PHYS 1211 F2020

Chapter 2: 1D Kinematic

September 9 and 11, 2020

Scalar and Vectors

Scalars have **magnitudes** but **no direction**. Examples are:

- Mass
- Speed
- Temperature

Vectors have **magnitude** and **direction**. Examples are:

- Weight
- Velocity
- Force

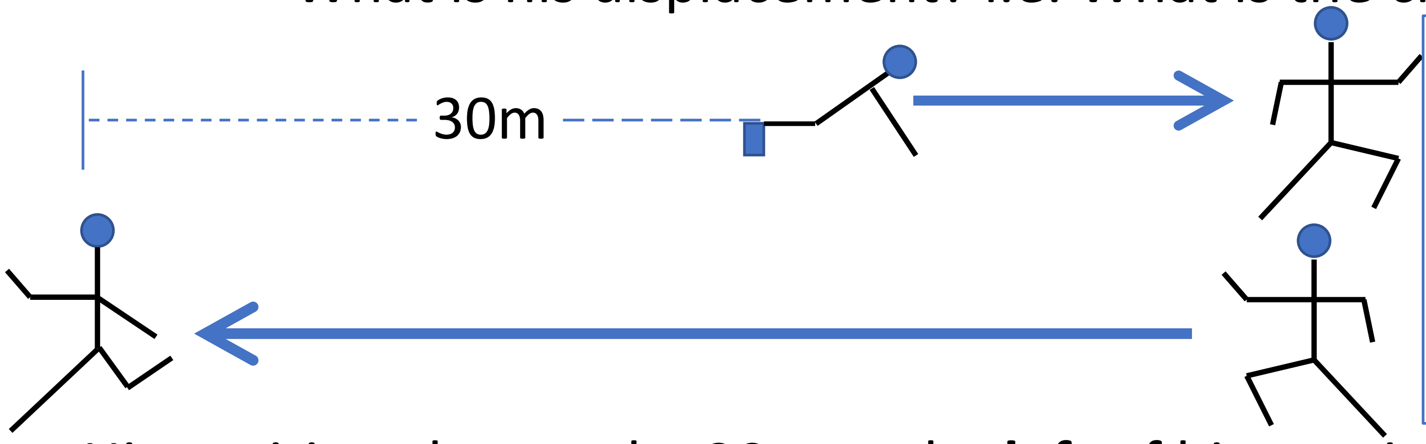
Scalar and Vectors

Adding scalars: Three rocks of mass $M_1 = 10kg, M_2 = 20kg, M_3 = 5kg$

- What is the Total Mass?
- $M = M_1 + M_2 + M_3 = 35kg$

Adding Vectors: Man runs 20 m right then he walks 50 m left.

- What is his displacement? i.e. What is the change in his position?



What is the total <u>distance traveled</u> ?

70 m

Distance Traveled is a **scalar**!

Displacement is a **vector**!

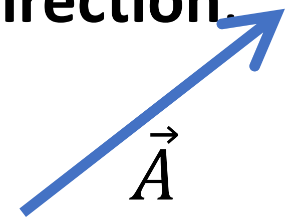
$-30m$ indicates a **left displacement**

His position changes by 30m to the **left** of his starting point.

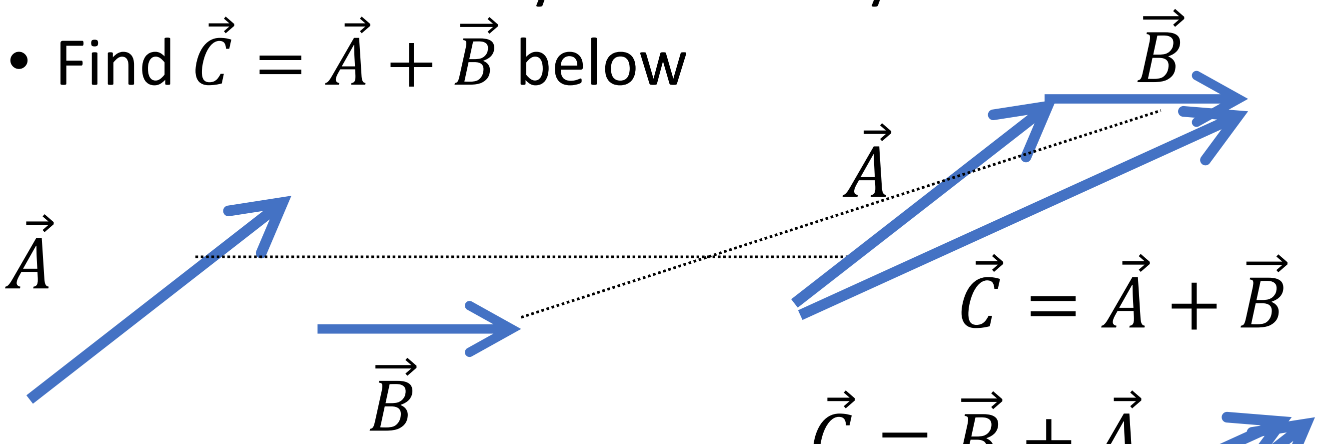
His displacement is $-30m$

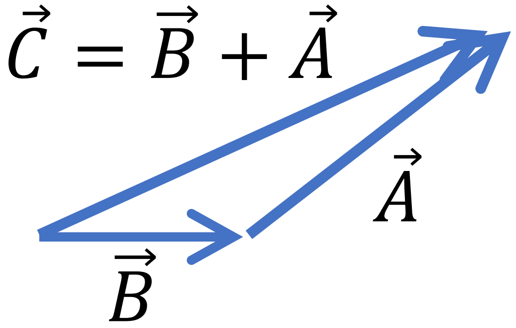
Adding Vectors in three dimensions (3D)

- A **Vector** is represented by an **arrow,** which specifies its **direction**.
- The **magnitude** corresponds to its **length**.



- A vector is usually denoted by a letter with an arrow on top.
- Find $\vec{C} = \vec{A} + \vec{B}$ below



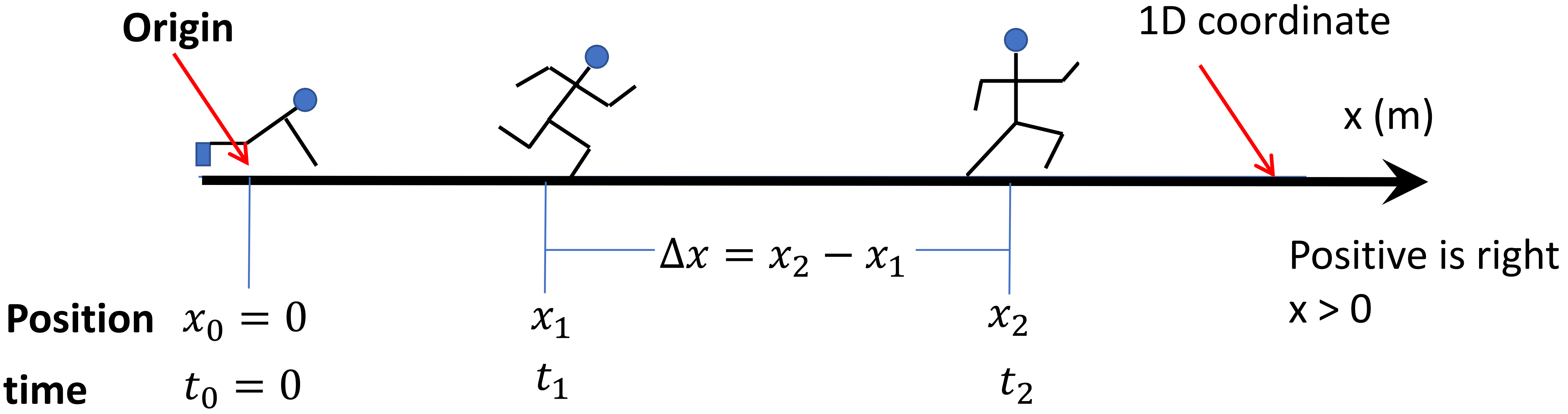


Vector addition is associative:
$\vec{A} + \vec{B} = \vec{B} + \vec{A}$

<u>Method for adding two vectors:</u>

1. Move $\vec{A}$ **without rotating**
2. Move **tail** of B, without rotating, to **head** of $\vec{A}$
3. Draw arrow from tail of $\vec{A}$ to head of $\vec{B}$ to find $\vec{C}$

Position, x, displacement, Δx, average velocity, v_{avg}, in One Dimension (1D)

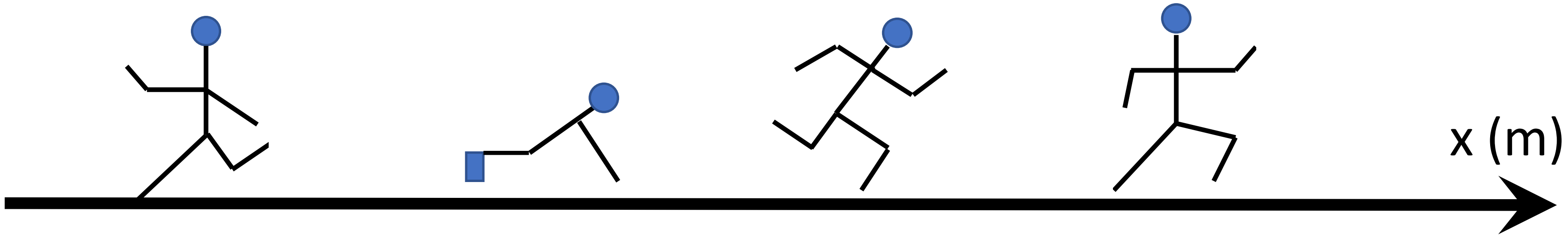


Displacement $\Delta x = x_2 - x_1$ defined for time interval $t_1 < t < t_2$

Average Velocity $v_{avg} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$ SI unit $\frac{m}{s}$ or $m \cdot s^{-1}$

Δ stands for change

Position, x, displacement, Δx, average velocity, v_{avg}, in One Dimension (1D). Examples.



Position	$x_3 = -2.0m$	$x_0 = 0$	$x_1 = 1.3m$	$x_2 = 2.0m$
time	$t_3 = 2.0s$	$t_0 = 0$	$t_1 = 0.2s$	$t_2 = 0.8s$

Examples:

1. Find v_{avg} in time interval $t_0 < t < t_2$

$$v_{avg} = \frac{x_2 - x_0}{t_2 - t_0} = \frac{2.0m - 0m}{0.8s - 0s} = 2.5\frac{m}{s}$$

$v_{avg} > 0$, right

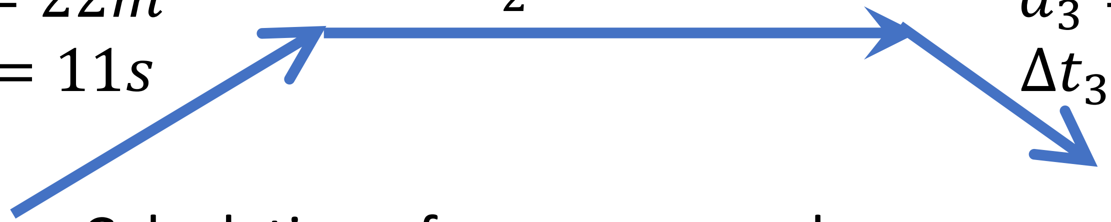
2. Find v_{avg} in time interval $t_1 < t < t_3$

$$v_{avg} = \frac{x_3 - x_1}{t_3 - t_1} = \frac{-2.0m - 1.3m}{2.0s - 0.2s} = -1.83\frac{m}{s}$$

$v_{avg} < 0$, left

Average Speed, s_{avg}

- $s_{avg} = \frac{distance\ traveled}{total\ time}$, compare this to textbook equation 2.1.3
- Example: A bird travel the path below:



<u>Calculation of average speed</u>

$$s_{avg} = \frac{d_1 + d_2 + d_3}{\Delta t_1 + \Delta t_2 + \Delta t_3}$$

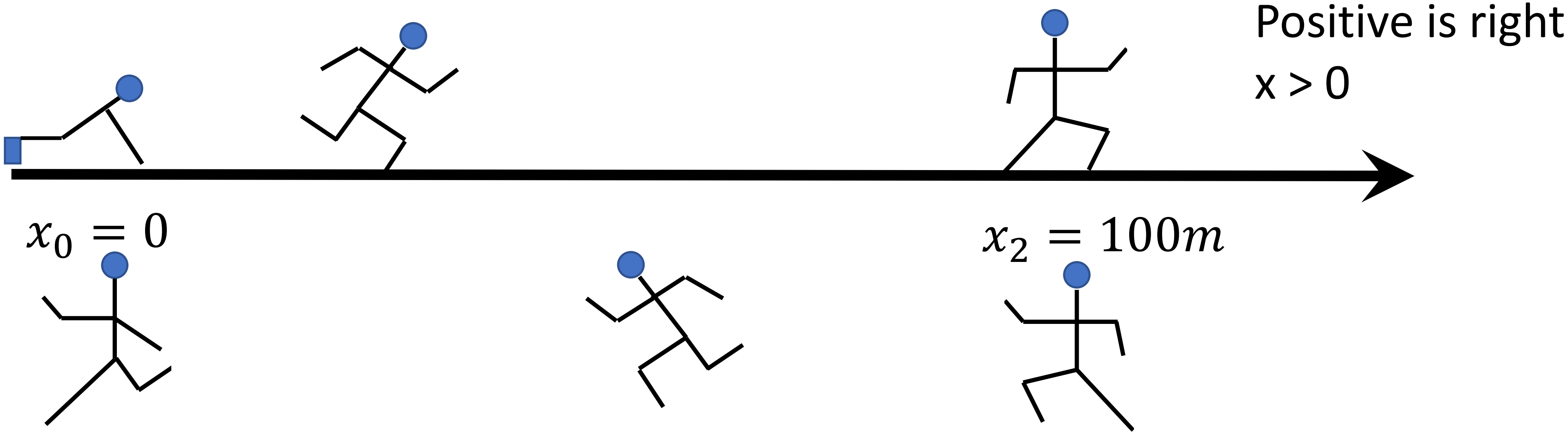
2 significant figures (SF)

$$s_{avg} = \frac{22m + 25m + 12m}{11s + 9.0s + 5.0s} = 2.36\frac{m}{s} = 2.4\frac{m}{s}$$

Comparing average velocity, v_{avg}, and average speed, s_{avg}, in One Dimension (1D)

A runner runs 100m in 10 s, then retraced his steps to origin in 11s

- What is his average velocity for the **whole 21s period**?
- Zero! $v_{avg} = \frac{\Delta x}{\Delta t}$ and his displacement is $\Delta x = 0$



Average speed $s_{avg} = \frac{100m+100m}{10s+11s} = 9.52\frac{m}{s} = 9.5\frac{m}{s}$

Instantaneous velocity, v, in one dimension (1D): A Calculus Argument

Average Velocity $v_{avg} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$

- In the limit of very small time interval, $\Delta t \to 0$, which also means $\Delta x \to 0$, which gives
- $v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$
- Instantaneous velocity, $v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$
- $\frac{dx}{dt}$ is the derivative of the position x(t) with respect to (wrt) time, t.
- Henceforth, in this course, **velocity**, v, means **instantaneous velocity**.

Instantaneous velocity, $\boldsymbol{v}$, in one dimension (1D): An example

A sprinter is running the 100-m dash. He completes it in 10s. The coach uses a device determines his position $x(t)$ as a function of time t:

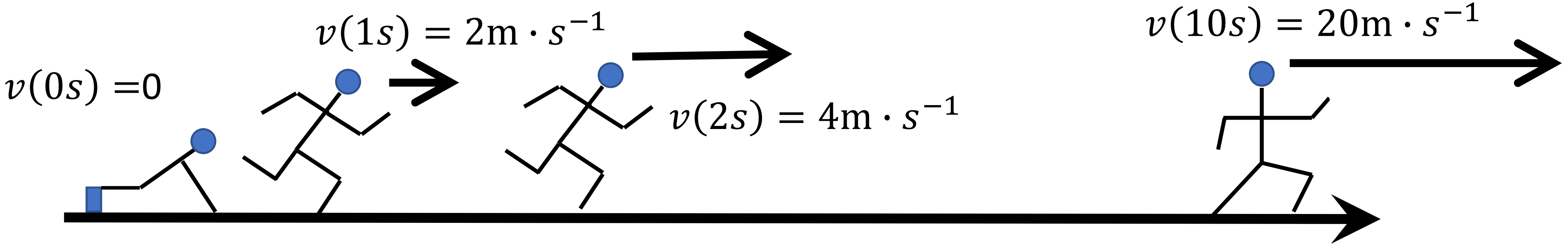
$$x(t) = \left(1\frac{m}{s^2}\right)t^2$$

Calculate his velocity at t= 0s, 1s, 2s, and at the instant he finish the sprint.

- Using differential Calculus: $v = \frac{dx}{dt} = \frac{d}{dt}\left(\left(1\frac{m}{s^2}\right)t^2\right) = \left(1\frac{m}{s^2}\right)2\times t^{2-1} = \left(2\frac{m}{s^2}\right)t$
- At t = 0s, $v(0s) = \left(2\frac{m}{s^2}\right)\times 0s = 0$
- At t = 1s, $x_1 = x(1s) = 1m, v(1s) = \left(2\frac{m}{s^2}\right)\times 1s = 2\frac{m}{s}$

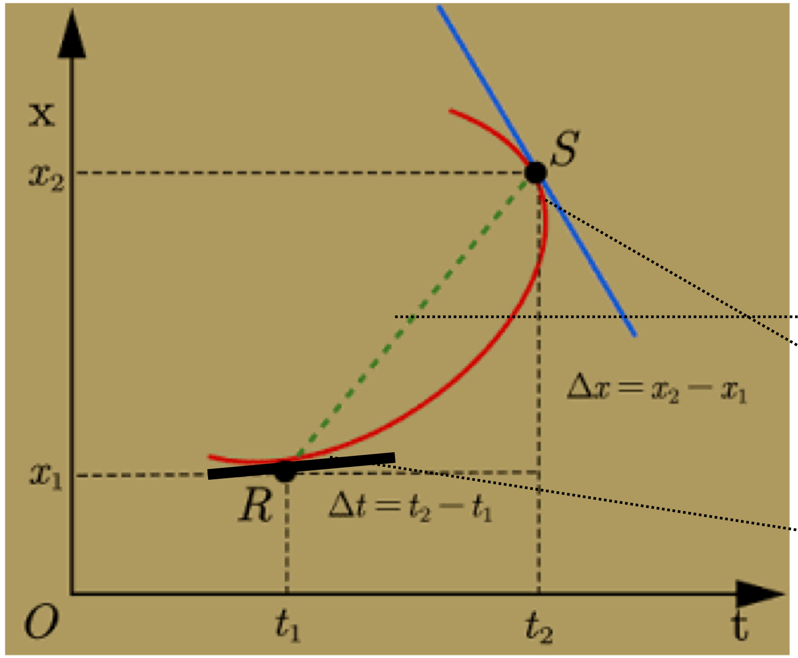
Average velocity

$v_{avg} = \frac{100m}{10s} = 10.\frac{m}{s}$



Position	$x_0 = 0$	$x_1 = 1.0m$	$x_2 = 4.0m$	$x_3 = 100m$	x (m)
time	$t_0 = 0$	$t_1 = 1s$	$t_2 = 2.0s$	$t_3 = 10.0s$	

Graphical Representation of 1D velocity



Average Velocity for interval RS:

<u>Slope</u> of <u>secant</u> joining <u>RS</u>

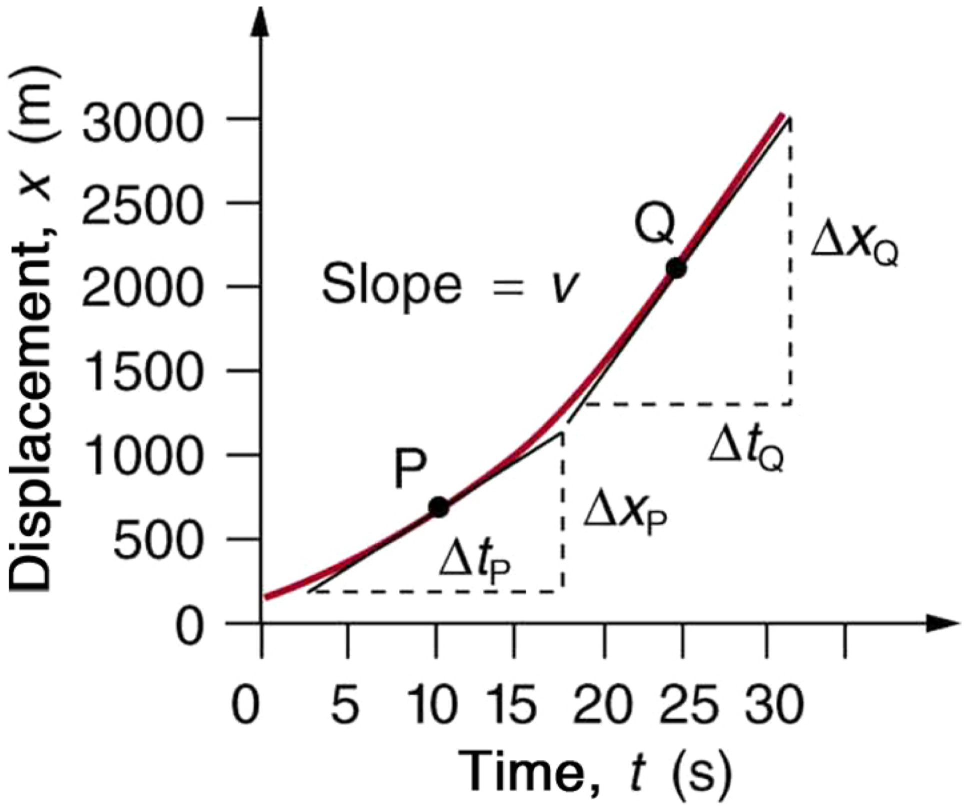
$$v_{avg} = \frac{\Delta x}{\Delta t} > 0$$

positive slope

Velocity (i.e. instantaneous velocity) is the **slope** of the **tangent** at the **instant** (time):

- At point R **slope** of tangent is **positive**, and velocity is **positive**, $v > 0$
- At point S slope of tangent is **negative**, and velocity is **negative**, $v < 0$

Graphical Representation of 1D velocity



t (s)	x (m)
0	200
5	338
10	600
15	988
20	1500
25	2138
30	2900

At point P:

Velocity, $v_P = ?$

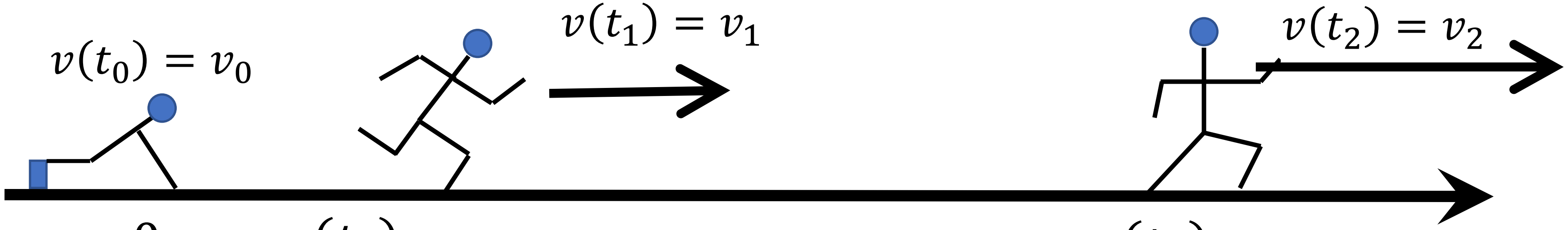
At point Q:

Velocity, $v_Q = ?$

Simple Question

- While driving a car at 90km/h, how far do you move while your eyes shut for 0.50s during a hard sneeze?
- By definition, $v_{avg} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$
- $\Delta x = v_{avg}\Delta t$
- $v_{avg} = 90\frac{km}{hr} \times \frac{1000m \cdot km^{-1}}{3600s \cdot hr^{-1}} = 25\frac{m}{s}$
- $\Delta x = v_{avg}\Delta t = 25\frac{m}{s} \times 0.5s = 12.5m \rightarrow 13m$

Average and Instantaneous acceleration in one dimension (1D)

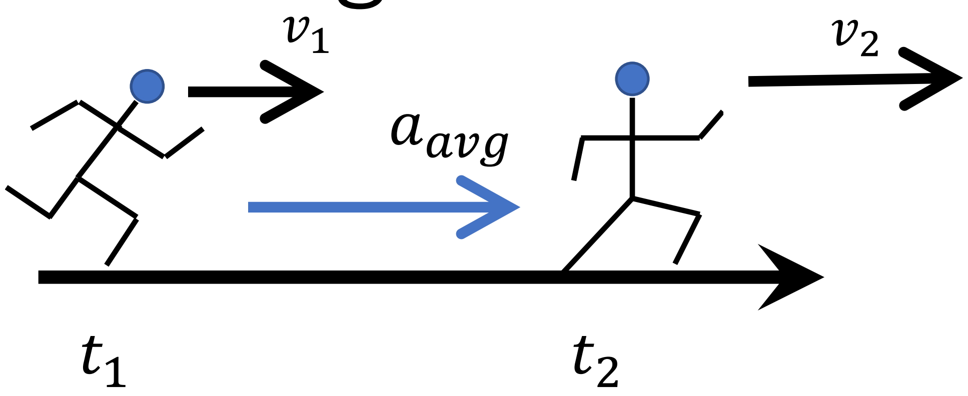


Position	$x_0 = 0$	$x(t_1) = x_1$	$x(t_2) = x_2$
time	$t_0 = 0$	t_1	t_2

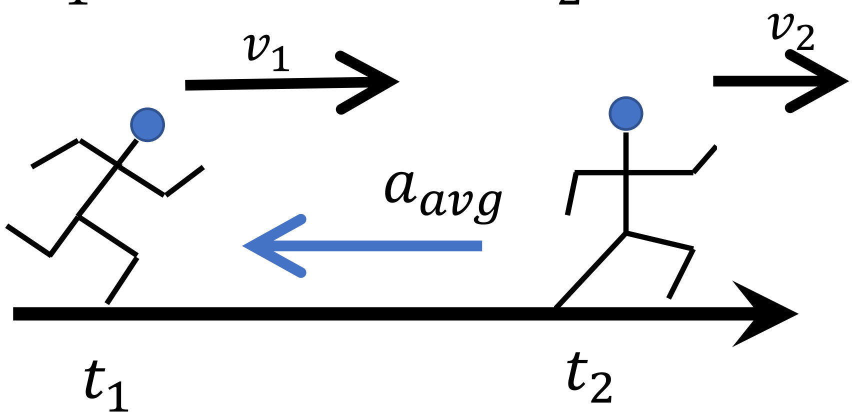
Acceleration is the rate of change of velocity

- Average Acceleration $a_{avg} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$, unit $\frac{m}{s^2}$
- In the limit of very small time interval, $\Delta t \rightarrow 0,\ \Delta x \rightarrow 0,$ which gives
- Instantaneous acceleration, $a = \lim_{\Delta t \rightarrow 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$
- Henceforth, in this course, acceleration means instantaneous acceleration

Average Acceleration examples:



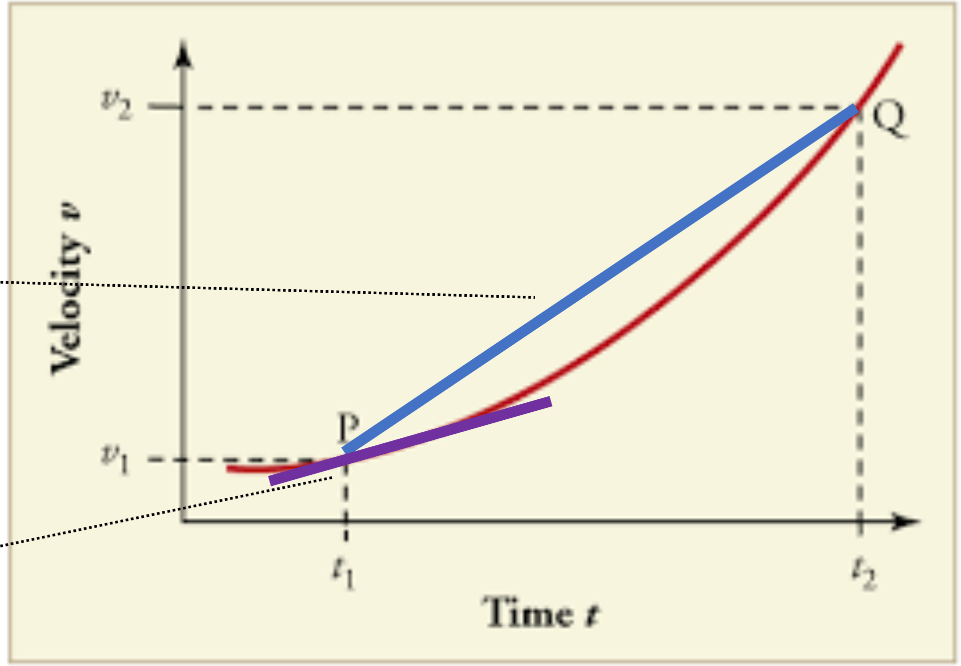
1. $v_2 > v_1, a_{avg} = \frac{v_2 - v_1}{t_2 - t_1} > 0$
   - $v_1 > 0, a_{avg} > 0$, speeding up



2. $v_1 > v_2, a_{avg} = \frac{v_2 - v_1}{t_2 - t_1} < 0$
   - $v_1 > 0, a_{avg} < 0$, slowing down

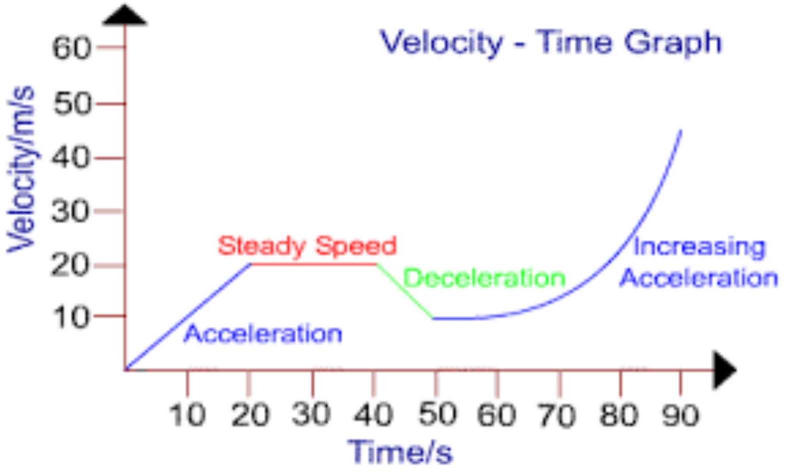
Acceleration by $v\ vs.\ t$ graph

- Average Acceleration, a_{avg}, for the PQ interval is <u>slope</u> of the <u>secant</u> joining PQ
- $a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$
- Acceleration, a, is the <u>slope</u> of the <u>tangent</u> at that point (i.e. at that time)
- Example is at point P

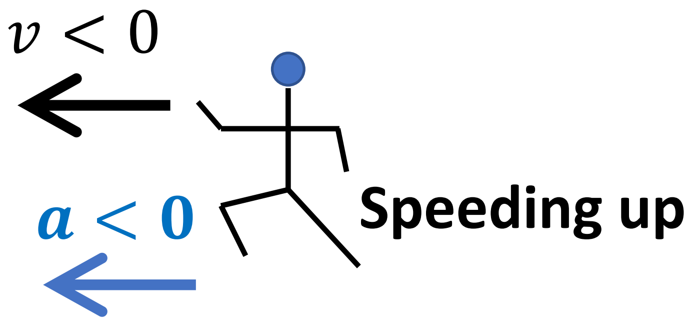


Acceleration by $v\ vs.\ t$ graph, speeding up and slowing down

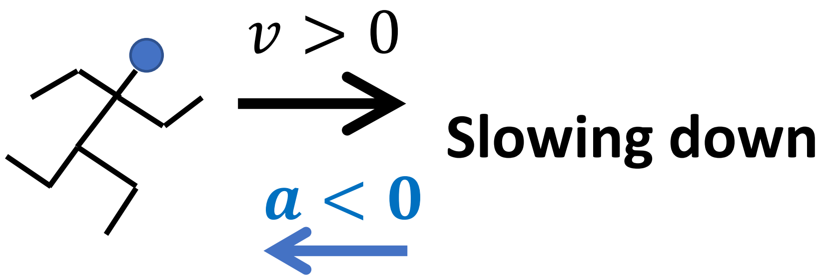
- Example of acceleration by graph



- If **signs** of acceleration, a, and velocity, v, are the **same**, then object is **speeding up**



- If **signs** of acceleration, a, and velocity, v, are the **different**, then object is **slowing down**



Kinematics with Calculus

Question Let x$= (At^2 - bt^3)$be the position of a particle moving in one dimension (1D), where A$= 2.2\frac{m}{s^2}$, and $b = 1.1\frac{m}{s^3}$

A) Find the position of the particle at *t = 1.2 s* and *t = 2.5 s*, and hence find the **average velocity** for the time interval *t = 1.2 s* and *t = 2.5 s*.

At *t = 1.2 s*, $x(1.2s) = 2.2\frac{m}{s^2}(1.2s)^2 - 1.1\frac{m}{s^3}(1.2s)^3 = 1.27m$

At *t = 2.5 s*, $x(2.5s) = 2.2\frac{m}{s^2}(2.5s)^2 - 1.1\frac{m}{s^3}(2.5s)^3 = -3.43m$

$$v_{avg} = \frac{x(2.5s) - x(1.2s)}{2.5s - 1.2s} = -3.62\frac{m}{s}$$

Significant Figure? $v_{avg} = -3.62\frac{m}{s}$, 2SF, but is 3SF or 4SF ok for test!

Kinematics with Calculus: Part B

<u>Question</u> Let x$= (At^2 - bt^3)$be the position of a particle moving in one dimension (1D), where A$= 2.2\,\frac{m}{s^2}$, and $b = 1.1\,\frac{m}{s^3}$

B) Find the velocity (i.e. instantaneous velocity) at t = 0 and t = 2.2 s

Find Derivative of position with time

$$v(t) = \frac{dx}{dt} = \frac{d(At^2 - bt^3)}{dt} = 2A \times t^{2-1} - 3b \times t^{3-1} = 2At - 3bt^2$$

Above equation give velocity at arbitrary time, t.

Velocity at specific time is found by substituting the specific time into equation.

At *t = 0 s*, $v(0s) = 2 \times 2.2\,\frac{m}{s^2}(0s) - 3 \times 1.1\,\frac{m}{s^3}(0s)^2 = 0$

t = 2.2 s, v(2.2s)=−6.29 m/s

Kinematics with Calculus: Part C and D

<u>Question</u> Let x$= (At^2 - bt^3)$be the position of a particle moving in one dimension (1D), where $\mathrm{A} = 2.2\,\frac{m}{s^2}$, and $b = 1.1\,\frac{m}{s^3}$

C) Find the **average acceleration** for time interval *t = 0 s* and *t = 2.2 s*.

Use Result of part B, $a_{avg} = \frac{v(2.2s) - v(0s)}{2.2s - 0s} = -2.86\,\frac{m}{s^2}$

D) Find Acceleration at *t = 0 s* and *t = 2.2 s*.

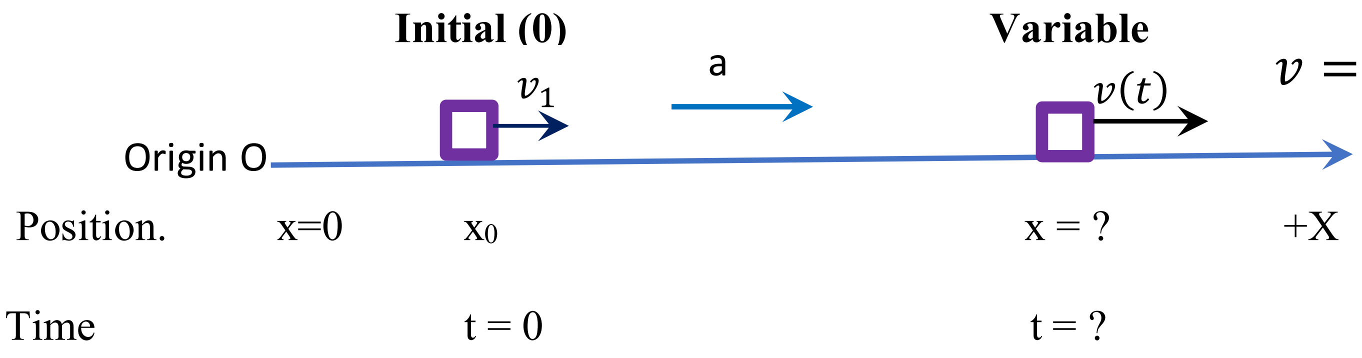
From **part B**, $v(t) = \frac{dx}{dt} = 2At - 3bt^2$

Acceleration at arbitrary time t, $a = \frac{dv}{dt} = 2A\times 1t^{1-1}3b\times 2\times t^{2-1} = 2A - 6bt$

At t = 0, $a(0) = 2A - 6b\times 0 = 2\times 2.2\,\frac{m}{s^2} = 4.4\,\frac{m}{s^2}$

At t = 2.2s, $a(2.2s) = 2\times 2.2\,\frac{m}{s^2} - 6\times 1.1\,\frac{m}{s^3}\times 2.2s = -10.12\,\frac{m}{s^2}$

Kinematics at constant Acceleration (1D), Section 2.4 of Textbook



$$v = v_0 + at \quad (2\text{-}41) \qquad x - x_0 = v_0 t + \frac{1}{2}at^2 \quad (2\text{-}42)$$

$$v^2 = v_0^2 + 2a(x - x_0) \quad (2\text{-}43)$$

$$x - x_0 = (1/2)(v + v_0)t \quad (2\text{-}44)$$

A great sprinter **accelerates** from rest at $2.5m \cdot s^{-2}$ until reaching a top speed of $15m \cdot s^{-1}$. He continues at this speed until he covers 100 m. How long does it take him to run 100m?

1. First find the time it takes him to reach top speed!
   - Use equation 2-41, $v_1 = v_0 + at_1$
   - $15m \cdot s^{-1} = 0 + 2.5m \cdot s^{-2} \times t_1 \rightarrow t_1 = 6s$
   - Find distance traveled during this time
   - Use equation 2-42, $x_1 - x_0 = v_0 t_1 + \frac{1}{2}at_1^2 = 45m$
2. Find time to travel rest of distance

- Rest of distance is $x_2 - x_1 = 100m - 45m = 55m$
- Since he runs at $15m \cdot s^{-1}$, it will take $t_2 = \frac{55m}{15m \cdot s^{-1}} = 3.67s \rightarrow t_1 + t_2 = 9.67s$

1D Kinematics Question

In *x vs. t* plot above, for which point (I, II, III, IV or V) is the object **moving right**, and **slowing down**?

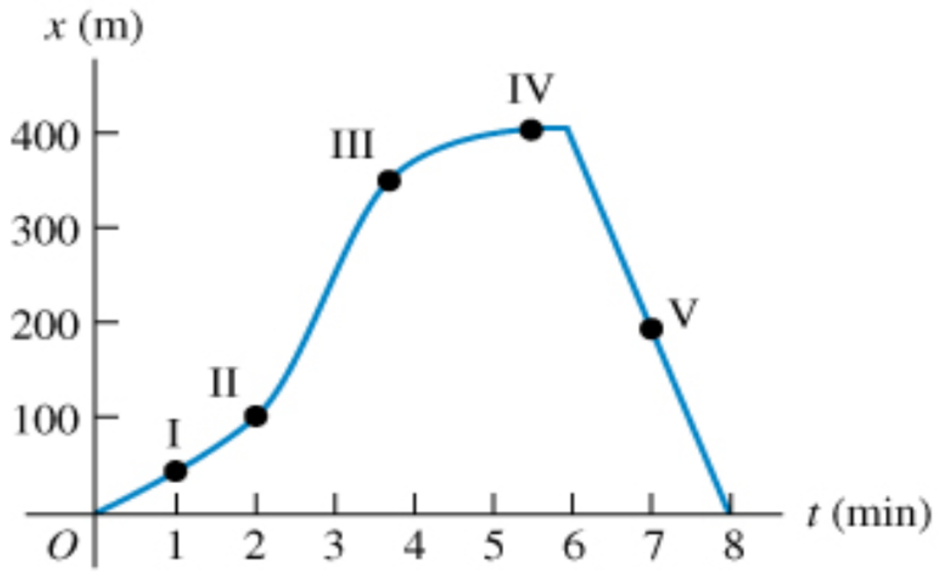
Explain your answers, briefly!

ANSWER: III

Slope must be positive $v > 0$: I, II or III

Only at point III is the slope decreasing, meaning that $a < 0$

When the **signs** of v and a are **opposite**, the object is **slowing down**.



1D Kinematics Question

A motorist makes a trip of 180 miles. For the first 90 miles she drives at a constant speed of 30 mph. At what constant speed must she drive the remaining distance if her average speed for the total trip is to be 40 mph:

a) 50 mph b) 55 mph c) 60 mph d) 45 mph e) 52.5 mph

<u>Solution:</u> C

Definition of average speed: $s_{avg} = \frac{Total\ Distance}{Total\ Time} = \frac{180\ miles}{t_1+t_2} = 40mph$

$Total\ Time = t_1 + t_2 = 4.5hr$

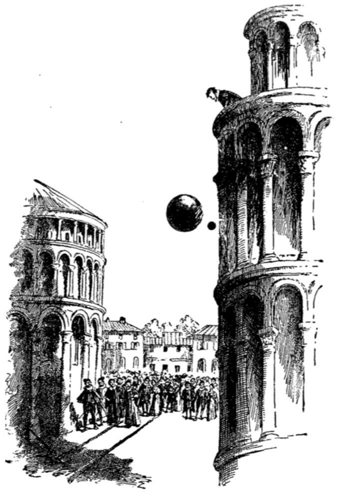
First 90 miles: $t_1 = \frac{90miles}{30mph} = 3hr \rightarrow t_2 = 1.5hr$

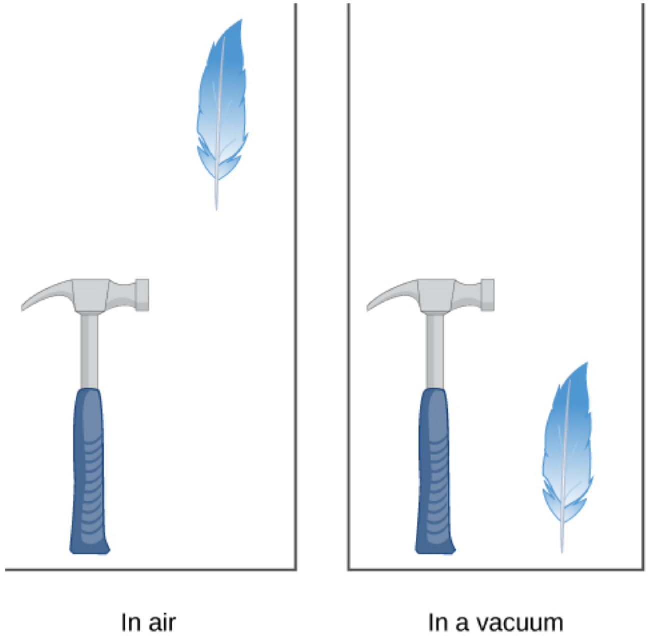
Second 90 miles: speed $s = \frac{90miles}{1.5hr} = 60mph$

Section 2.6 Free Fall

- A **small rock** falls (accelerates) as quickly as a **large rock**.
- See the Galileo ”Leaning tower of Pisa” experiments in the 1590s

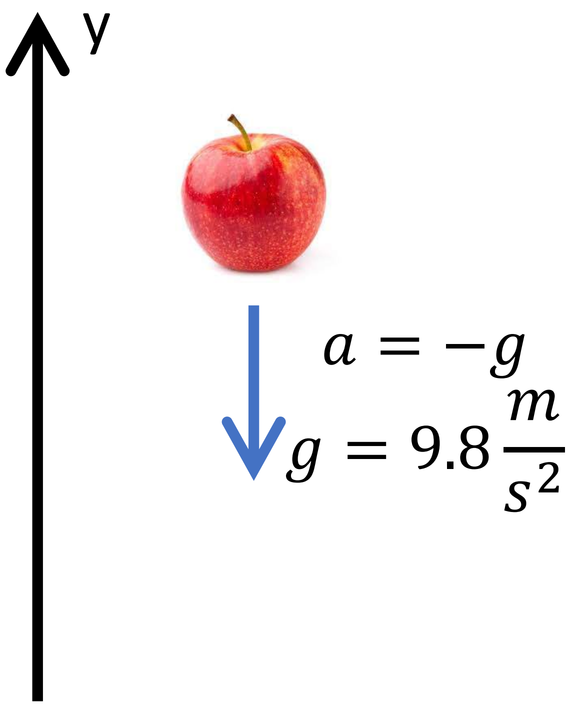
- **Without air**, a **feather** would fall as quickly as a **hammer**!





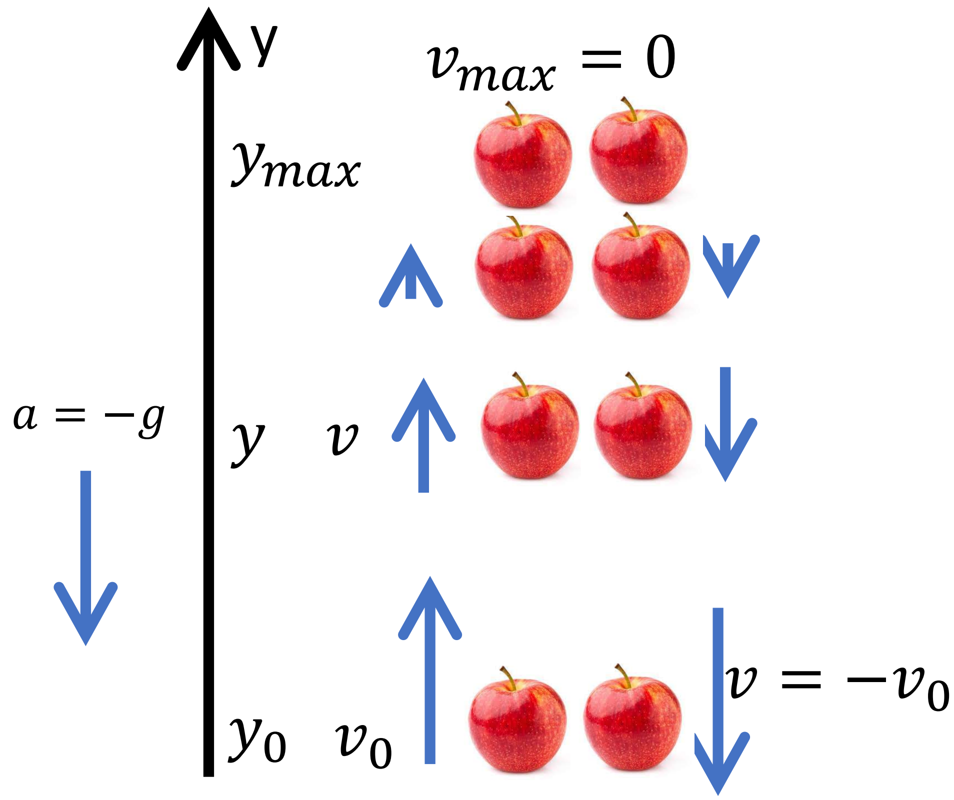
Section 2.6: Free Fall

- An object near the earth's surface falls towards the ground with an at a rate of $g = 9.8\frac{m}{s^2}$.
- From the textbook: The constant-acceleration equations we developed in Section 2.4 also apply to free fall near Earth's surface because the acceleration is a constant *g*. They apply for an object in vertical flight, either up or down (as long as we can neglect the effects of air). However, note that for free fall:
    1. The directions of motion are now along a vertical *y* axis instead of the *x* axis, with the positive direction of *y* upward. (This is important for later chapters when combined horizontal and vertical motions are examined.)
    2. The free-fall acceleration is negative—that is, downward on the *y* axis, toward Earth's center—and so the acceleration has the value $-g$ in the constant-acceleration equations. (*Heads up:* The symbol *g* is a quick way of writing the positive number 9.8 m/s^2. Indicating the downward direction with a minus sign is separate.)



Free Fall Kinematic Equation: **Apple** thrown **straight up** at speed v_0

- Vertical coordinate, y(t), with up as positive (+)
- Origin $t = t_0 = 0$, $y_0 = 0$, velocity $v = v_0$
- Arbitrary time t, position $y(t)$, velocity $v(t)$



<u>Free Fall Equations</u>

$v = v_0 - gt$ (2-41)

$y - y_0 = v_0 t - \frac{1}{2} g t^2$ (2-42)

$v^2 = v_0^2 - 2g(y - y_0)$ (2-43)

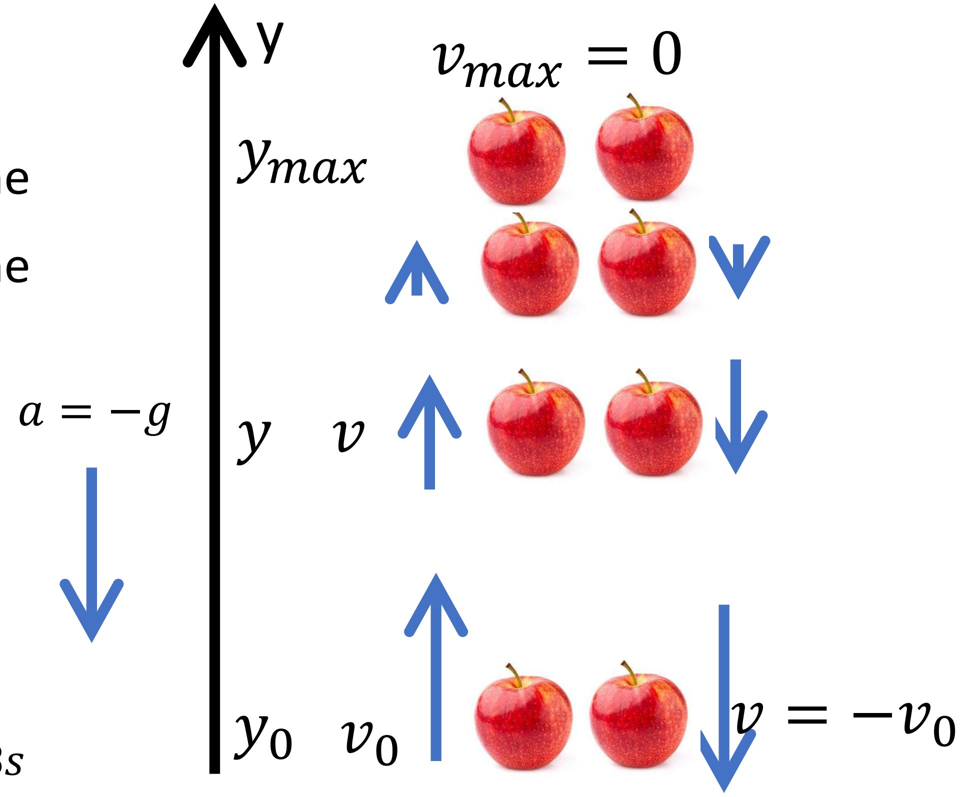
$y - y_0 = \frac{1}{2}(v + v_0)t$ (2-44)

<u>Important points</u>

1. The apple is at the same height h twice, first on the way up, then on the way down
2. The above is an example of a **motion diagram**

Free Fall Example: **Apple** thrown **straight up** at speed v_0

Example: An Apple is thrown up with a speed of 3.0 m/s. **A)** Draw the path of the ball **B)** Calculate the height above the initial position when the speed of the ball is 1.0 m/s. **C)** Calculate the time when the speed of the ball is 1.0 m/s. **D)** Find Maximum height



<u>Solution</u>

B) Use 2.43, $v^2 = v_0^2 - 2g(y - y_0) \rightarrow$

$$(1.0m \cdot s^{-1})^2 = (3.0m \cdot s^{-1})^2 - 2\times 9.8m \cdot s^{-2}(y - y_0)$$

Using $y_0 = 0, y = 0.408m = 0.41m$

C) Use 2.41, $v = v_0 - gt \rightarrow$

- $1.0m \cdot s^{-1} = 3.0m \cdot s^{-1} - 9.8m \cdot s^{-2}\times t \rightarrow t = 0.204s$
- On **way down**, $-1.0m \cdot s^{-1} = 3.0m \cdot s^{-1} - 9.8m \cdot s^{-2}\times t \rightarrow t = 0.408s$

D) Use 2.43, with final velocity being **zero**!

$$v^2 = v_0^2 - 2g(y - y_0) \rightarrow 0 = (3.0m \cdot s^{-1})^2 - 2(9.8m \cdot s^{-2})y_{max}$$

$$y_{max} = 0.46m$$

$$v = v_0 - gt \quad \text{(2-41)}$$

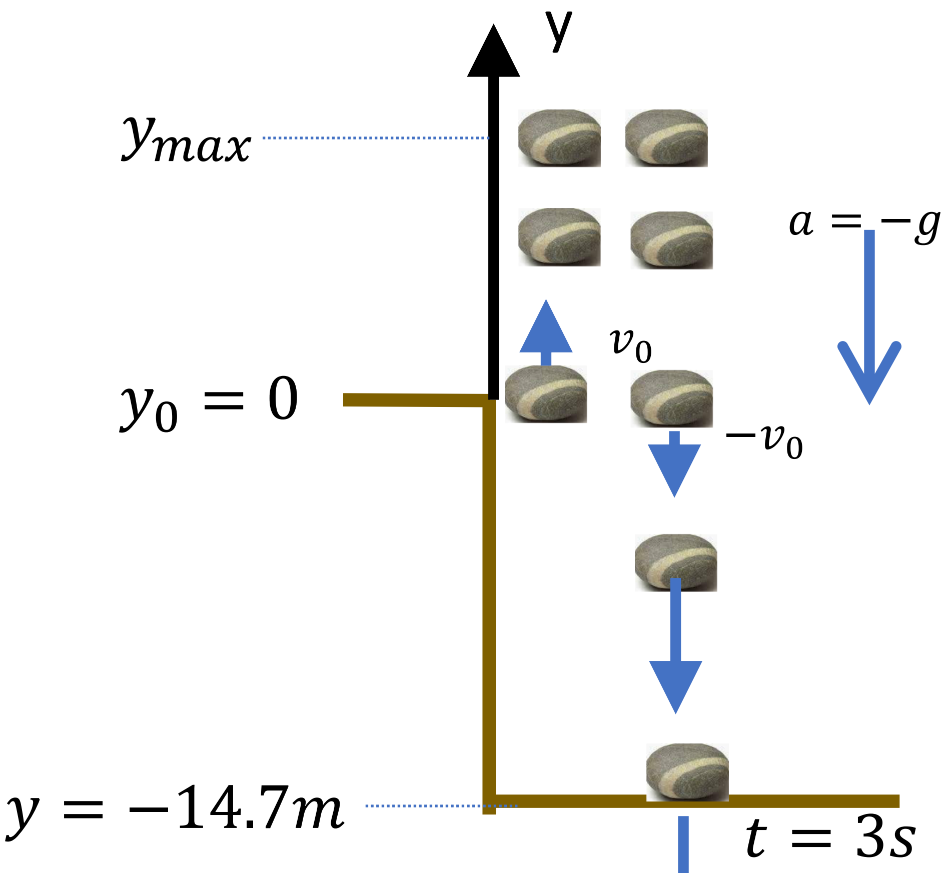
$$y - y_0 = v_0 t - \frac{1}{2}gt^2 \quad \text{(2-42)}$$

$$v^2 = v_0^2 - 2g(y - y_0) \quad \text{(2-43)}$$

A tossed pebble problem

A 2-kg pebble is on top of a 14.7 m high cliff. It is **thrown straight up** with a **speed** of $9.8m \cdot s^{-1}$. **A)** Draw a **motion diagram** of the pebble till it hits the ground 14.7 m below. **B)** Find the time it takes for the pebble to hit the ground. **C)** Find velocity at bottom.

- Use one of:
- $v = v_0 - gt$ (2-61)
- $y - y_0 = v_0 t - \frac{1}{2} g t^2$ (2-62)
- $v^2 = v_0^2 - 2g(y - y_0)$ (2-63)



Solution Part B

Use 2-62 with $y_0 = 0$, $v_0 = 9.8m \cdot s^{-1}$
Use $y = -14.7m$

$$y - y_0 = v_0 t - \frac{1}{2} g t^2 \rightarrow -14.7m = 9.8\frac{m}{s^2} t - 4.9\frac{m}{s^2} t^2$$

Divide both sides by -4.9 and rearranging we have

$$t^2 - 2t - 3 = 0$$

factoring quadratic equation, $(t - 3s)(t + 1s) = 0$
Solution : $t = 3s, -1s$

PHYSICAL SOLUTION:

$$t = 3s$$

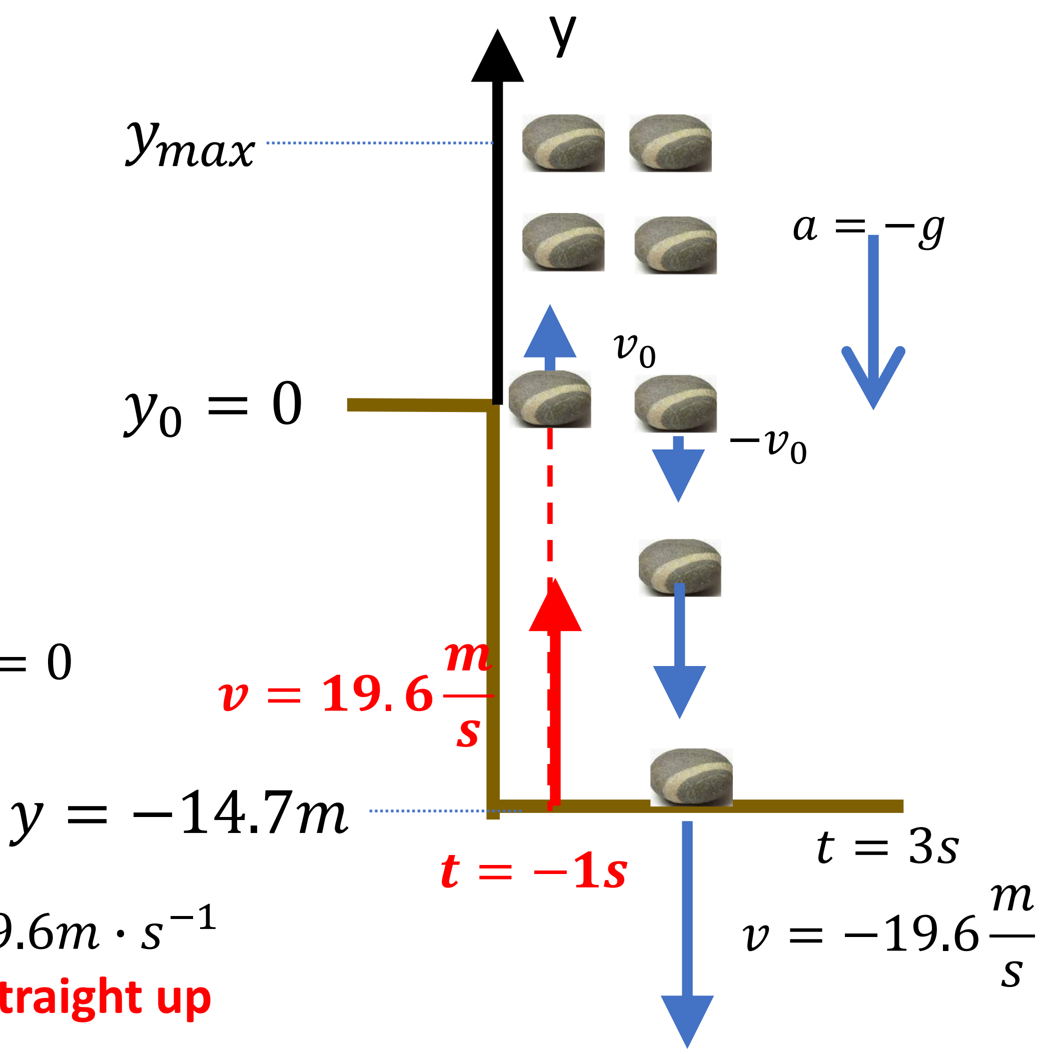
Part C: Use 2-61, $v = v_0 - gt$

$$v = 9.8m \cdot s^{-1} - (9.8m \cdot s^{-2}) \times 3s = -19.6m \cdot s^{-1}$$

A tossed pebble problem: The meaning of **negative time** solution.

A 2-kg pebble is on top of a 14.7 m high cliff. It is **thrown straight up** with a **speed** of $9.8 m \cdot s^{-1}$. **A)** Draw a **motion diagram** of the pebble till it hits the ground 14.7 m below. **B)** Find the time it takes for the pebble to hit the ground. **C)** Find velocity at bottom.

- $v = v_0 - gt$ (2-61)
- $y - y_0 = v_0 t - \frac{1}{2} g t^2$ (2-62)



Solution Part B

$y - y_0 = v_0 t - \frac{1}{2} g t^2 \rightarrow t^2 - 2t + 3 = 0 \rightarrow (t - 3s)(t + 1s) = 0$

Solution : $t = 3s, -1s$

Positive time: $t = 3s, v = 19.6 m \cdot s^{-1}$

Negative Time: $\boldsymbol{t = -1s}$, see extrapolation

Velocity: $v = v_0 - gt = 9.8 m \cdot s^{-1} - (9.8 m \cdot s^{-2}) \times 3s = -19.6 m \cdot s^{-1}$

$\boldsymbol{t = -1s}$ corresponds to scenario where the pebble is thrown **straight up** with a speed of $\boldsymbol{19.6 \frac{m}{s}}$ at time $\boldsymbol{t = -1s}$.