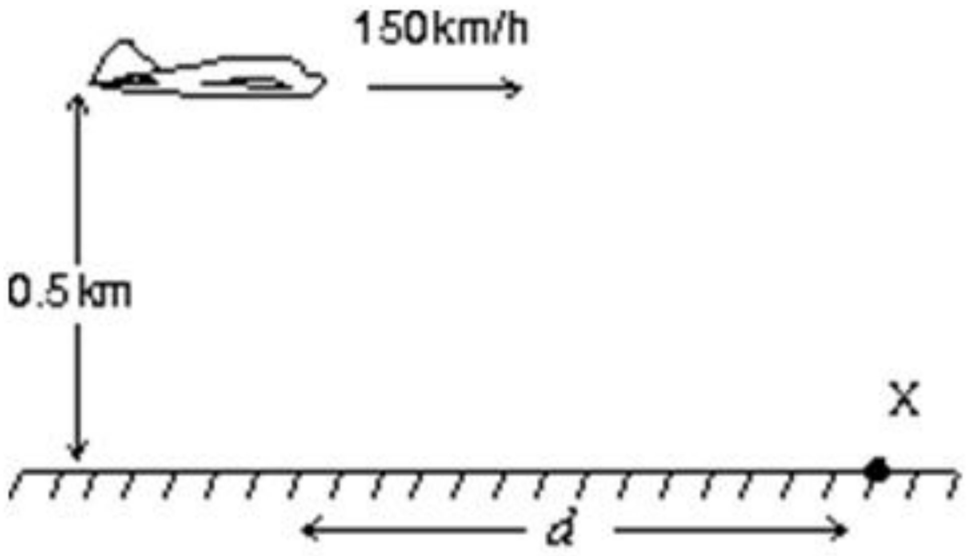
- $x = x_0 + v_{0x}t$, **E1**

<u>Vertical (y)</u>

- y= $y_0 + v_{0y}t - \frac{1}{2}gt^2$, **E2**
- $v_y^2 = v_{0y}^2 - 2g(y - y_0)$, **E3**, $v_y = v_{0y} - gt$, **E4**

Simple Projectile Problem

- The airplane shown is in level flight at an altitude of 0.50 km and a speed of 150 km/h. At what distance d should it release a heavy bomb to hit the target X? Take g = 10 m/s^2.
- **A)** 150m; **B)** 295 m; **C)** 417 m; **D)** 1500 m; **E)** 15000 m.



<u>Horizontal (x)</u>

$x = x_0 + v_{0x}t$, **E1**

<u>Vertical (y)</u>

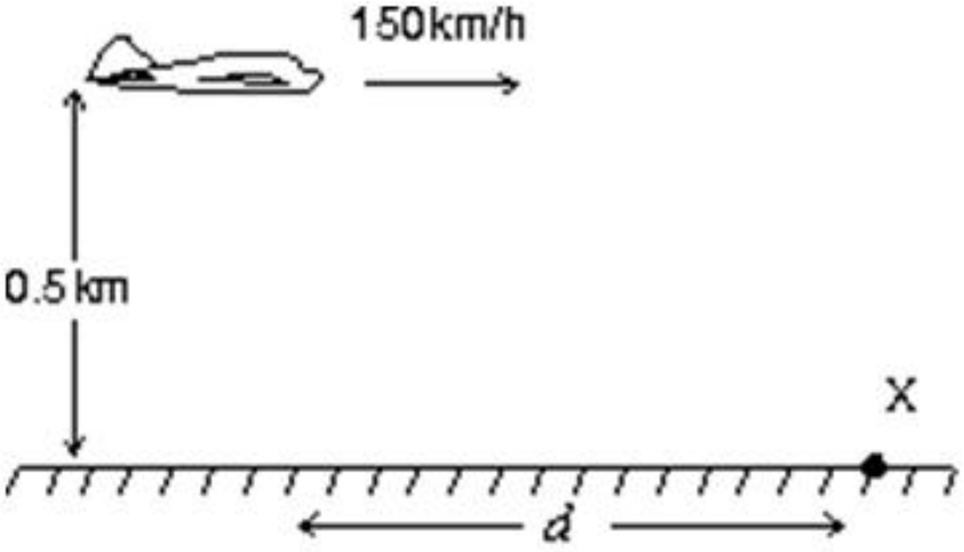
$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$, **E2**

$v_y^2 = v_{0y}^2 - 2g(y - y_0)$, **E3**

$v_y = v_{0y} - gt$, **E4**

Simple Projectile Problem: Solution

- The airplane shown is in level flight at an altitude of 0.50 km and a speed of 150 km/h. At what distance d should it release a heavy bomb to hit the target X? Take g = 10 m/s^2.
- **A)** 150m; **B)** 295 m; **C)** 417 m; **D)** 1500 m; **E)** 15000 m.



<u>Solution:</u>

Consider **Vertical** and use **E2**, to find time bomb falls 500m (0.5 km):

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$

$$-500m = 0 + 0 \times t - 5\frac{m}{s^2}t^2 \rightarrow t = 10s$$

<u>Horizontal Range</u>

Initial Horizontal velocity $v_{ox} = 41.7m \cdot s^{-1}$

Horizontal Range: **E1** $x = v_{ox}t = 417m$

<u>Horizontal (x)</u> $x = x_0 + v_{0x}t$, **E1**

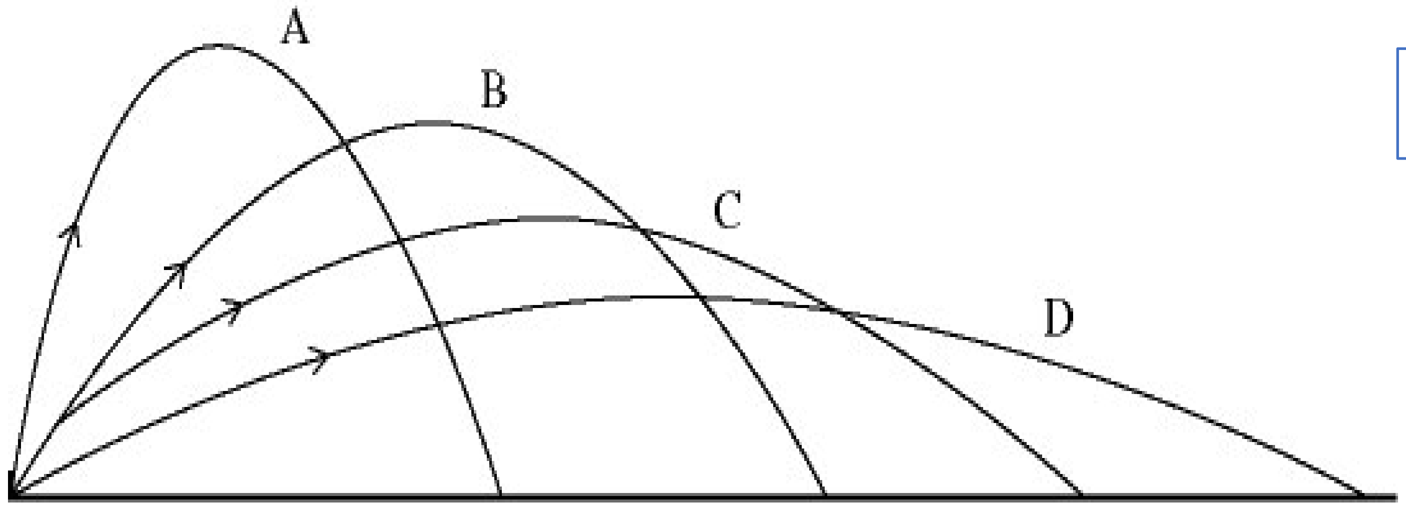
<u>Vertical (y)</u>

$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$, **E2** ; $v_y^2 = v_{0y}^2 - 2g(y - y_0)$, **E3**

$v_y = v_{0y} - gt$, **E4**

Multiple Choice

- The figure below (top of next page) shows trajectories of four artillery shells. Each fired with the same initial speed. Which trajectory remains in the air for the longest time? Circle the right answer. **Hint**: ask yourself how to throw a ball so that it remains in the air for the longest period.

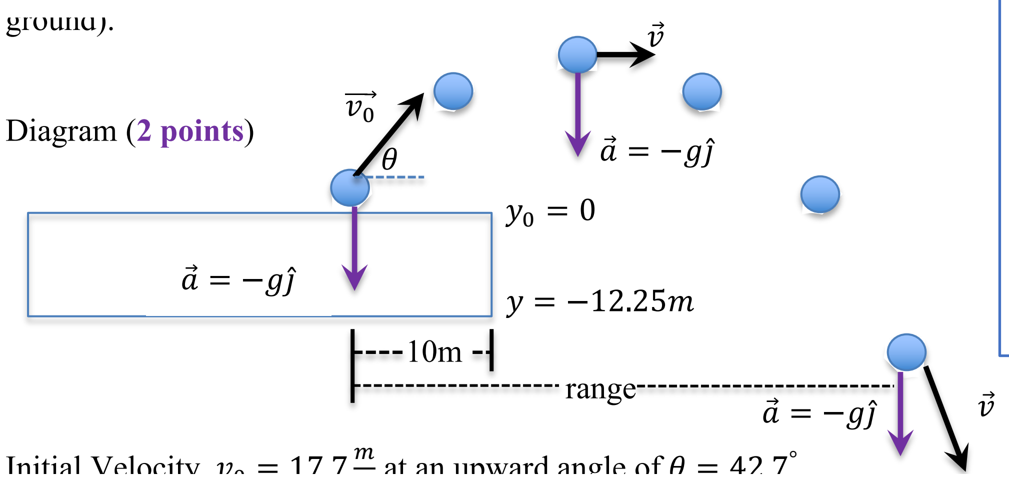


Answer: A

Previous Test Question

- (**10 points**) A child is standing **10m** from the edge of a cliff. He kicks a soccer ball is kicked on the ground with an initial speed of 17.7 m/s at an upward angle of 42.7° above the horizontal. The cliff is 12.25 m high.

A) Draw the path of trajectory (or a motion diagram) of the soccer the ball, which shows the **direction** of the **velocity** and **acceleration** at the following points: i) the instant it leaves the ground; ii) its maximum height; iii) the instant when it hits the ground. **Calculate** the **x** and **y** component of the initial velocity



<u>Solution of A)</u>

Initial Velocity, $v_0 = 17.7\frac{m}{s}$ at $\theta = 42.7^{\circ}$

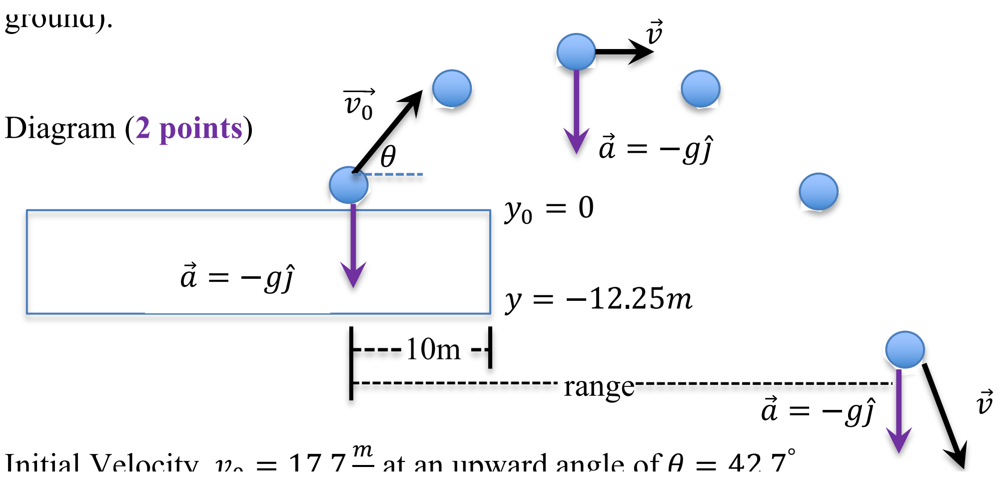
x-com, $v_{0x} = v_0 cos\theta = 13\frac{m}{s}$ **(1 point)**

y-com, $v_{0y} = v_0 sin\theta = 12\frac{m}{s}$ **(1 point)**

Previous Test Questions: Solution part B

- (**10 points**) A child is standing **10m** from the edge of a cliff. He kicks a soccer ball is kicked on the ground with an initial speed of 17.7 m/s at an upward angle of 42.7° above the horizontal. The cliff is 12.25 m high.

B) Calculate the **time** it takes the ball to **hit** the **ground**. Does it land on the bottom of the cliff? **HINT:** The **answer** is **yes**, but you must show this using projectile equations. **Partial Answer:** t = 3.23 s



Solution of B)

First, we must show that the ball does **not** land on the cliff. Since it must travel 10m **horizontally** to the cliff edge, the time it takes is $10m = v_{0x}t \rightarrow t = 0.77s$. We used **E1**.

Using **E2**, the vertical position is $y = v_{0y}t - \frac{1}{2}gt^2 = 12\frac{m}{s}\times 0.77s - \frac{1}{2}\left(9.8\frac{m}{s^2}\right)(0.77s)^2 = 6.33m.$

Hence at the cliff edge it is **above** top of the cliff. (**2 points**)

Horizontal (x) $x = x_0 + v_{0x}t$, **E1**

Vertical (y)

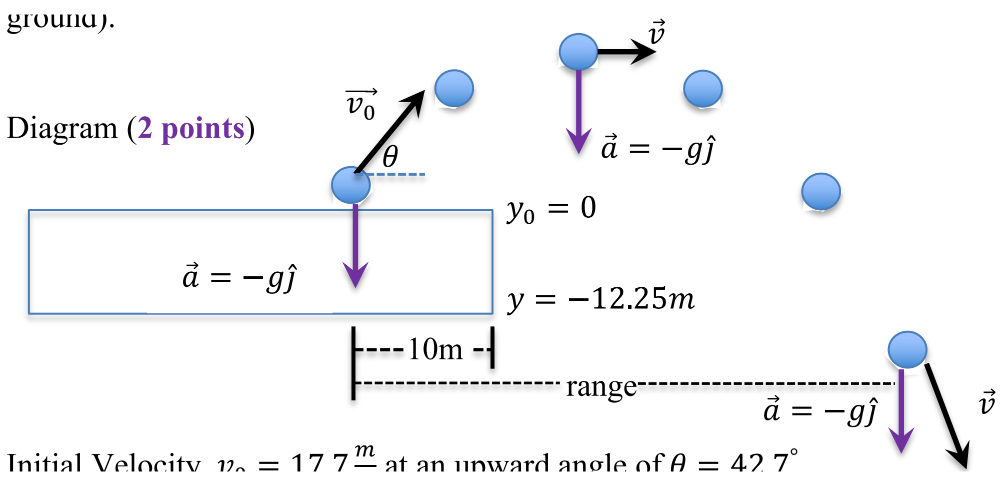
$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$, **E2** ; $v_y^2 = v_{0y}^2 - 2g(y - y_0)$, **E3**

$v_y = v_{0y} - gt$, **E4**

Previous Test Questions: Solution part B, Continued

- (**10 points**) A child is standing **10m** from the edge of a cliff. He kicks a soccer ball is kicked on the ground with an initial speed of 17.7 m/s at an upward angle of 42.7° above the horizontal. The cliff is 12.25 m high.

B) Calculate the **time** it takes the ball to **hit** the **ground**. Does it land on the bottom of the cliff? **HINT:** The **answer** is **yes**, but you must show this using projectile equations. **Partial Answer:** t = 3.23 s



Solution of B) Continued

We have shown that the ball will clear the cliff's edge and fall to the ground

To find time to the ground use **E2**

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$$

$$-12.25m = 12\frac{m}{s}t - 4.9\frac{m}{s^2}t^2 = 0 \rightarrow t^2 - 2.45t - 2.5 = 0$$

Solution, $t = \frac{2.45 \pm \sqrt{2.45^2 + 4 \times 2.5}}{2} = \frac{2.45 \pm 4}{2}s = 3.23s \text{ and } -0.78s.$

Physical Solution is t = 3.23 s (**3 points**)

Horizontal (x) $x = x_0 + v_{0x}t$, **E1**

Vertical (y)

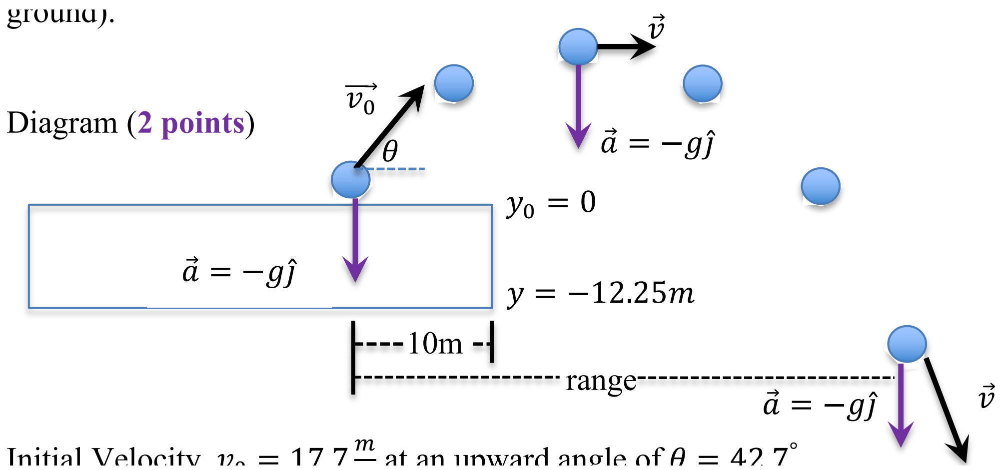
$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$, **E2** ; $v_y^2 = v_{0y}^2 - 2g(y - y_0)$, **E3**

$v_y = v_{0y} - gt$, **E4**

Previous Test Questions: Solution part C

- (**10 points**) A child is standing **10m** from the edge of a cliff. He kicks a soccer ball is kicked on the ground with an initial speed of 17.7 m/s at an upward angle of 42.7° above the horizontal. The cliff is 12.25 m high.

C) Hence calculate the **horizontal distance** by the ball traveled just before it hits the ground at the bottom of the cliff, i.e. calculate the **range**.



Solution of C)

To find **range** to the ground use **E1**

Also from part A) $v_{0x} = v_0 cos\theta = 13\frac{m}{s}$, $v_{0y} = v_0 sin\theta = 12\frac{m}{s}$

From part B, t = 3.23 s

Range is $x = v_{0x}t = 13\frac{m}{s}\times 3.23s = 42m$. (**1 point**)

Horizontal (x) $x = x_0 + v_{0x}t$, **E1**

Vertical (y)

$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$, **E2** ; $v_y^2 = v_{0y}^2 - 2g(y - y_0)$, **E3**

$v_y = v_{0y} - gt$, **E4**

Projectile Motion: Equation of Motion revisited

- At $t_0 = 0$, position $x_0 = 0$ and $y_0 = 0$, ball with initial velocity $\vec{v}_0$
- Initial velocity components:
- X-comp, $v_{0x} = v_0 \cos\theta_0$ **E13**
- Y-comp, $v_{0y} = v_0 \sin\theta_0$ **E15**

Equations to find **position**, $x(t)$, $y(t)$

and **velocity**, $\vec{v}(t) = v_x(t)\hat{\imath} + v_y(t)\hat{\jmath}$ at **time t.**

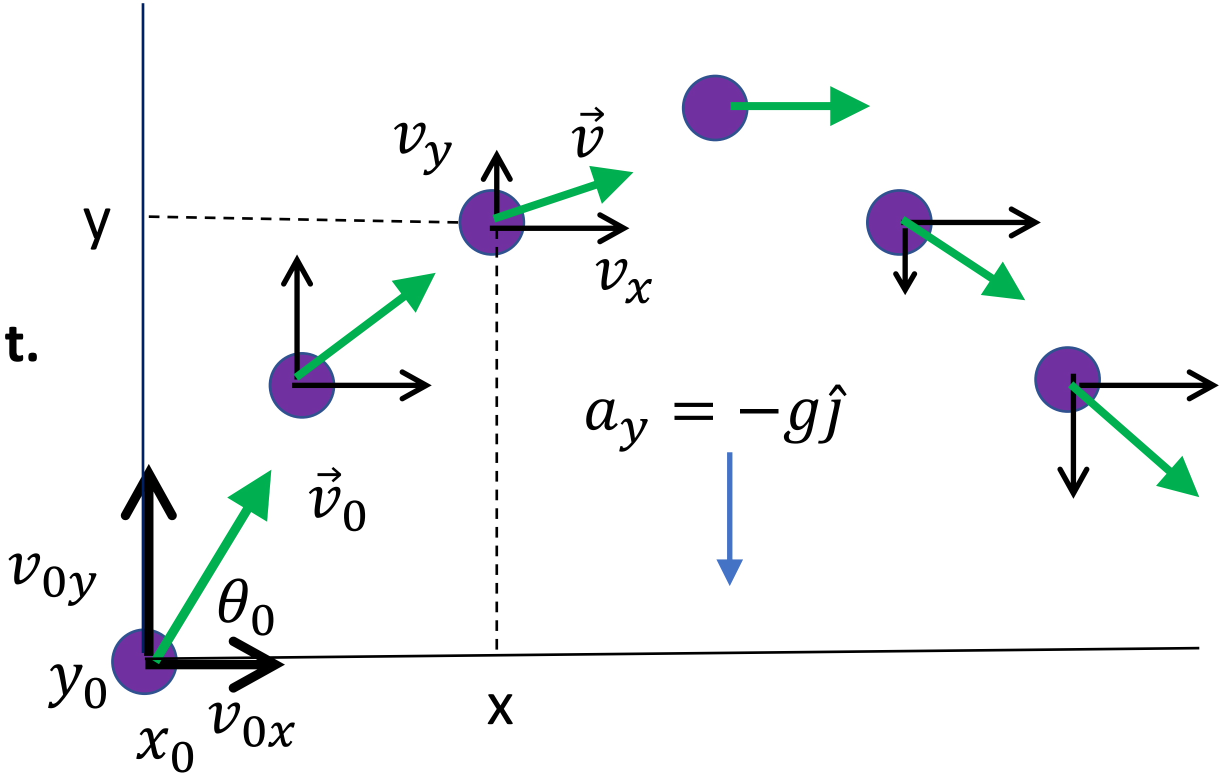
<u>Horizontal (x)</u>

- $x = x_0 + v_{0x}t$, **E14**

<u>Vertical (y)</u>

$v_y = v_{0y} - gt$, **E16**

- $y = y_0 + v_{0y}t - \frac{1}{2}gt^2$, **E17**
- $v_y^2 = v_{0y}^2 - 2g(y - y_0)$, **E18**,



Equations for Midterm 1

- 1D Kinematics:
- $v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$ **E1**; $v = \frac{dx}{dt}$ **E2**; $s_{avg} = \frac{total\ distance}{total\ time}$ **E3**
- $a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$ **E4**; $a = \frac{dv}{dt}$ **E5**
- $v = v_0 + at$ **E6**; $x = x_0 + v_0 t + \frac{1}{2}at^2$ **E7**; $v^2 = v_0^2 + 2a(x - x_0)$ **E8**
- 2D Kinematics:
- $\vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{x_2 - x_1}{t_2 - t_1}\hat{\imath} + \frac{y_2 - y_1}{t_2 - t_1}\hat{\jmath} + \frac{z_2 - z_1}{t_2 - t_1}\hat{k} = v_{avg,x}\hat{\imath} + v_{avg,y}\hat{\jmath} + v_{avg,z}\hat{k}$ **E9**;
- $\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath} + \frac{dz}{dt}\hat{k} = v_x\hat{\imath} + v_y\hat{\jmath} + v_z\hat{k}$ **E10**
- $\vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{v_{2x} - v_{1x}}{t_2 - t_1}\hat{\imath} + \frac{v_{2y} - v_{1y}}{t_2 - t_1}\hat{\jmath} + \frac{v_{2z} - v_{1z}}{t_2 - t_1}\hat{k} = a_{avg,x}\hat{\imath} + a_{avg,y}\hat{\jmath} + a_{avg,z}\hat{k}$ **E11**
- $\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{\imath} + \frac{dv_y}{dt}\hat{\jmath} + \frac{dv_z}{dt}\hat{k} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$ **E12**
- Projectile Motion
- **Horizontal:** $v_{0x} = v_0 \cos\theta_0$ **E13**; $x = x_0 + v_{0x}t$ **E14**
- **Vertical:** $v_{0y} = v_0 \sin\theta_0$ **E15**; $v_y = v_{0y} - gt$ **E16**; $y = y_0 + v_{0y}t - \frac{1}{2}gt^2$ **E17**;
- $v_y^2 = v_{0y}^2 - 2g(y - y_0)$ **E18**

Tossed ball from rising Balloon

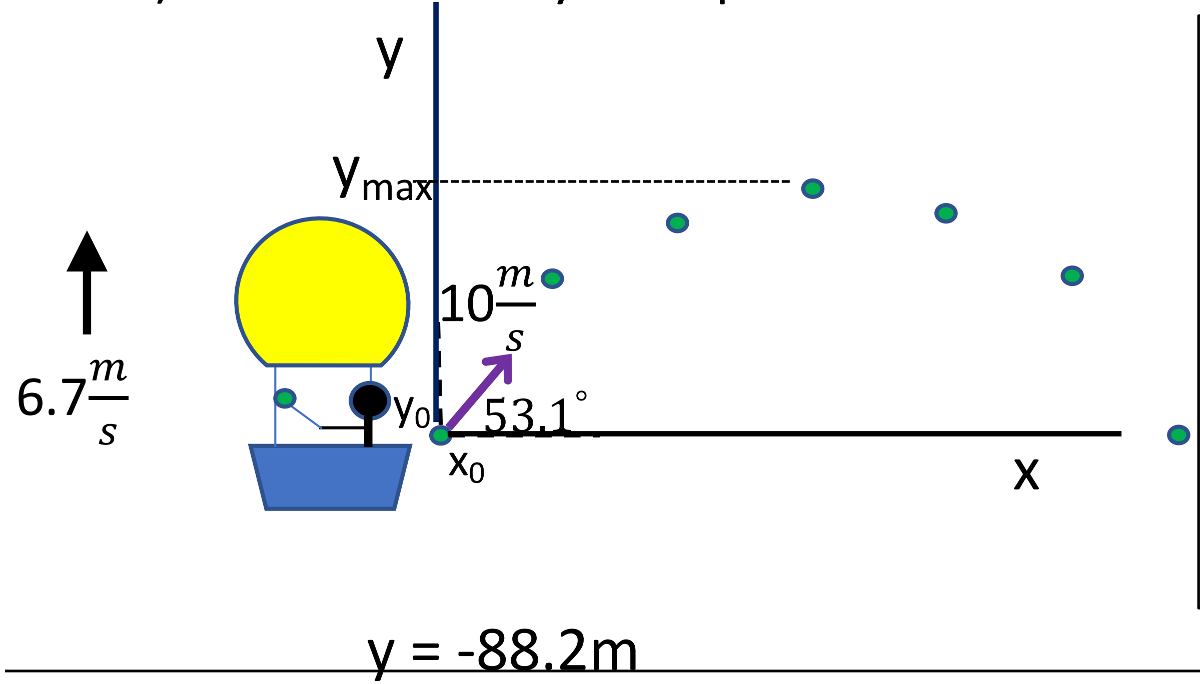
A balloon is rising at a **constant** speed of $6.7\,\frac{m}{s}$. At 88.2 m above the ground, a ball is launch from the balloon at $10\,\frac{m}{s}$ (with respect to himself) at 53.1° above the horizontal.

A) Find the maximum height of the ball.

B) At the **maximum height**, what is the **speed** and **acceleration** of the ball?

C) Find the range (horizontal distance from the launch point) of the ball.

D) Find the velocity and speed of the ball when it hits the ground.



<u>Solution of Part A</u>

Initial Velocity component:

Use **E13** $v_{0x} = 10\times\cos 53.1^{\circ} = 6\,\frac{m}{s}$

Use **E15** $v_{0y} = 10\times\sin 53.1^{\circ} = 8\,\frac{m}{s}$

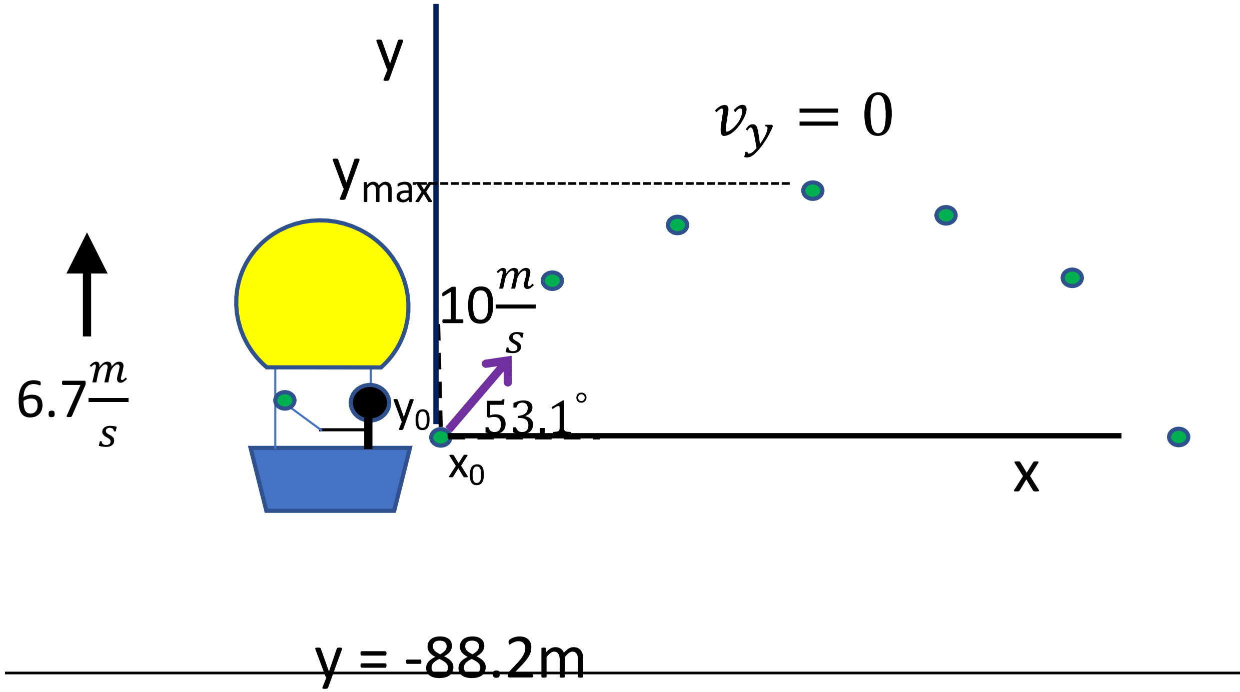
But this **neglect** the balloon's **vertical velocity**

$$v_{0y} = 10\times\sin 53.1^{\circ} + 6.7\,\frac{m}{s} = 14.7\,\frac{m}{s}$$

Tossed ball from rising Balloon: Part A

A balloon is rising at a **constant** speed of $6.7\frac{m}{s}$. At 88.2 m above the ground, a ball is launch from the balloon at $10\frac{m}{s}$ (with respect to himself) at 53.1° above the horizontal.

A) Find the maximum height of the ball.



<u>Solution of Part A</u> (continued)

Initial Velocity component:

$$v_{0x} = 6\frac{m}{s};\ v_{0y} = 14.7\frac{m}{s}$$

At the Maximum height….??

Use **E18**, $v_y^2 = 0 = v_{0y}^2 - 2g(y - y_0)$

$$0 = v_{0y}^2 - 2gy_{max}$$

$$y_{max} = \frac{\left(14.7\frac{m}{s}\right)^2}{2\times 9.8m\cdot s^{-2}} = 11.025m = 11m$$

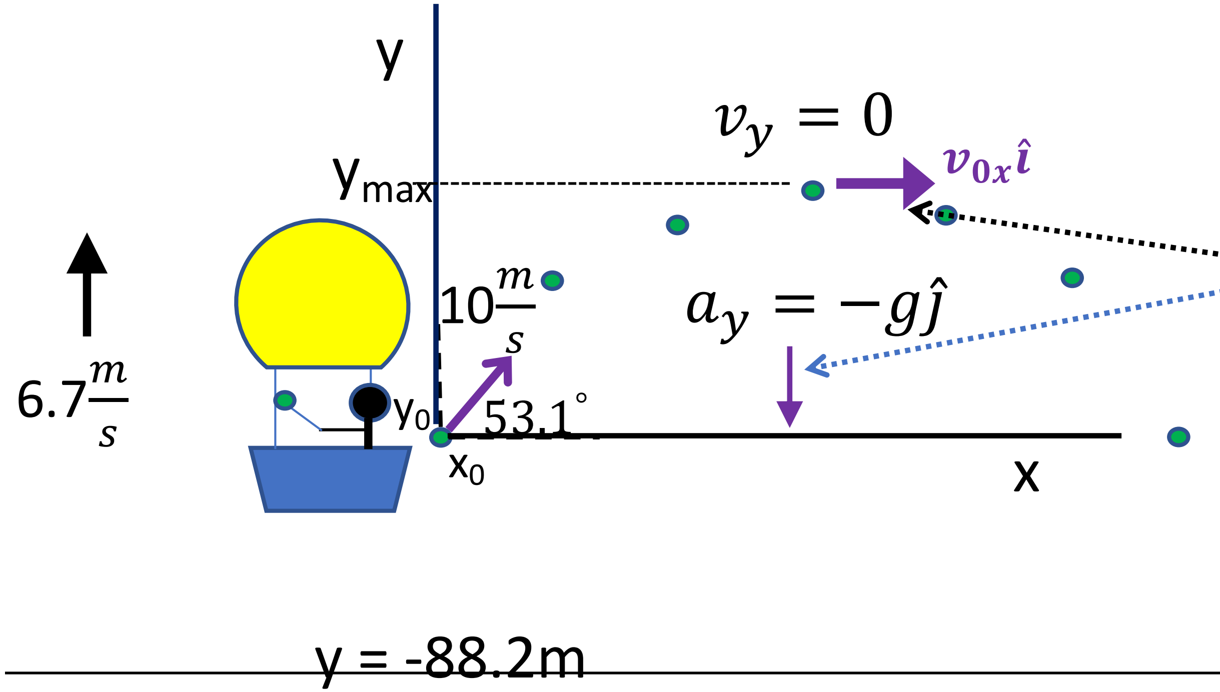
Maximum height is 11m + 88.2 m = **99.2 m above** the **ground**.

Tossed ball from rising Balloon: Part B

A balloon is rising at a **constant** speed of $6.7\,\frac{m}{s}$. At 88.2 m above the ground, a ball is launch from the balloon at $10\,\frac{m}{s}$ (with respect to himself) at 53.1° above the horizontal.

A) Find the maximum height of the ball.

B) At the **maximum height**, what is the **speed** and **acceleration** of the ball?



Alternate Solution of Part A, Use **E16** to find the time to y_{max}, then use **E17** to find y_{max}

Solution of Part B

Acceleration is $g = 9.8\,\frac{m}{s^2}$ **down** towards the ground.

Speed is simply the x-comp, $v_{0x} = 6\,\frac{m}{s}$

Tossed ball from rising Balloon: Part C

A balloon is rising at a **constant** speed of $6.7\,\frac{m}{s}$. At 88.2 m above the ground, a ball is launch from the balloon at $10\,\frac{m}{s}$ (with respect to himself) at 53.1°above the horizontal.

C) Find the **range** (horizontal distance from the launch point) of the ball.

<u>Solution of C</u>

From A, $v_{0x} = 6\,\frac{m}{s};\ v_{0y} = 14.7\,\frac{m}{s}$

Use **E17**, $y = y_0 + v_{0y}t - \frac{1}{2}gt^2$

$$-88.2m = 0 + 14.7\,\frac{m}{s}t - 4.9\,\frac{m}{s^2}t^2$$

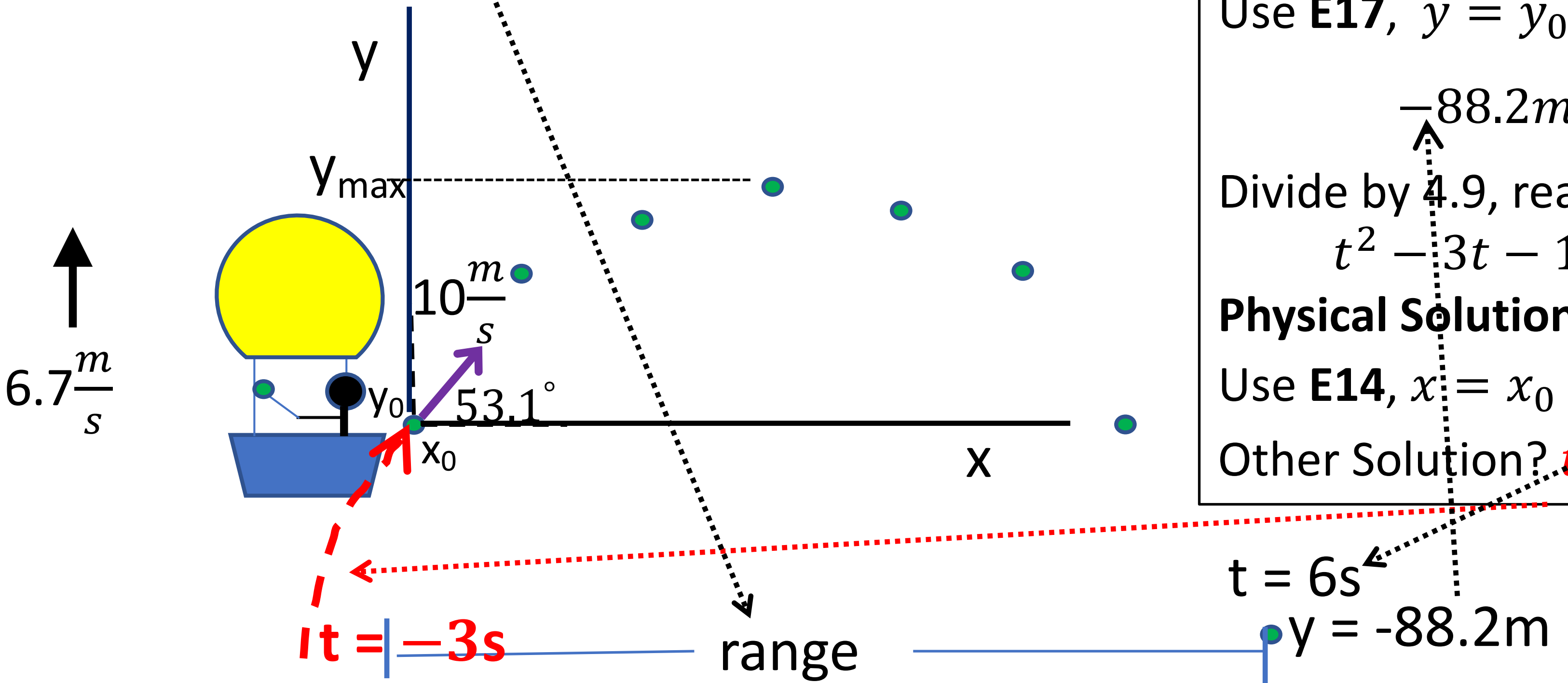
Divide by 4.9, rearrange and omit unit:

$$t^2 - 3t - 18 = 0 \rightarrow (t - 6s)(t + 3s) = 0$$

Physical Solution: $t = 6s$

Use **E14**, $x = x_0 + v_{0x}t$, range $= 6\,\frac{m}{s}\times 6s = 36m$

Other Solution? $\boldsymbol{t = -3s}$



Tossed ball from rising Balloon: Part D

A balloon is rising at a **constant** speed of $6.7\frac{m}{s}$. At 88.2 m above the ground, a ball is launch from the balloon at $10\frac{m}{s}$ (with respect to himself) at 53.1° above the horizontal.

D) Find the velocity and speed of the ball when it hits the ground.

Solution of D From A, $v_{0x} = 6\frac{m}{s}$; $v_{0y} = 14.7\frac{m}{s}$

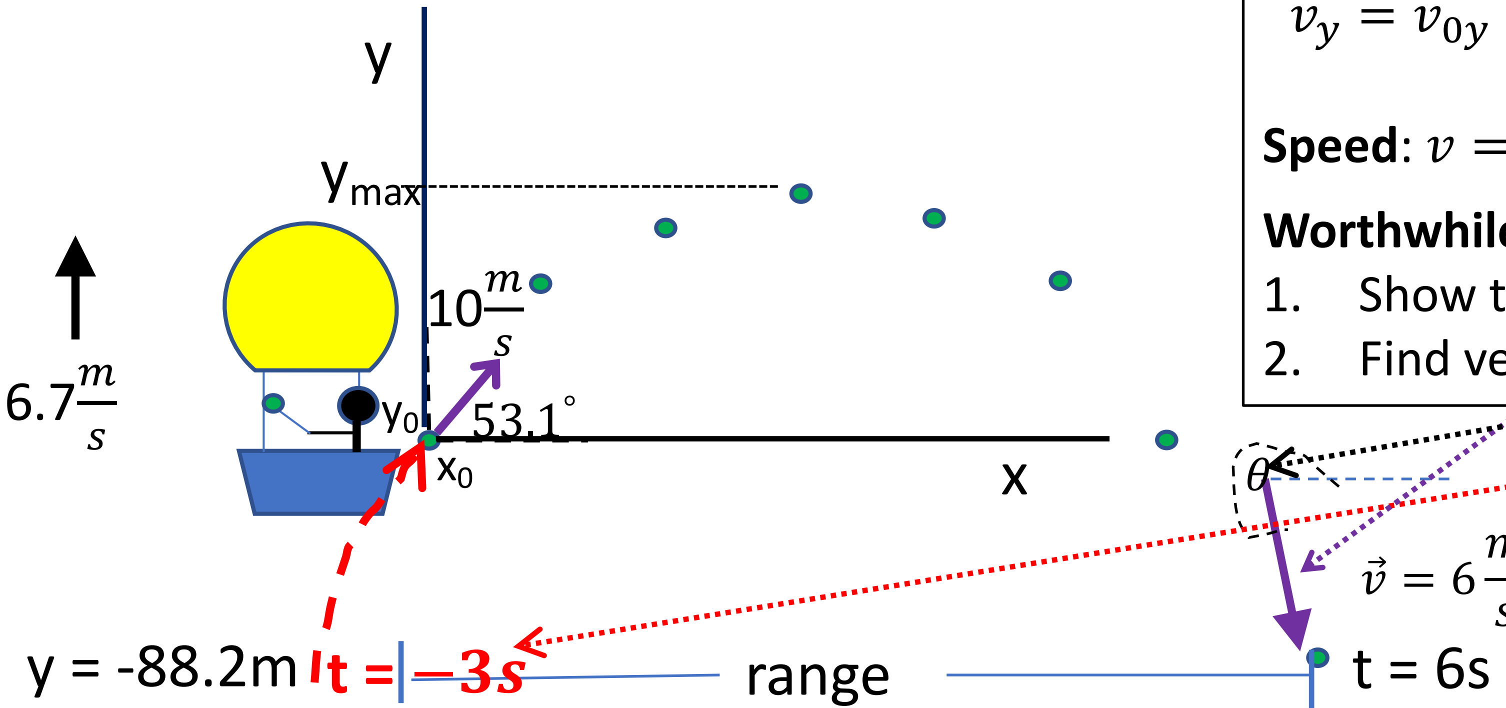
Use **E16**, $v_y = v_{0y} - gt$, and t = 6s

$$v_y = v_{0y} - gt = -44.1\frac{m}{s} \rightarrow \vec{v} = 6\frac{m}{s}\hat{\imath} - 44.1\frac{m}{s}\hat{\jmath}$$

Speed: $v = |\vec{v}| = \sqrt{\left(6\frac{m}{s}\right)^2 + \left(-44.1\frac{m}{s}\right)^2} = 44.5\frac{m}{s}$

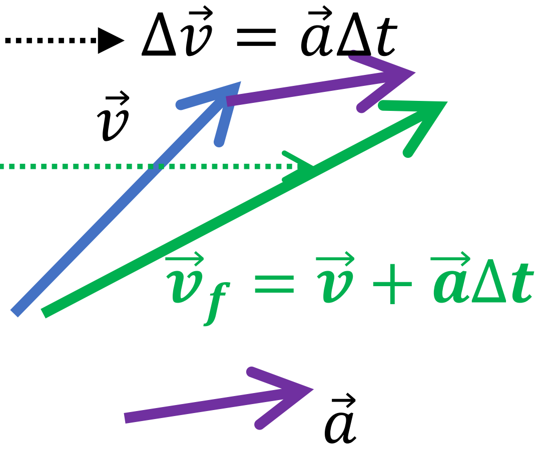
Worthwhile exercise:

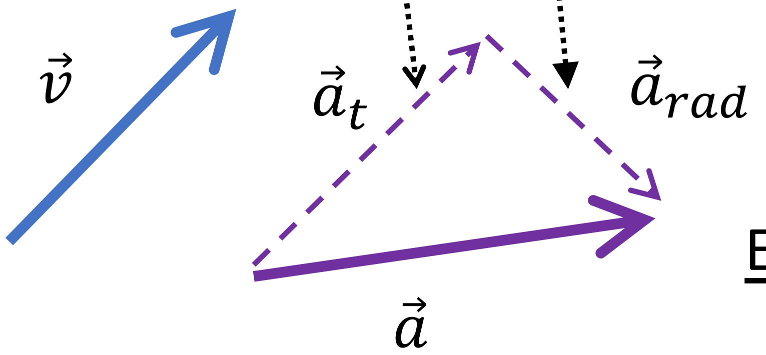
1. Show that for **final velocity**, at t = 6s, $\theta = 278^{\circ}$
2. Find velocity for other solution, **t = $-3s$**



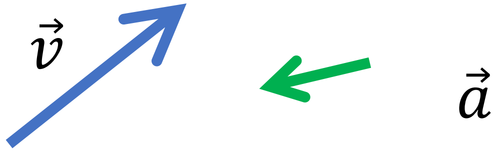
Tangential, $\vec{a}_t$, and Radial, $\vec{a}_{rad}$ component of Acceleration.

- **Tangential Component**, $\vec{a}_t$, is **parallel** to the velocity $\vec{v}$
- **Radial Component**, $\vec{a}_{rad}$, is **perpendicular** to the velocity $\vec{v}$
- How does the velocity change?
- In time Δt, the velocity will change by $\Delta\vec{v} = \vec{a}\Delta t$
- Final velocity is $\boldsymbol{\vec{v}_f = \vec{v} + \vec{a}\Delta t}$
- System on right is **speeding up** and **turning right**!



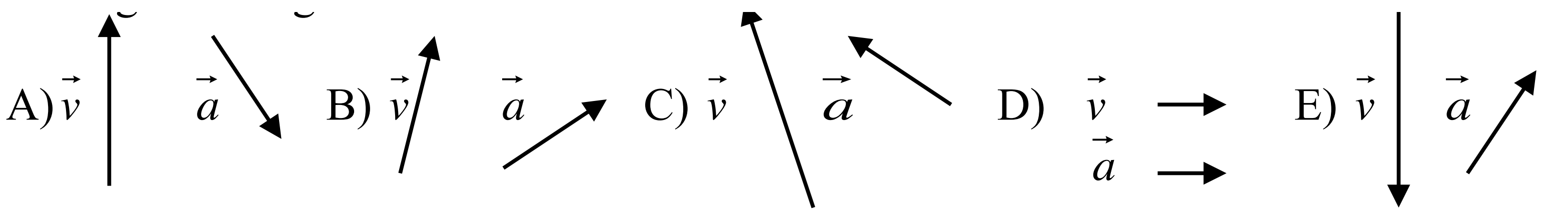


<u>Example of **Slowing down** and **turning left**</u>



A Multiple Choice

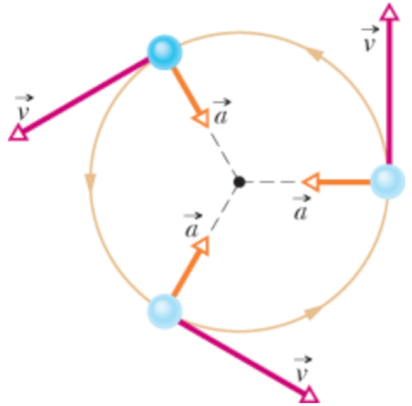
- Shown Below are the velocity and acceleration vectors for an object in several different types of motion. In which case is the object **slowing down** and **turning left**?

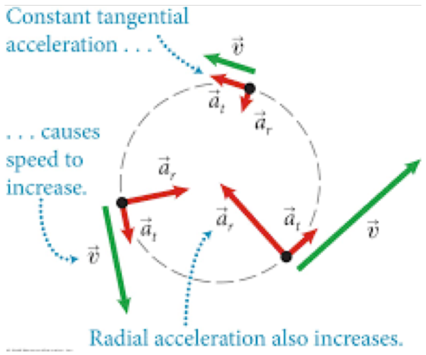


ANSWER: E

Circular Motion Section 4.5

- An object that follows an circular path of radius r, at constant speed, v is said to be in a **uniform circular motion** as shown in the Figure below:





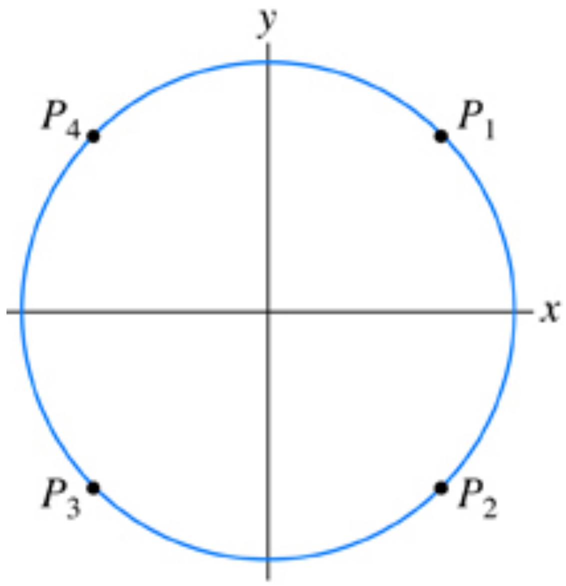
The **acceleration**, $\vec{a}$ of an **object** in **uniform circular motion** is perpendicular to its velocity, $\vec{v}$.
The **acceleration**, $\vec{a}$ points to the center of the circle and has a **magnitude** of:

$$a = \frac{v^2}{r}$$

The figure on the left is an **example** of a **circular motion** at **variable speed**.

Another Multiple choice

- An object is moving around in a circle at constant speed in the **clockwise** direction (shown below), from P_1 to P_2 to P_3 to P_4 and back to P_1. At which point does the object have a **negative x-velocity** and a **negative y-velocity**? Circle the correct answer.
- a) P_1 b) P_2 c) P_3 d) P_4 e) none of these answers



ANSWER: B