More about Free Fall and Introduction to Vectors

Chapter 2 and 3

A multiple choice

A projectile is shot vertically upward with a given initial velocity. It reaches a maximum height of 100 m. If on a second shot the initial velocity is doubled, then the projectile will reach a maximum height of:

A) 70.7 m; B) 141.4 m; C) 200 m; D) 241 m; E) 400 m

A multiple choice: the Solution

A projectile is shot vertically upward with a given initial velocity. It reaches a maximum height of 100 m. If on a second shot the initial velocity is doubled, then the projectile will reach a maximum height of:

A) 70.7 m; B) 141.4 m; C) 200 m; D) 241 m; E) 400 m

$v = v_0 - gt \quad (2\text{-}41) \qquad y - y_0 = v_0 t - \frac{1}{2} g t^2 \quad (2\text{-}42) \qquad v^2 = v_0^2 - 2g(y - y_0) \quad (2\text{-}43)$

Solution

Use 2.43, $v^2 = v_{01}^2 - 2g(y - y_0)$, on **first shot**, with $(y - y_0) = h_1 = 100m$

v_{01} is unknown! And final velocity at maximum height is $y = 0$

$$0 = v_{01}^2 - 2gh_1 \rightarrow h_1 = \frac{v_{01}^2}{2g}$$

For second shot, $v_{02} = 2v_{01}$, so the second height, $h_2 = \frac{v_{02}^2}{2g} = \frac{(2v_{01})^2}{2g} = 4\frac{v_{01}^2}{2g}$

Height, $h_2 = \frac{v_{02}^2}{2g} = \frac{(2v_{01})^2}{2g} = 4\frac{v_{01}^2}{2g} = 400m$

Another Free-Fall Problem

- A **rock** is thrown **directly downward** from the edge of the roof of a building that is 34.6 meters tall. The rock misses the building on its way down, and is observed to strike the ground 1.50 seconds after being thrown. Take the acceleration due to gravity to have magnitude 9.80 m/s^2 and neglect any effect due to air resistance. Find the initial velocity of the rock.

$v = v_0 - gt \quad (2\text{-}41)$ $y - y_0 = v_0 t - \frac{1}{2} g t^2 \quad (2\text{-}42)$ $v^2 = v_0^2 - 2g(y - y_0) \quad (2\text{-}43)$

<u>Solution</u>

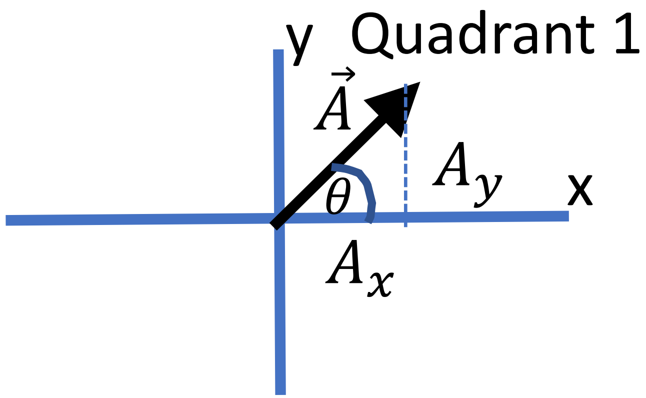
Use 2.42, $y = v_0 t - \frac{1}{2} g t^2$, where we used $y_0 = 0$

Use $y = -34.6m$ an $t = 1.50s$

$$-34.6m = v_0(1.5s) - \frac{1}{2} 9.8 \frac{m}{s^2} (1.5s)^2$$

$$v_0 = \frac{-34.6m + \frac{1}{2} 9.8 \frac{m}{s^2} (1.5s)^2}{1.5s} = -15.7 \frac{m}{s}$$

Vector Component in 2D



$$0^\circ < \theta < 90^\circ$$

$$A_x = A\cos\theta > 0, \qquad A_y = A\sin\theta > 0$$

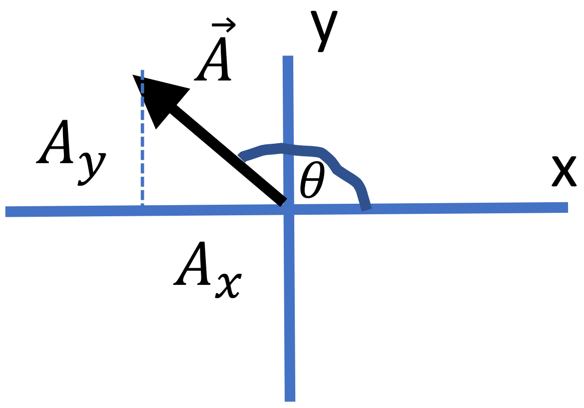
Pythagorean theorem

Magnitude of $\vec{A}$, $A = \sqrt{A_x^2 + A_y^2}$

Example: $\vec{A} = A \; at \; \theta \rightarrow A = 2.0, at\ 30^\circ$,

$$\vec{A} = (A_x, A_y) = (1.73,1)$$

Quadrant 2



$$90^\circ < \theta < 180^\circ$$

$$A_x = A\cos\theta < 0, \qquad A_y = A\sin\theta > 0$$

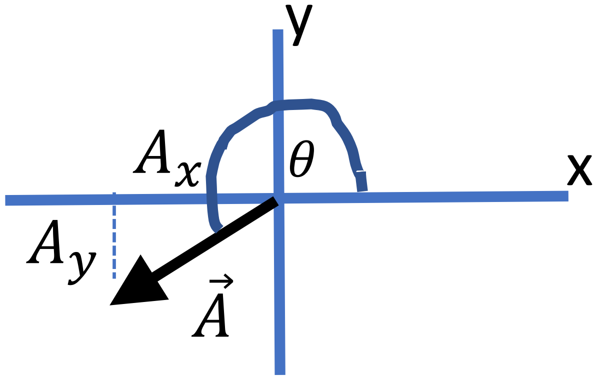
Pythagorean theorem

Magnitude of $\vec{A}$, $A = \sqrt{A_x^2 + A_y^2}$

Example: $\vec{A} = A \; at \; \theta \rightarrow A = 2.0, at\ 120^\circ$,

$$\vec{A} = (A_x, A_y) = (-1,1.73)$$

Vector Component in 2D



Quadrant 3

$180^\circ < \theta < 270^\circ$

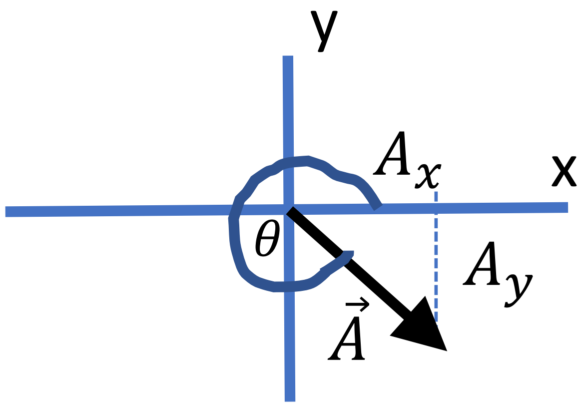
$A_x = Acos\theta < 0, \quad A_y = Asin\theta < 0$

Pythagorean theorem

Magnitude of $\vec{A}$, $A = \sqrt{A_x^2 + A_y^2}$

Example: $\vec{A} = A\ at\ \theta \rightarrow A = 3.0, at\ 225^\circ,$

$\vec{A} = (A_x, A_y) = (-2.12, -2.12)$



Quadrant 4

$270^\circ < \theta < 360^\circ$

$A_x = Acos\theta > 0, \quad A_y = Asin\theta < 0$

Pythagorean theorem

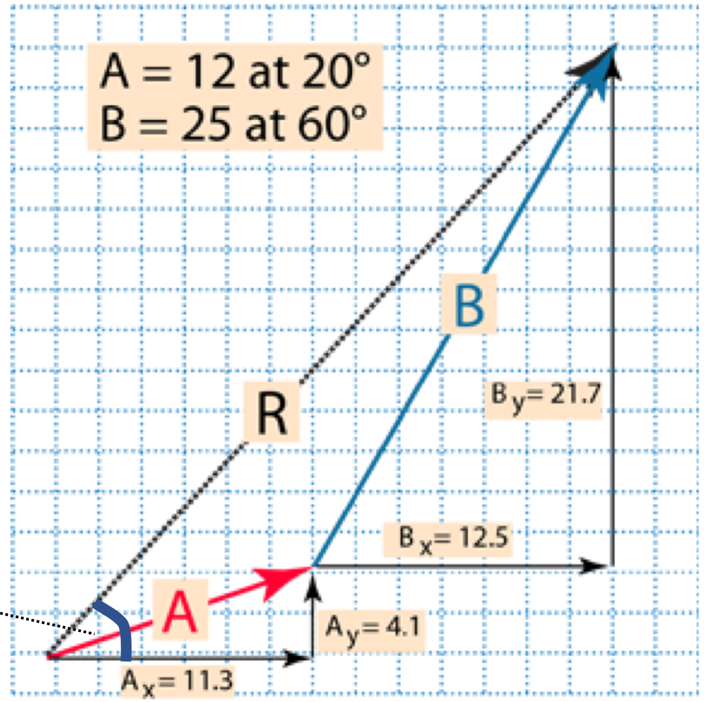
Magnitude of $\vec{A}$, $A = \sqrt{A_x^2 + A_y^2}$

Example: $\vec{A} = A\ at\ \theta \rightarrow A = 3.3, at\ 277^\circ,$

$\vec{A} = (A_x, A_y) = (0.4, -3.27)$

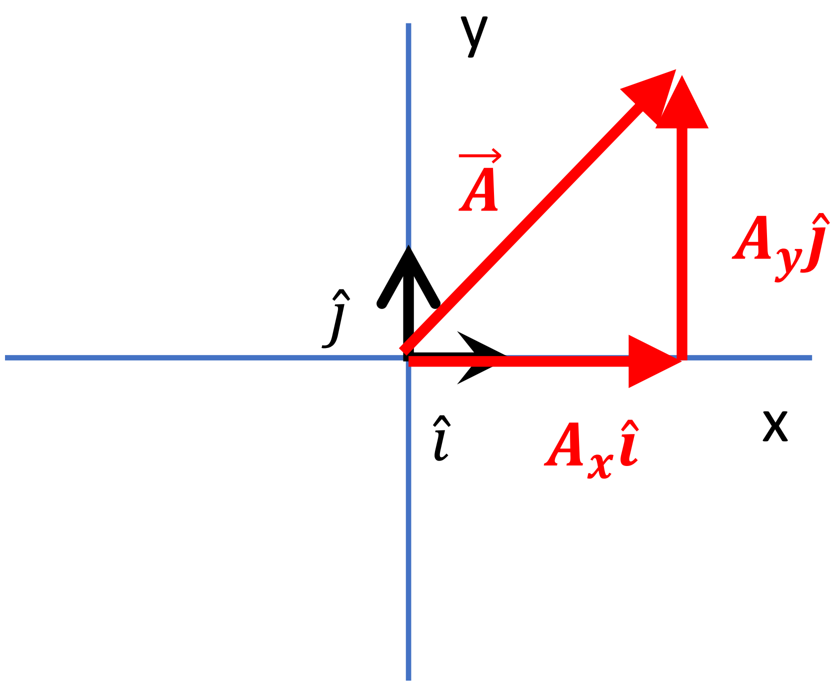
Vector Addition: Traditional Notation

- $\vec{A} = (A_x, A_y), \vec{B} = (B_x, B_y)$
- Addition: $\vec{R} = \vec{A} + \vec{B} \rightarrow (R_x, R_y) = (A_x, A_y) + (B_x, B_y)$
  - X-component: $R_x = A_x + B_x$; Y-component: $R_y = A_y + B_y$
- Example to the right:
  - $A_x = 12 \cos 20^\circ = 11.3$, $B_x = 25 \cos 60^\circ = 12.5$
  - $R_x = A_x + B_x = 23.8$
  - $A_y = 12 \sin 20^\circ = 4.1$; $B_y = 25 \sin 60^\circ = 21.7$
  - $R_y = A_y + B_y = 25.8$
- $\vec{R} = (R_x, R_y) \rightarrow$
  - $Magnitude\ R = \sqrt{23.8^2 + 25.8^2} = 35.1$
  - $\theta = tan^{-1}\left(\frac{R_y}{R_x}\right) = 47.3^\circ$
- $\vec{R} = 35\ at\ 47^\circ$



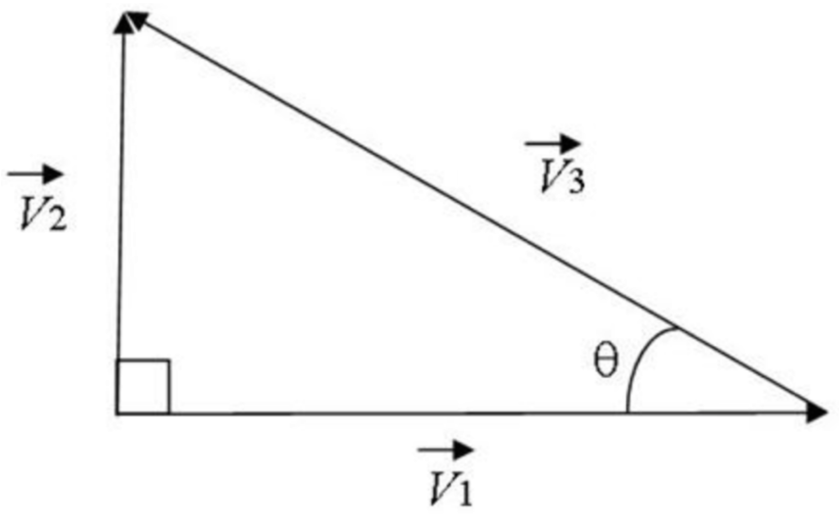
Unit Vector Notation in two dimensions (2D)

- Unit vector: $\hat{\imath} = (1,0)$
- Unit vector: $\hat{\jmath} = (0,1)$
- $\vec{A} = (A_x, A_y) =$
- $\boldsymbol{\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath}}$
- The vector $\vec{A}$ is the sum of the vectors $A_x\hat{\imath}$ and $A_y\hat{\jmath}$



Brain Teaser

- The Vector $\vec{V}_3$ in the diagram **below** is equal to:



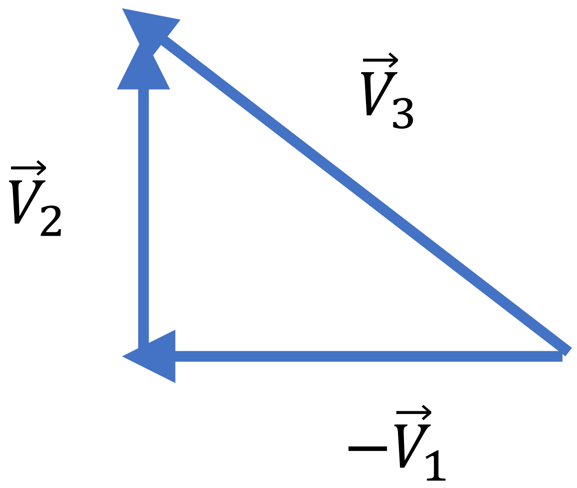
a) $\vec{V}_1 + \vec{V}_2$

b) $\vec{V}_1 / \cos\theta$

c) $\vec{V}_2 - \vec{V}_1$

d) $\vec{V}_1 - \vec{V}_2$

e) $\vec{V}_1 \cos\theta$



Answer C

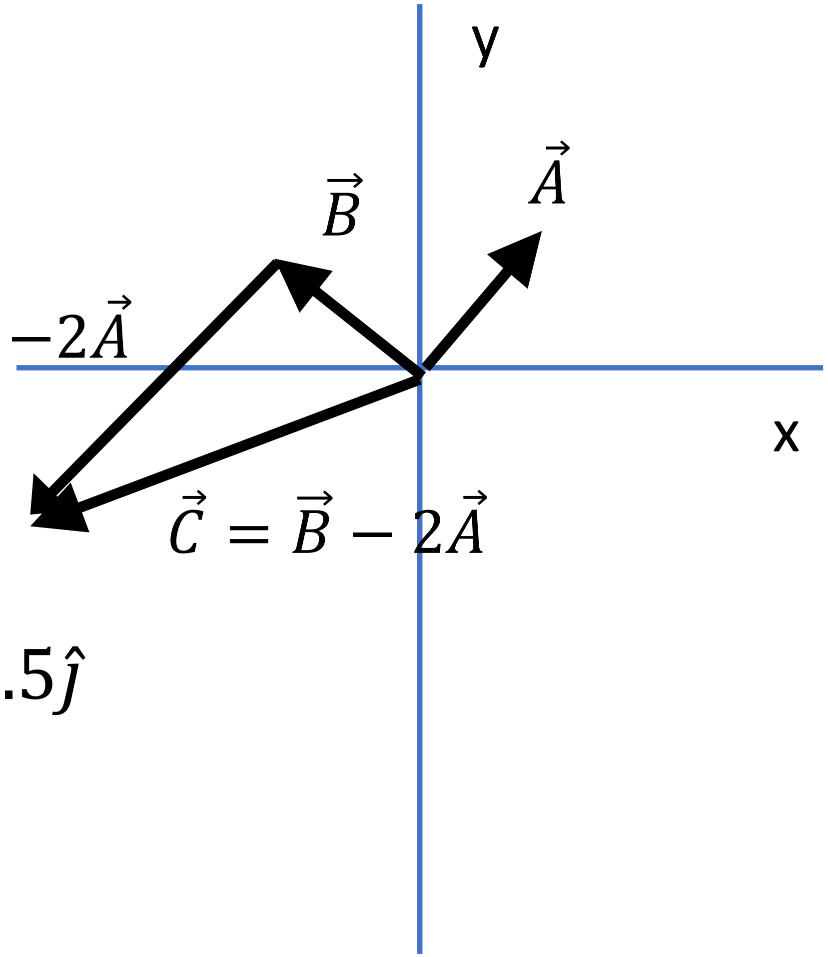
Advice: Study this problem

Adding and subtracting using unit vector notation

- $\vec{A} = (0.5, 1.0)$
- $\vec{B} = (-1.0, 0.5)$
- $\vec{C} = -2\vec{A} + \vec{B} = \vec{B} - 2\vec{A}$
- $C_x = -2 \times 0.5 - 1.0 = -2.0$
- $C_y = -2 \times 1.0 + 0.5 = -1.5$

Answer: $\vec{C} = (-2.0, -1.5)$

Or in unit vector notation $\vec{C} = -2.0\hat{\imath} - 1.5\hat{\jmath}$



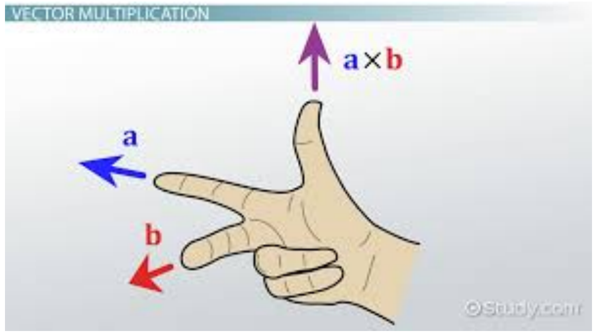
Homework for Friday, and reminder

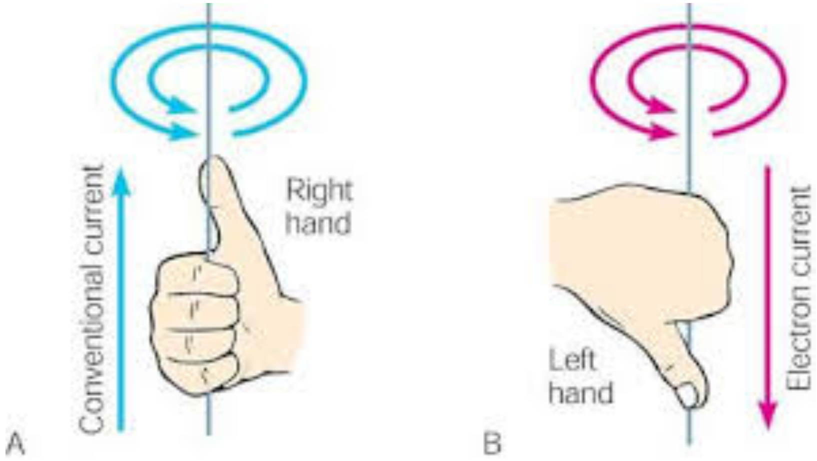
- For the classes of Friday September 18, 2020
  Read Chapter 3: section 3.1 (Vectors), 3.2 (Unit Vectors, and adding by components), 3.3 (Multiplying Vectors)
  Focus on the **Right-Hand Rule**
  Read Chapter 4: Kinematic in 3D

Lab: Tuesday September 22, 2020, Experiment 2, Graphical Analysis

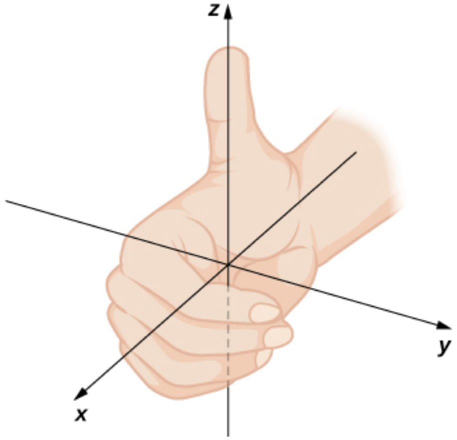
- **Section F1B: 8:30 AM to 11:30 AM, CB2010**
- **Section F2B: 2:30 PM to 5:30 PM, CB 2010**
- **Section F3B: 7:00 PM to 10:00 PM, CB2010**
- **Students in section F1A, F2A, F3A, FD4 must hand in their Experiment 2 lab reports on September 22, 2020**

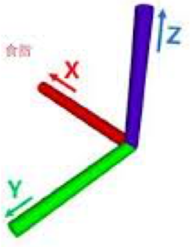
Study the Right-Hand Rule

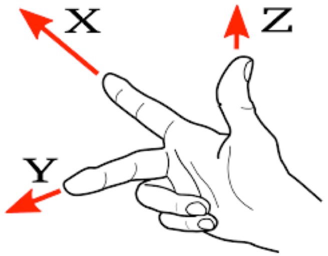




Three-Dimension (3D) Cartesian Coordinate

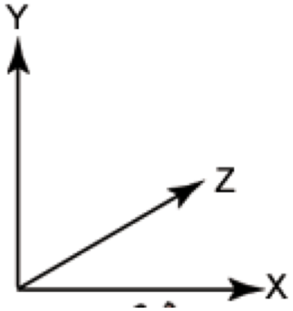


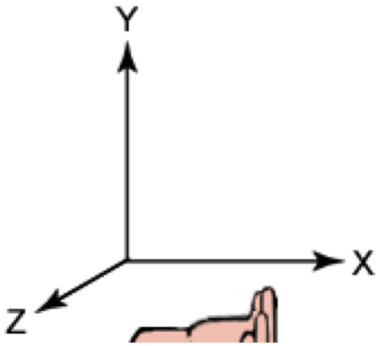




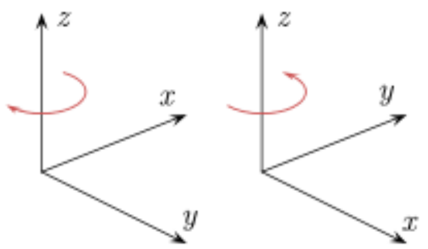
Which coordinate is right handed?

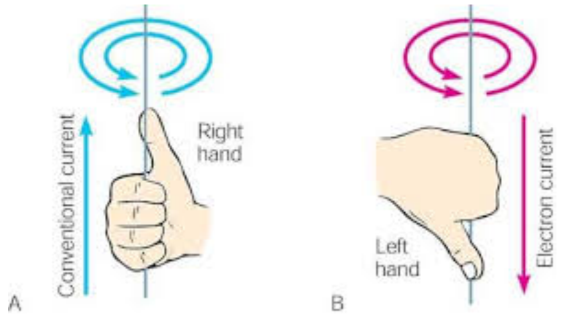
- The right one



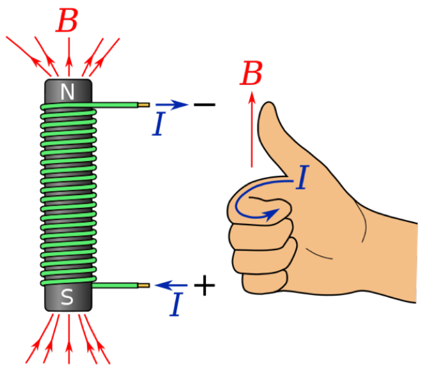


Some more right/left hand coordinate systems and **Chirality**

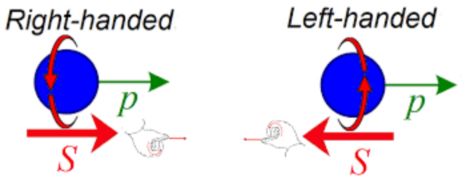




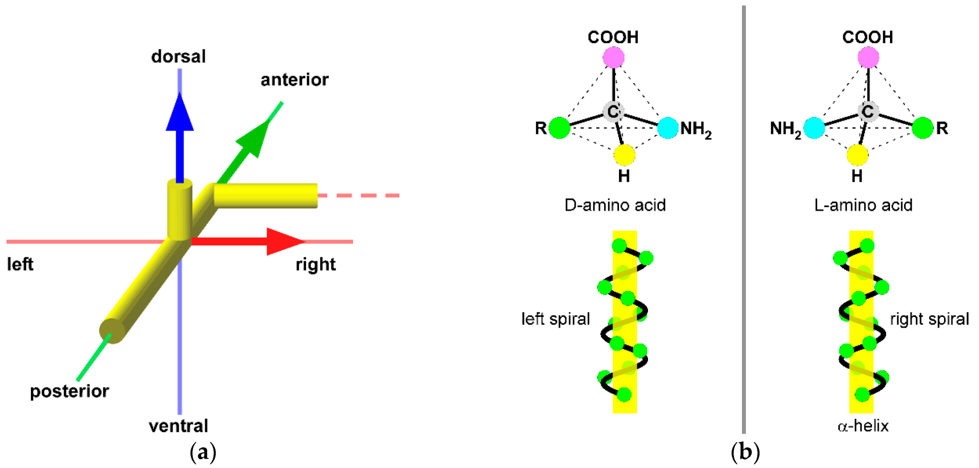
<u>In physics Handedness is associated with Chirality</u>



Magnetic Field



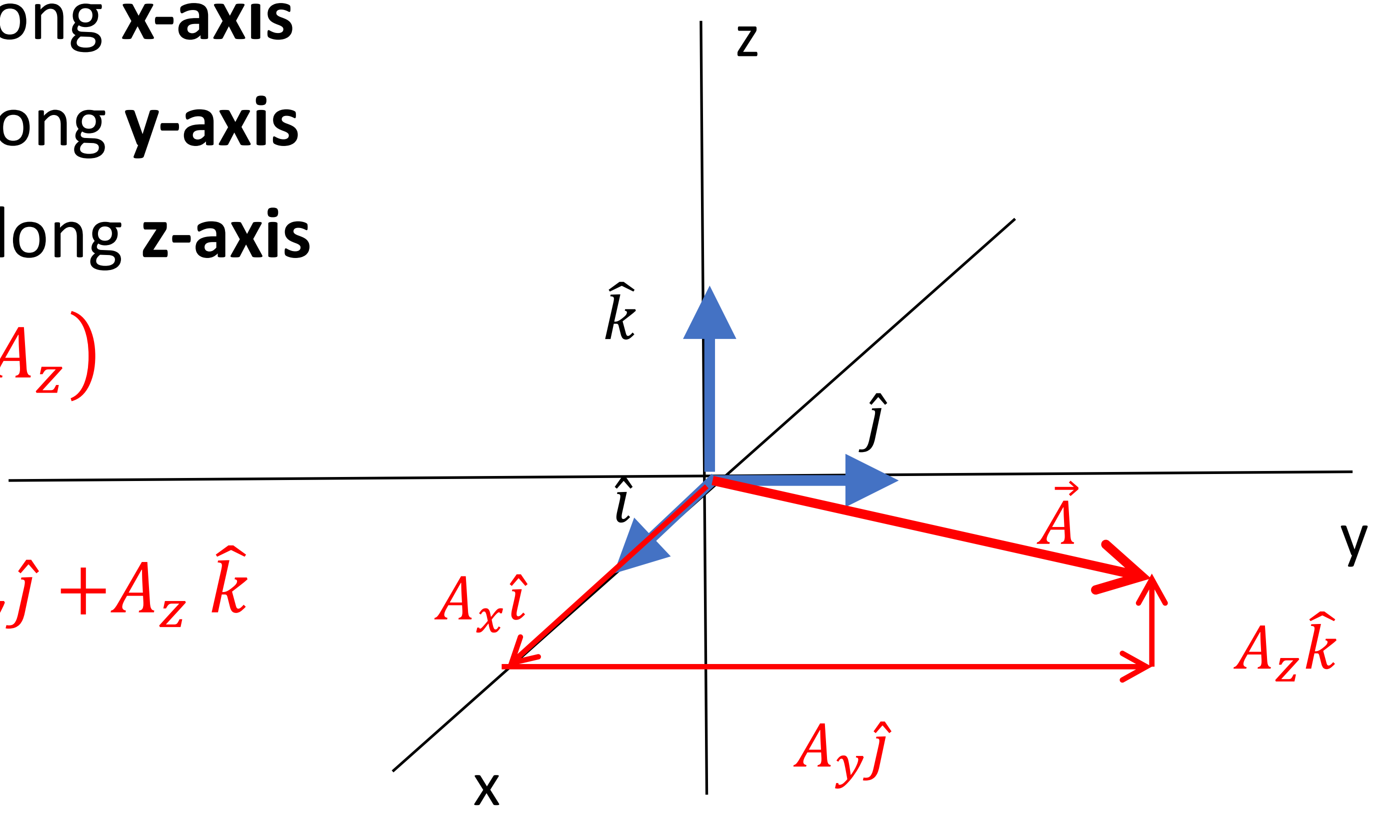
Parity of neutrino and anti-neutrino



Biology and Chemistry

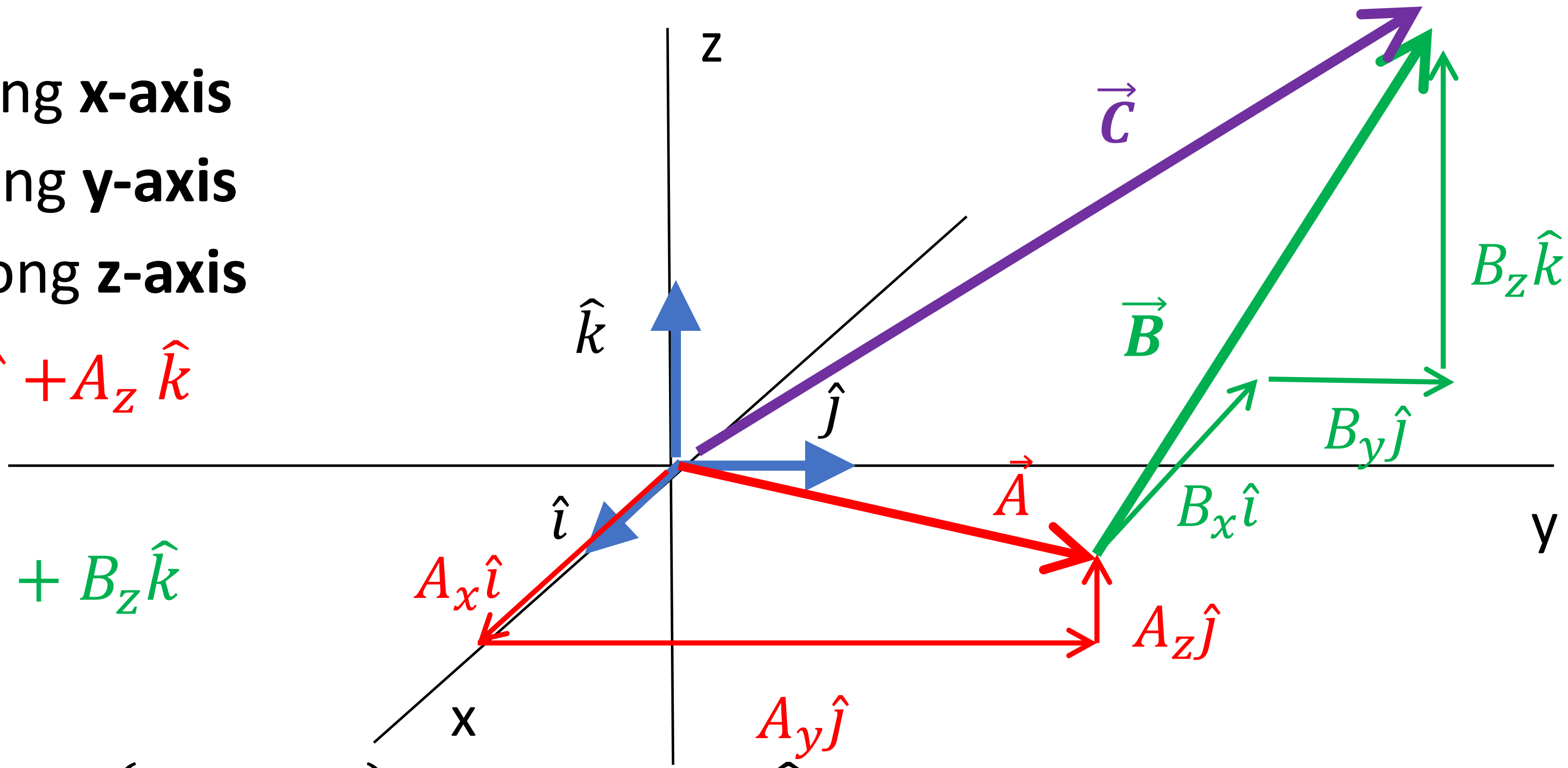
Unit Vector Representation in Three Dimensions (3D)

- $\hat{\imath} = (1,0,0)$ along **x-axis**
- $\hat{\jmath} = (0,1,0)$ along **y-axis**
- $\hat{k} = (0,0,1)$ along **z-axis**
- $\vec{A} = (A_x, A_y, A_z)$
- $\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\,\hat{k}$



Adding Vectors in Three Dimensions (3D)

- $\hat{\imath} = (1,0,0)$ along **x-axis**
- $\hat{\jmath} = (0,1,0)$ along **y-axis**
- $\hat{k} = (0,0,1)$ along **z-axis**
- $\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$



- $\vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$
- $\vec{C} = \vec{A} + \vec{B}$
- $\vec{C} = (A_x + B_x)\hat{\imath} + \left(A_y + B_y\right)\hat{\jmath} + (A_z + B_z)\hat{k}$
- $\vec{\boldsymbol{C}} = \quad \boldsymbol{C_x}\,\hat{\boldsymbol{\imath}} \quad + \quad \boldsymbol{C_y}\hat{\boldsymbol{\jmath}} \quad + \quad \boldsymbol{C_z}\hat{\boldsymbol{k}}$

Example on Unit Vector addition

- Let $\vec{A}_1 + 2.0\vec{A}_2 = 5.0\vec{A}_3$ and $\vec{A}_1 - \vec{A}_2 = \vec{A}_3$ and $\vec{A}_3 = 2.0\hat{\imath} + 3.0\hat{\jmath} - 5\hat{k}$. Find $\vec{A}_1 and\ \vec{A}_2$

<u>Method of Solution</u>

1. Note $\vec{A}_1 = A_{1x}\hat{\imath} + A_{1y}\hat{\jmath} + A_{1z}\hat{k}$ and $\vec{A}_2 = A_{2x}\hat{\imath} + A_{2y}\hat{\jmath} + A_{2z}\hat{k}$
2. Label $\vec{A}_1 + 2.0\vec{A}_2 = 5.0\vec{A}_3$ **E1**, $\vec{A}_1 - \vec{A}_2 = \vec{A}_3$ **E2**
3. Use Algebra to eliminate one of the **two unknown vectors**, $\vec{A}_1$ or $\vec{A}_2$

<u>Execution</u>:

Abbreviation: LHS stands for Left hand side; RHS stands for Right hand side

- **E1** + 2 x **E2**→ $\left(\vec{A}_1 + 2.0\vec{A}_2\right) + 2\left(\vec{A}_1 - \vec{A}_2\right) = 5.0\vec{A}_3 \quad + \quad 2\times\vec{A}_3$
- LHS of E1 2 X LHS of E2 RHS of E1 2 X LHS of E2
- $3\vec{A}_1 = 7.0\vec{A}_3$
- But we know $\vec{A}_3 = 2.0\hat{\imath} + 3.0\hat{\jmath} - 5\hat{k}$

Example on Unit Vector addition, Continued

- Let $\vec{A}_1 + 2.0\vec{A}_2 = 5.0\vec{A}_3$ and $\vec{A}_1 - \vec{A}_2 = \vec{A}_3$ and $\vec{A}_3 = 2.0\hat{\imath} + 3.0\hat{\jmath} - 5\hat{k}$. Find $\vec{A}_1 and\ \vec{A}_2$

1. Note $\vec{A}_1 = A_{1x}\hat{\imath} + A_{1y}\hat{\jmath} + A_{1z}\hat{k}$ and $\vec{A}_2 = A_{2x}\hat{\imath} + A_{2y}\hat{\jmath} + A_{2z}\hat{k}$
2. Label $\vec{A}_1 + 2.0\vec{A}_2 = 5.0\vec{A}_3$ **E1**, $\vec{A}_1 - \vec{A}_2 = \vec{A}_3$ **E2**

<u>Execution of Solution</u>: Continued

- $3\vec{A}_1 = 7.0\vec{A}_3$
- But we know $\vec{A}_3 = 2.0\hat{\imath} + 3.0\hat{\jmath} - 5\hat{k}$
- $3\vec{A}_1 = 7.0\vec{A}_3 \rightarrow 3\left(A_{1x}\hat{\imath} + A_{1y}\hat{\jmath} + A_{1z}\hat{k}\right) = 7\left(2.0\hat{\imath} + 3.0\hat{\jmath} - 5\hat{k}\right)$
- $3A_{1x}\hat{\imath} + 3A_{1y}\hat{\jmath} + 3A_{1z}\hat{k} = 14.0\hat{\imath} + 21.0\hat{\jmath} - 35\hat{k}$
- $3A_{1x} = 14 \rightarrow A_{1x} = \frac{14}{3}; A_{1y} = 7.0\hat{\jmath}; A_{1z} = -\frac{35}{3} \rightarrow \vec{A}_1 = \frac{14}{3}\hat{\imath} + 7\hat{\jmath} - \frac{35}{3}\hat{k}$

<u>**Exercise**</u> Use one of **E1** or **E2** to find $\vec{A}_2$

Example on Unit Vector addition, Verified

- Let $\vec{A}_1 + 2.0\vec{A}_2 = 5.0\vec{A}_3$ and $\vec{A}_1 - \vec{A}_2 = \vec{A}_3$ and $\vec{A}_3 = 2.0\hat{\imath} + 3.0\hat{\jmath} - 5\hat{k}$. Find $\vec{A}_1 and\ \vec{A}_2$

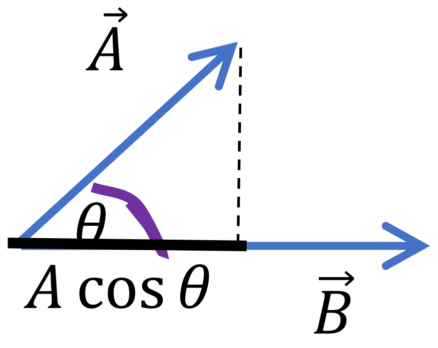
1. Note $\vec{A}_1 = A_{1x}\hat{\imath} + A_{1y}\hat{\jmath} + A_{1z}\hat{k}$ and $\vec{A}_2 = A_{2x}\hat{\imath} + A_{2y}\hat{\jmath} + A_{2z}\hat{k}$
2. Label $\vec{A}_1 + 2.0\vec{A}_2 = 5.0\vec{A}_3$ **E1**, $\vec{A}_1 - \vec{A}_2 = \vec{A}_3$ **E2**

<u>Execution of Solution</u>: Continued

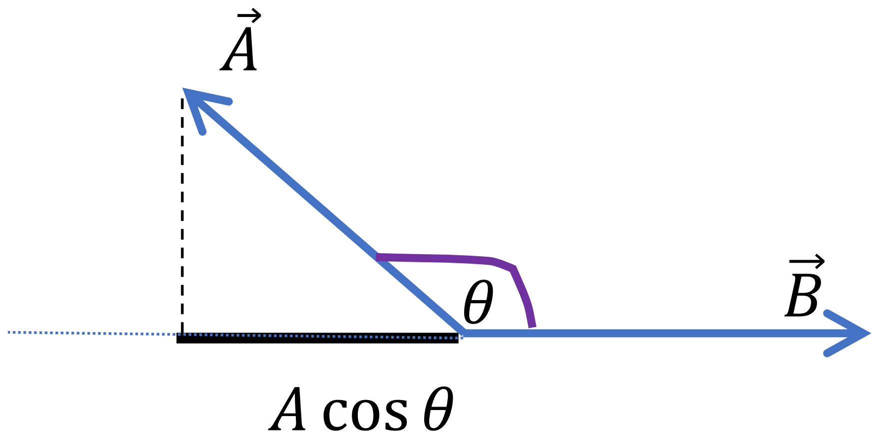
- $\vec{A}_1 = \frac{14}{3}\hat{\imath} + 7\hat{\jmath} - \frac{35}{3}\hat{k}$ and $\vec{A}_3 = 2.0\hat{\imath} + 3.0\hat{\jmath} - 5\hat{k}$
- Use **E2**, $\vec{A}_2 = \vec{A}_1 - \vec{A}_3 \rightarrow$
- $A_{2x}\hat{\imath} + A_{2y}\hat{\jmath} + A_{2z} = \frac{14}{3}\hat{\imath} + 7\hat{\jmath} - \frac{35}{3}\hat{k} - \left(2.0\hat{\imath} + 3.0\hat{\jmath} - 5\hat{k}\right)$
- $\vec{A}_2 = \frac{8}{3}\hat{\imath} + 4.0\hat{\jmath} - \frac{20}{3}\hat{k}$
- Verify **E1**,
- $\vec{A}_1 + 2.0\vec{A}_2 = \frac{14}{3}\hat{\imath} + 7\hat{\jmath} - \frac{35}{3}\hat{k} + 2\times\left(\frac{8}{3}\hat{\imath} + 4.0\hat{\jmath} - \frac{20}{3}\hat{k}\right) = 10.0\hat{\imath} + 15.0\hat{\jmath} - 25\hat{k} = 5.0\vec{A}_3$

Scalar (dot) Product

- Geometrical Definition: $\vec{A} \cdot \vec{B} = AB\cos\theta\,, 0^{\circ} < \theta < 180^{\circ}$
- $A = |\vec{A}|$; B= $|\vec{B}|$ are the magnitude of the Vectors $\vec{A}$ and $\vec{B}$, respectively
- $A\cos\theta$ is the **scalar projection** of the vector $\vec{A}$ in the direction of the vector $\vec{B}$, $A_B = A\cos\theta$



Here $0^{\circ} < \theta < 90^{\circ}, A\cos\theta > 0$



Here $90^{\circ} < \theta < 180^{\circ}, A\cos\theta < 0$

Scalar Product: Algebraic Definition

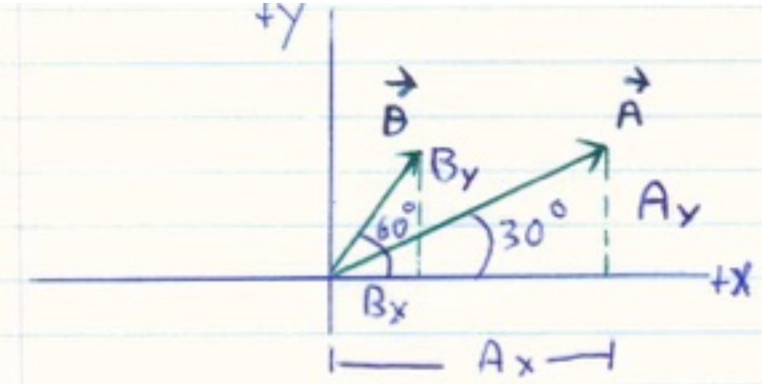
- Consider two vectors:
  - $\vec{A} \ = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$
  - $\vec{B} \ = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$
- $\vec{A} \cdot \vec{B} = A_x\, B_x + A_y\, B_y + A_z\, B_z$

More Formally:

- $\vec{A} \cdot \vec{B} = \left(A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}\right) \cdot \left(B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}\right)$
- $\vec{A} \cdot \vec{B} = \left(A_x\, B_x\hat{\imath} \cdot \hat{\imath} + A_y\, B_y\hat{\jmath} \cdot \hat{\jmath} + A_z\, B_z\hat{k} \cdot \hat{k}\right) + \hat{\imath} \cdot \hat{\jmath}\left(A_x\, B_y\right) + \hat{\jmath} \cdot \hat{\imath}\left(A_y\, B_x\right) + \hat{\imath} \cdot \hat{k}(A_x\, B_z) + \hat{k} \cdot \hat{\imath}(A_z\, B_x) + \hat{\jmath} \cdot \hat{k}\left(A_y\, B_z\right) + \hat{k} \cdot \hat{\jmath}\left(A_z\, B_y\right)$
- $\hat{\imath} \cdot \hat{\imath} = \hat{\jmath} \cdot \hat{\jmath} = \hat{k} \cdot \hat{k} = 1;\ \hat{\imath} \cdot \hat{\jmath} = \hat{\jmath} \cdot \hat{\imath} = \hat{\imath} \cdot \hat{k} = \hat{k} \cdot \hat{\imath} = \hat{\jmath} \cdot \hat{k} = \hat{k} \cdot \hat{\jmath} = 0$
- Connclusion $\rightarrow \vec{A} \cdot \vec{B} = A_x\, B_x + A_y\, B_y + A_z\, B_z$

Scalar Product Example

$\vec{A}$, $A = 4.0$, $\theta = 30^\circ$
$\vec{B}$, $B = 2.0$, $\theta = 60^\circ$



CAN SEE that ϕ between $\vec{A}$ & $\vec{B}$ is 30°!
VERIFY BY DOT PRODUCT.

$A_x = 4.0 \cos 30^\circ = 4.0 \times \frac{\sqrt{3}}{2} = 2.0\sqrt{3}$

$A_y = 4.0 \sin 30^\circ = 4.0 \times \frac{1}{2} = 2.0$

$B_x = 2.0 \cos 60^\circ = 2.0 \times \frac{1}{2} = 1.0$

$B_y = 2.0 \sin 60^\circ = 2.0 \times \frac{\sqrt{3}}{2} = \sqrt{3}$

$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y = (2\sqrt{3})(1.0) + (2.0)(\sqrt{3}) = 4.0\sqrt{3}$

$\vec{A} \cdot \vec{B} = AB \cos\phi$ divide through by AB

$\cos\phi = \frac{\vec{A} \cdot \vec{B}}{AB} = \frac{4\sqrt{3}}{4 \times 2} = \frac{\sqrt{3}}{2}$

$\phi = \arccos\left(\frac{\sqrt{3}}{2}\right) = 30^\circ$