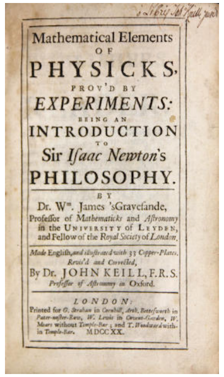
PHYS 1211 F2020 Lecture Monday October 26, 2020

Chapter 7

Introduction to work and Kinetic Energy

Kinetic Energy: 7.1

- Willem 's Gravesande 1722 published the result of experiments in which brass balls are dropped into soft clay from varying heights.
- Penetration depth is proportional to squared speed



Mathematical Elements
OF
PHYSICKS,
PROV'D BY
EXPERIMENTS:
BEING AN
INTRODUCTION
TO
Sir *Isaac Newton's*
PHILOSOPHY.

BY
Dr. Wm. James 'sGravesande,
Professor of *Mathematicks* and *Astronomy*
in the UNIVERSITY of LEYDEN,
and Fellow of the *Royal Society* of *London*.

Made English, and illustrated with 33 *Copper-Plates.*
Revis'd and Corrected,
By Dr. JOHN KEILL, F.R.S.
Professor of Astronomy in Oxford.

LONDON:
Printed for *G. Strahan* in *Cornhill*, *Arth. Bettesworth* in *Pater-noster-Row*, *W. Lewis* in *Covent-Garden*, *W. Mears* without *Temple-Bar*; and *T. Woodward* within *Temple-Bar*. MDCCXX.

Émilie du Châtelet concluded energy $\sim mv^2$

Kinetic Energy: $K = \frac{1}{2}mv^2$

Unit: $1\,Joule = 1J = 1kg \cdot m^2 \cdot s^{-2}$

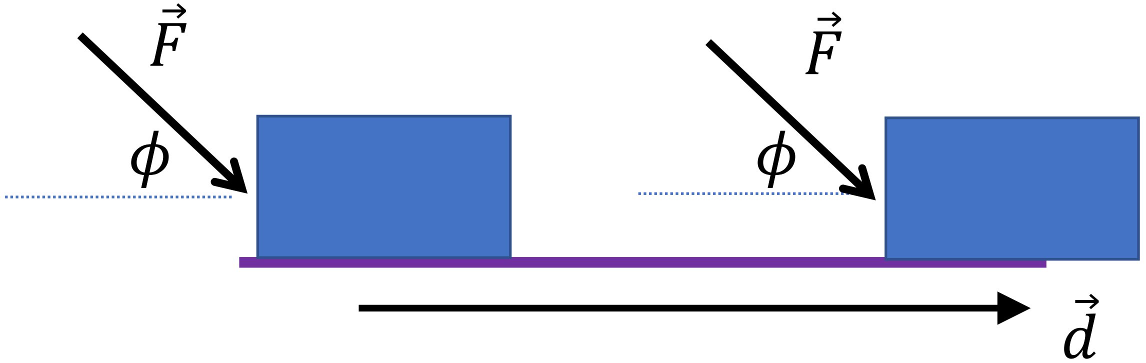
Kinetic Energy

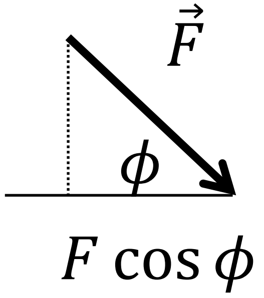
- Cat running at 10 m/s: $K = \frac{1}{2}mv^2 = \frac{1}{2}1.5kg\left(10\frac{m}{s}\right)^2 = 75J$
- Man running at 10 m/s: $K = \frac{1}{2}mv^2 = \frac{1}{2}70kg\left(10\frac{m}{s}\right)^2 = 3500J$
- Car moving at 50 km/hr:

$$K = \frac{1}{2}mv^2 = \frac{1}{2}1000kg\left(50\frac{km\times 1000m \cdot km^{-1}}{hr\times 3600s \cdot hr^{-1}}\right)^2 = 96450J$$

Work

- A moving object is acted on by a constant force, $\vec{F}$, during a period where its displacement is $\vec{d}$.





- The work done during this period is:

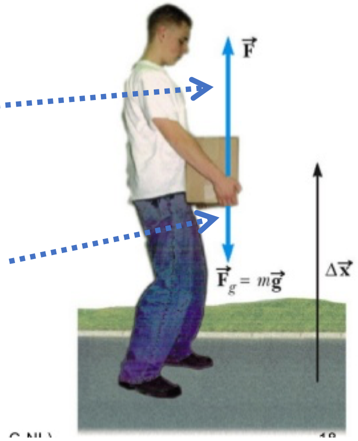
$$W = \vec{F} \cdot \vec{d} \qquad (7.2.7)$$

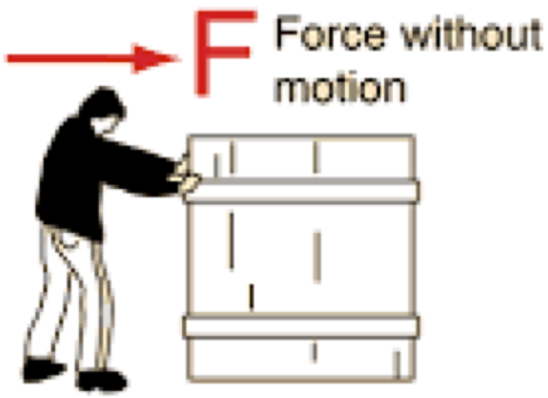
$$W = |\vec{F}||\vec{d}|\cos\phi = (\mathrm{F}\cos\phi)d = Fd\cos\phi \; (7.2.6)$$

Unit of work, $N \cdot m = \frac{kg \cdot m}{s^2} \times m = \frac{kg \cdot m^2}{s^2} = J$

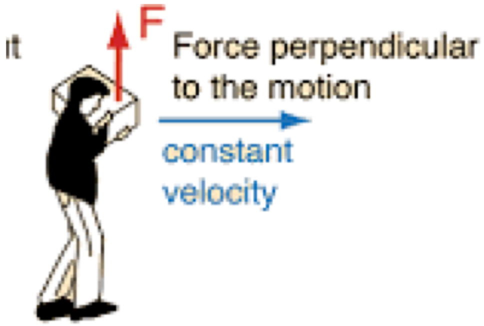
Work Examples

- Work is positive (+) when lifting box
- Work is positive (–) when lowering box





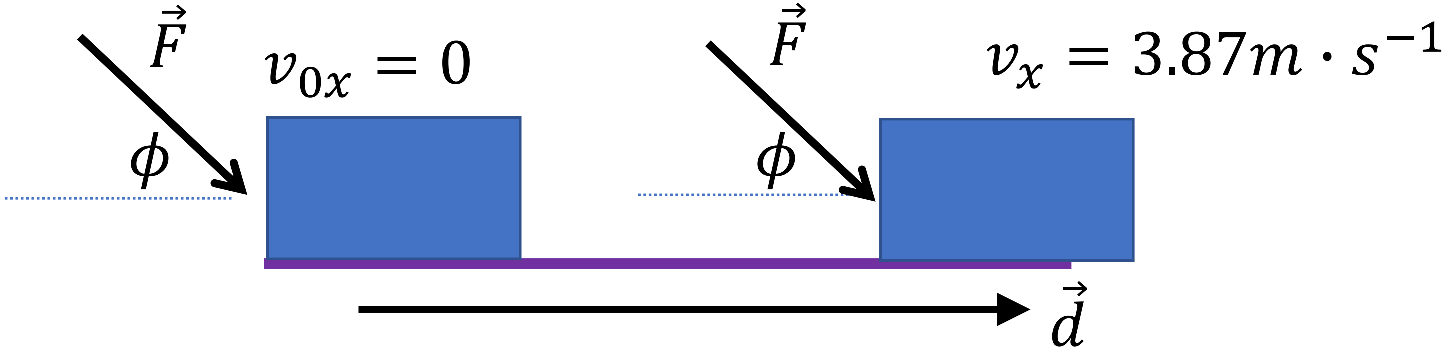
When a force is exerted on an object which does not move, no work is done on the object.

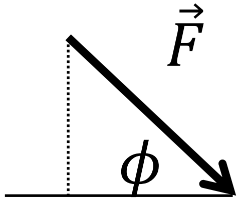


When an object is carried at constant velocity by a force which acts at right angles to the motion, no work is done on the object.

Work and Kinetic Energy

- A Box of mass m = 2 kg is push by a force $F = 10N$ at $\phi = 60^{\circ}$ for a displacement of $d = 3m$, as shown below. If the box was initially at rest calculate the work done on the box by the force, and the change in kinetic energy of the box. There is no friction.





$F_x = F\cos\phi$

The work done during this period is: $W = |\vec{F}||\vec{d}|\cos\phi = Fd\cos\phi = 15J$

Use Newton 2nd on the horizontal (x) comp, $F_x = F\cos 60^{\circ} = ma \rightarrow a = \frac{10N\times 0.5}{2kg} = 2.5\frac{m}{s^2}$

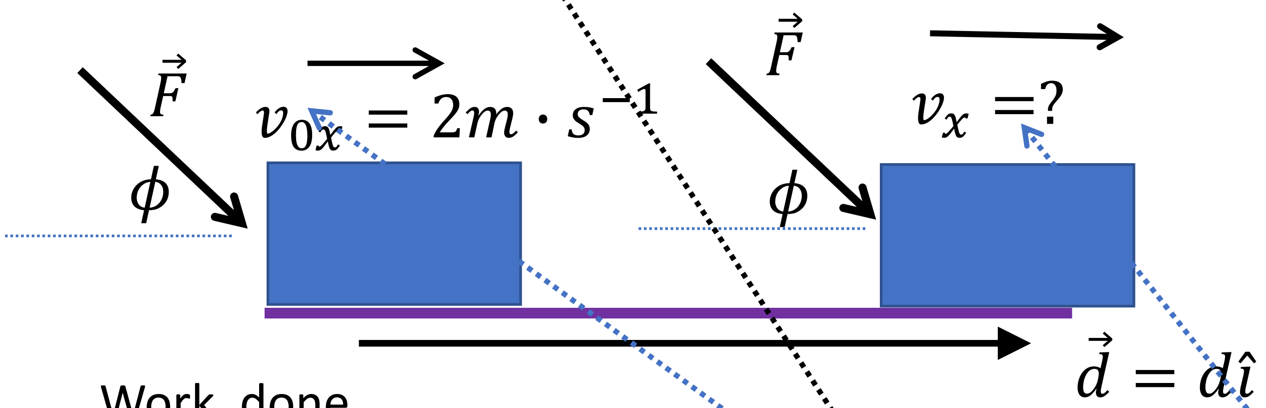
Using Kinematic Equations, $v_x^2 = v_{0x}^2 + 2ad \rightarrow v_x = \sqrt{2ad} = 3.87m \cdot s^{-1}$

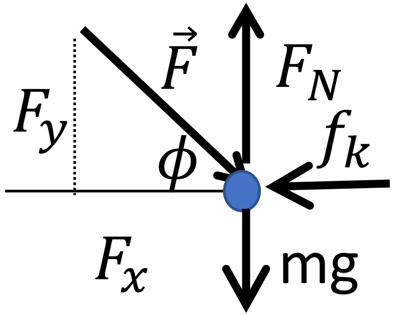
Change in Kinetic energy:$\Delta K = \frac{1}{2}mv_x^2 - \frac{1}{2}mv_{0x}^2 = \frac{1}{2}2kg\times(3.87m \cdot s^{-1})^2 = 15J$

Work energy Theorem: $W = \Delta K$

Work and Kinetic Energy

- A Box of mass m = 2 kg is push by a force $F = 10N$ at $\phi = 60^{\circ}$ for a displacement of $d = 3m$, as shown below. If the box was initially moving at $v_{0x} = 2m \cdot s^{-1}$. If the coefficient of friction is $\mu_s = 0.2$ and $\mu_k = 0.15$, calculate the work done by all the forces acting on the box. Calculate the final speed of the box.





Y-com

$$F_{net,y} = F_y + F_N - mg = 0$$
$$-F\sin\phi + F_N - mg = 0$$
$$F_N = 8.66N + 19.6N = 28.3N$$
$$f_k = F_N\mu_k = 4.23N$$

<u>Work done</u>

$\vec{F}$: $W_F = \vec{F} \cdot \vec{d} = F_x d = Fd\cos\phi = 15J$

Gravity, mg: $W_g = -mg\hat{\jmath} \cdot \vec{d} = 0$

Normal force, F_N : $W_N = F_N\hat{\jmath} \cdot \vec{d} = 0$

Friction: $W_{f_k} = \vec{f_k} \cdot \vec{d} = -4.23N \times 3m = -12.7J$

Total work: $W = W_F + W_g + W_N + W_{f_k} = 2.31J$

Work Energy Theorem: W $= \Delta K \rightarrow 2.31J = \frac{1}{2}mv_x^2 - \frac{1}{2}mv_{0x}^2 \rightarrow v_x = 2.5m \cdot s^{-1}$

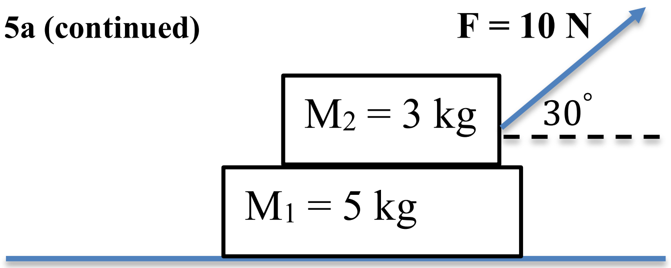
Verify with Newton's law

$$F_{net,x} = F_x - f_k = ma$$
$$a = \frac{5N - 4.23N}{2kg} = 0.385\frac{m}{s^2}$$
$$v_x^2 = v_{0x}^2 + 2ad \rightarrow v_x = 2.5m \cdot s^{-1}$$

Question 5 of Midterm 2 of F2019

(**10 points**) In the figure, **Block 1** (M_1 = 5 kg) rests on a frictionless surface, and **Block 2** (M_2 = 3 kg) rests on **Block 1**. The friction coefficients between **Block 2** and **Block 1** are: $\mu_k = 0.1$, $\mu_s = 0.2$. A force F = 10 N is applied at 30° to the **Horizontal**, as shown.

Question 5a (continued)



A) Assume that when the force is applied to **Block 2** there is no **slipping**, and the two blocks accelerate together. Use Newton's second law to calculate the **magnitude** of the **acceleration**, a.

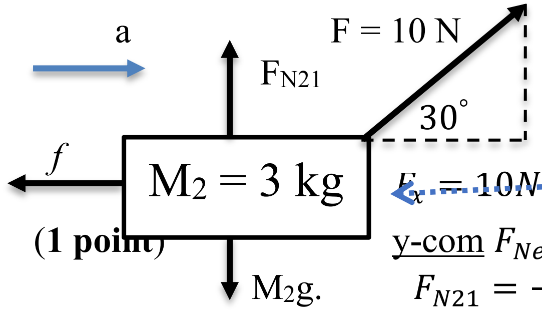
acceleration, a.

$$F_{Net,x} = F_x = (M_1 + M_2)a \rightarrow 10N \times cos30^{\circ} = 8kg \times a \rightarrow a = 1.0825\frac{m}{s^2}$$ (**2 points**)

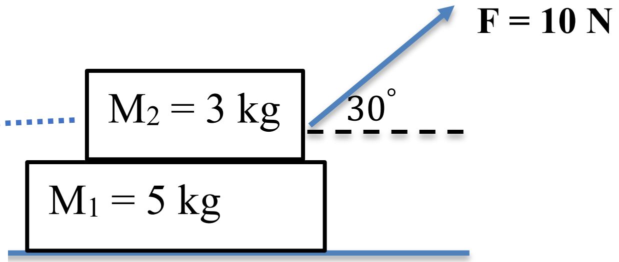
Question 5 of Midterm 2 of F2019, part b

(**10 points**) In the figure, **Block 1** (M_1 = 5 kg) rests on a frictionless surface, and **Block 2** (M_2 = 3 kg) rests on **Block 1**. The friction coefficients between **Block 2** and **Block 1** are: $\mu_k = 0.1$, $\mu_s = 0.2$. A force F = 10 N is applied at 30° to the **Horizontal**, as shown.

B) Draw a FBD on **Block 2**, which includes the **4 forces** acting on it. Calculate the **normal force** of on **Block 2** due to **Block 1**, F_{N21}, and the maximum static friction, $f_{s,max}$. Using the **magnitude** of the **acceleration**, a, from part **A**, and applying Newton's second law to the horizontal component, calculate the **magnitude** and **direction** of the friction force acting on **Block 2**. Is the assumption of no slipping in part **A** correct?



$F_y = 10N \times sin30^\circ = 5N$



$F_x = 10N \times cos30^\circ = 8.67N$

y-com $F_{Net,y} = F_{N21} + F_y - M_2g = 0$

$F_{N21} = -F_y + M_2g = -5N + 29.4N = 24.4N$

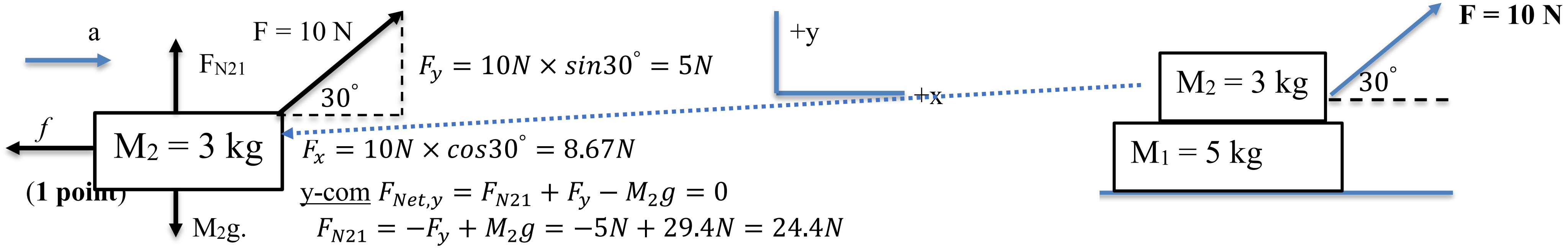
Maximum Static Friction: $f_{s,max} = F_{N21}\mu_s = 24.4N \times 0.2 = 4.88N$ (**2 points**)

x-comp $F_{Net,x} = F_x - f = M_2a \rightarrow f = F_x - M_2a = 8.67N - 3kg \times 1.0825\frac{m}{s^2} = 5.42N$

Since $f > f_{s,max}$, the blocks will slip! The assumption of part A is wrong. (**1 point**)

Question 5 of Midterm 2 of F2019, part c

(**10 points**) In the figure, **Block 1** (M_1 = 5 kg) rests on a frictionless surface, and **Block 2** (M_2 = 3 kg) rests on **Block 1**. The friction coefficients between **Block 2** and **Block 1** are: $\mu_k = 0.1$, $\mu_s = 0.2$. A force F = 10 N is applied at 30° to the **Horizontal**, as shown.



C) In part **B)** you will find that the **Block 2** slips on **Block 1**. Calculate the force of kinetic friction, f_k. Use the FBD of part B and Newton's second law to calculate the **acceleration** of **Block 2**. **Note**: your answer will not be the same as in part A.

Kinetic Friction: $f_k = F_{N21}\mu_k = 24.4N \times 0.1 = 2.44N$ (**1 point**)

x-comp $F_{Net,x} = F_x - f_k = M_2 a \rightarrow a = \frac{F_x - f_k}{M_2} = \frac{8.67N - 2.44N}{3kg} = 2.08\frac{m}{s^2}$ (**1 point**)

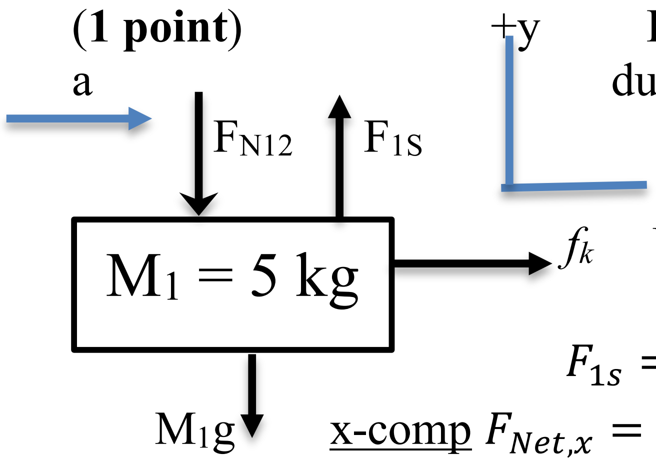
Block 2, $\vec{a}_2 = 2.08\frac{m}{s^2}\hat{\imath}$

Question 5 of Midterm 2 of F2019, part d

(**10 points**) In the figure, **Block 1** (M_1 = 5 kg) rests on a frictionless surface, and **Block 2** (M_2 = 3 kg) rests on **Block 1**. The friction coefficients between **Block 2** and **Block 1** are: $\mu_k = 0.1$, $\mu_s = 0.2$. A force F = 10 N is applied at 30° to the **Horizontal**, as shown.

D) Draw a FBD of **Block 1**, and use f_k and Newton's second law to calculate the **acceleration** of **Block 1**.

(**1 point**)



In the diagram, F_{N12} is the normal force on **Block 1** due to **Block 2**. Using Newton's 3rd law:

$F_{N12} = F_{N21} = 24.4N$ No grades for this part

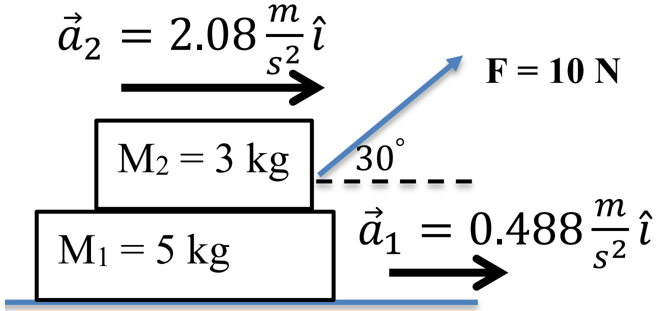
+x. F_{1s} is the normal force of the surface on **Block 1**

Using 2nd law on y-component

<u>y-com</u> $F_{Net,y} = F_{1s} - F_{N12} - M_1 g = 0$

$F_{1s} = F_{N12} + M_1 g = 24.4N + 49N = 73.4N$ No grades

<u>x-comp</u> $F_{Net,x} = f_k = M_1 a \rightarrow a = \frac{f_k}{M_1} = \frac{2.44N}{5kg} = 0.488\frac{m}{s^2}$ (**1 point**)



A Multiple choice

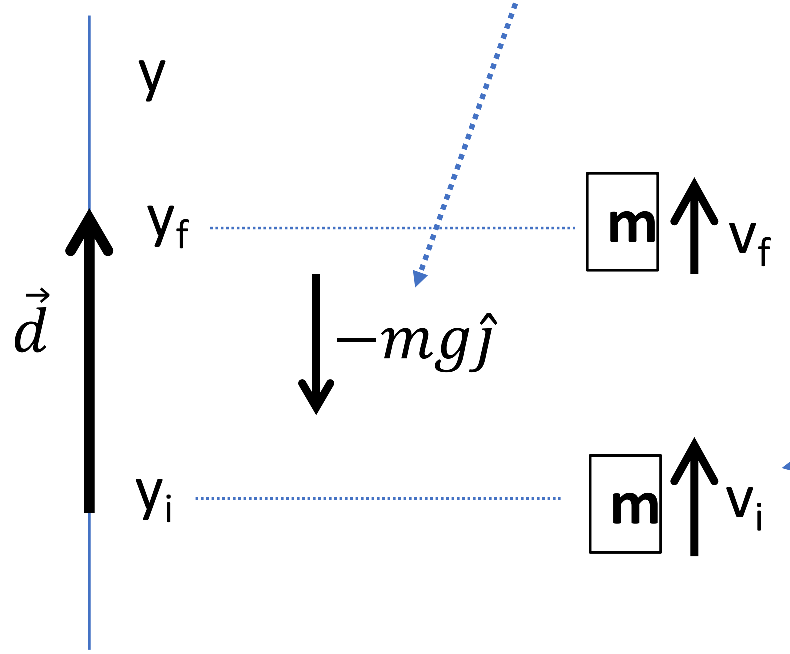
- An object is constrained by a cord to move in a circular path of radius 0.5 m on a horizontal frictionless surface. The cord will break if its tension exceeds 16 N. The maximum kinetic energy the object can have is:
- A) 4J B) 8J C) 16J D) 32J E) 64J

Centripetal Force T =$\frac{mv^2}{R} \rightarrow \frac{mv^2}{2} = \frac{1}{2}TR = 4J$

A

Work by Gravity: Part I

- Ball of mass m, going **up** vertical **displacement** $\vec{d} = \Delta y \hat{j} = (y_f - y_i)\hat{j}$
- i stands for **initial** and f stands for **final**
- **Force** of **Gravity**, $\vec{F}_g = -mg\hat{j}$



Work Done: $W_g = \vec{F}_g \cdot \vec{d}$

$W_g = -mg\hat{j} \cdot \Delta y \hat{j} = -mg\Delta y(\hat{j} \cdot \hat{j}) = -mg\Delta y$

$W_g < 0$, since $y_f > y_i \rightarrow \Delta y = y_f - y_i > 0$

Work-Energy Theorem:

$$W_g = \Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

v_i is initial velocity, v_f is final velocity

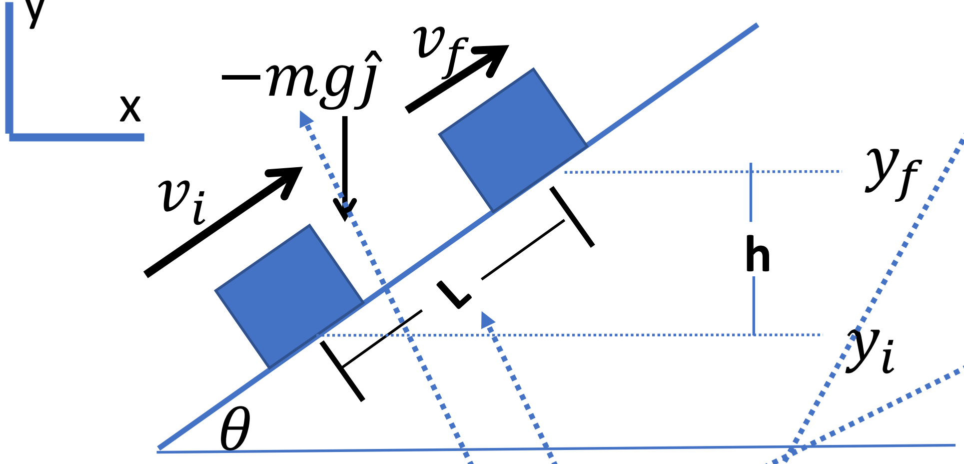
$$W_g = -\cancel{m}g\Delta y = \frac{1}{2}\cancel{m}v_f^2 - \frac{1}{2}\cancel{m}v_i^2$$

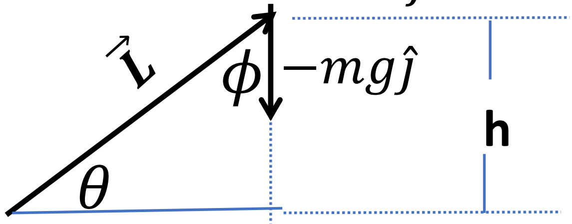
Rearranging, $v_f^2 = v_i^2 - 2g\Delta y$

Same as projectile equation E18 of midterm 1

Work by Gravity Example

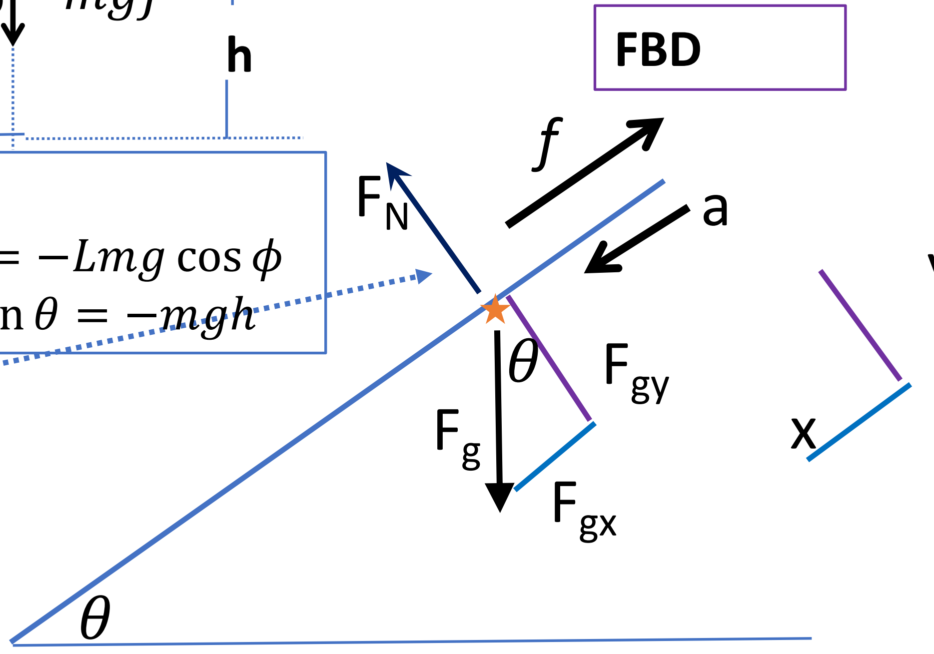
- A **box** of **mass m = 8 kg** is **moving** at $v_i = 10m \cdot s^{-1}$ on a $\theta = 53.1^{\circ}$ frictionless incline.
- After it has traveled L = 0.5m, what is its final speed, v_f. Use **Work-energy theorem**





Aside

$$\vec{L} \cdot (-mg\hat{j}) = -Lmg\cos\phi$$
$$= -mgL\sin\theta = -mgh$$



Work by gravity:

$$W_g = \vec{F}_g \cdot \vec{L} = -mg\hat{j} \cdot \vec{L} = -mgL\sin\theta = -mgh$$

Work by normal force?

ZERO!

Work-Energy theorem. Total work:

$$W = \Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = -mgh$$

$$v_f = \sqrt{v_i^2 - 2gh} = 9.6m \cdot s^{-1}$$

Also

$$h = L\sin 53.1^{\circ} = 0.4m$$

Work by Gravity: Arbitrary Path

Below is a path of an object that moves in a curve path from point **1** to point **2**.

The work done by gravity is calculated by dividing the path into infinitesimal path of length

$$d\vec{l} = dl\cos\phi\,\hat{\imath} + dl\sin\phi\,\hat{\jmath}$$

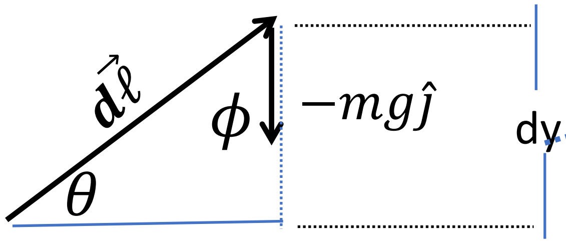
The work done by gravity on path $d\vec{l}$ is

$$dW_g = \vec{F}_G \cdot d\vec{l} = -mg\hat{\jmath} \cdot (dl\sin\phi\,\hat{\imath} + dl\cos\phi\,\hat{\jmath}) = -mgdy$$

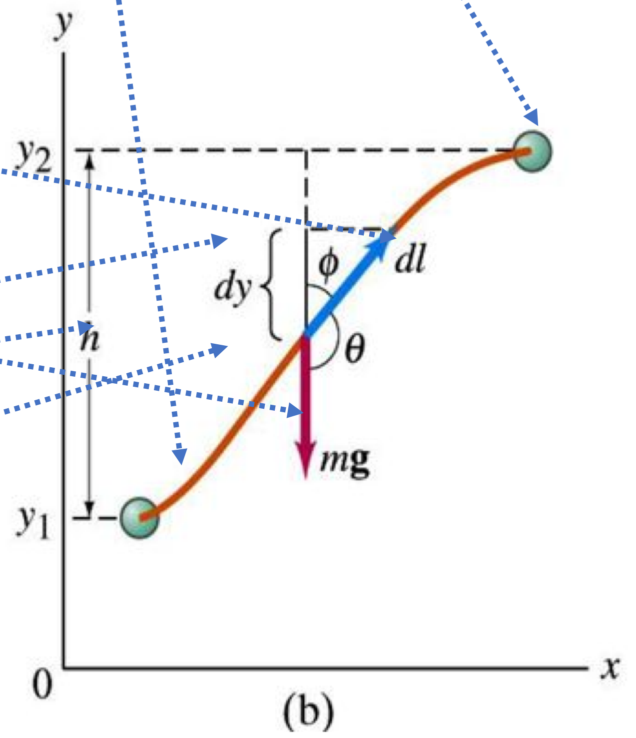
The total work done is

$$W_g = \sum dW_g = \int_1^2 -mgdy = -mgh$$

Where $h = y_2 - y_1$ is the **change** in **height**.

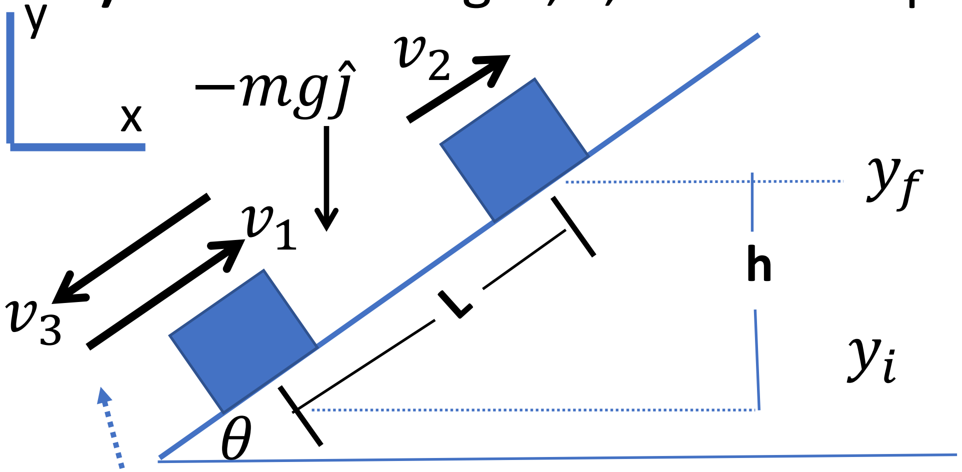


$$\boxed{\begin{gathered} -mg\hat{\jmath} \cdot (dl\sin\phi\,\hat{\imath} + dl\cos\phi\,\hat{\jmath}) = \\ -mgdl\sin\phi\,\hat{\jmath}\cdot\hat{\imath} - mgdl\cos\phi\,\hat{\jmath}\cdot\hat{\jmath} = \\ -mgdl\cos\phi = -mgdl\sin\theta = -mgdy \\ \text{Since } \phi = \frac{\pi}{2} - \theta \rightarrow \cos\phi = \sin\theta \end{gathered}}$$



Work by Gravity on incline with no Friction

- A **box** of **mass m = 8 kg** is **moving** at $v_1 = 10m \cdot s^{-1}$ on a $\theta = 53.1^\circ$ **frictionless** incline.
- **A)** Find max length, L, it travels up incline. **B)** Find speed when it returns to its initial height.



Way up:

Total Work done: $W = W_g + W_{F_N} = -mgh + 0$

Why is work done by normal force, $W_{F_N} = 0$?

Work-Energy Theorem: $W = -mgh = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$

$h = L\sin\theta$, $\quad \boldsymbol{v_2 = 0}$

$$-mgL\sin\theta = -\frac{1}{2}mv_1^2 \rightarrow L = \frac{1}{2}\frac{(10m \cdot s^{-1})^2}{\sin 53.1^\circ \, 9.8m \cdot s^{-2}}$$

$$L = 6.38m$$

Way down:

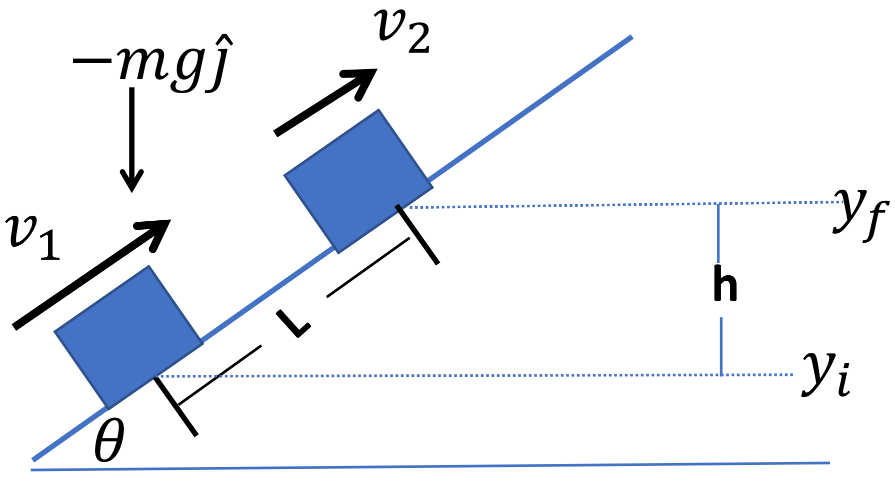
Total Work done: $W = W_g + W_{F_N} = +mgh + 0 = \frac{1}{2}mv_3^2 - \frac{1}{2}mv_2^2 = \frac{1}{2}mv_3^2 - 0$

$$mgL\sin\theta = \frac{1}{2}mv_3^2 \rightarrow v_3 = \sqrt{2gL\sin\theta} = \sqrt{2 \times 9.8m \cdot s^{-2} \times 6.38m \times \sin 53.1^\circ} = 10\frac{m}{s}$$

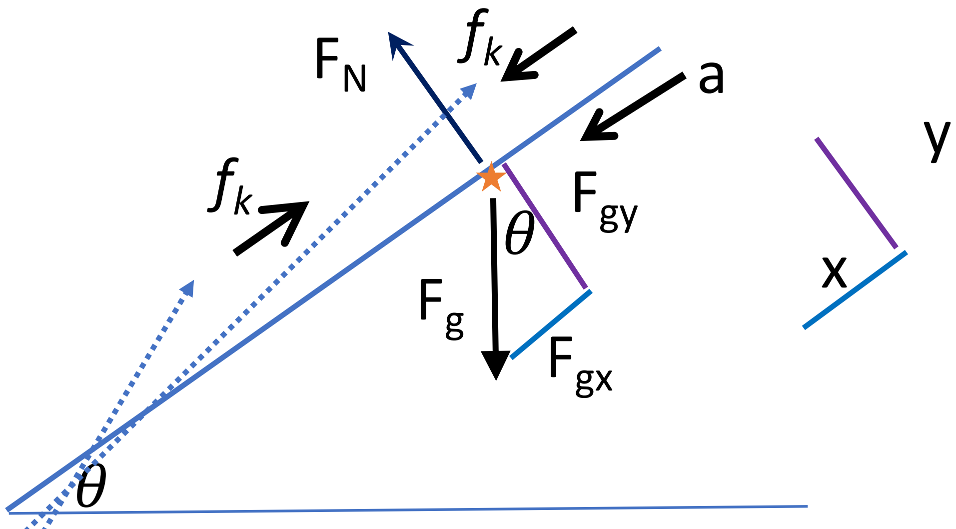
$v_1 = v_3$, Conservation of Kinetic Energy

Work by Gravity on incline with friction, Part I

- A **box** of **mass m = 8 kg** is **moving up** $v_1 = 10m \cdot s^{-1}$ on a $\theta = 53.1^{\circ}$ frictionless incline, with $\mu_k = 0.4$ and $\mu_s = 0.5$.
- **A)** Find max length, L, it travels up incline. **B)** Find speed when it returns to its initial height.



Direction of friction, $\vec{f}$, is **opposite** direction of motion … or **possible motion**



First Draw FBD

Y-com: $F_{Net,y} = F_N + F_{gy} = 0$

$$F_N - mg\cos 53.1 = 0 \rightarrow F_N = 47.04N$$

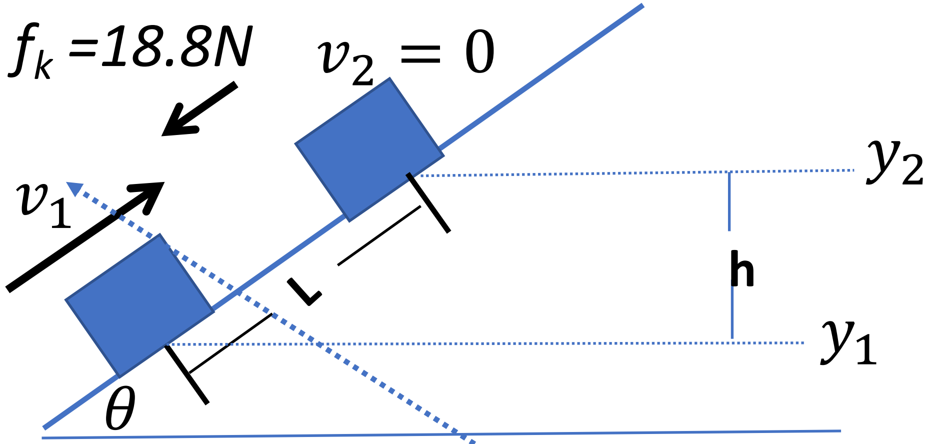
Kinetic Friction, $\underline{f_k = f = F_N\mu_k = 18.8N}$

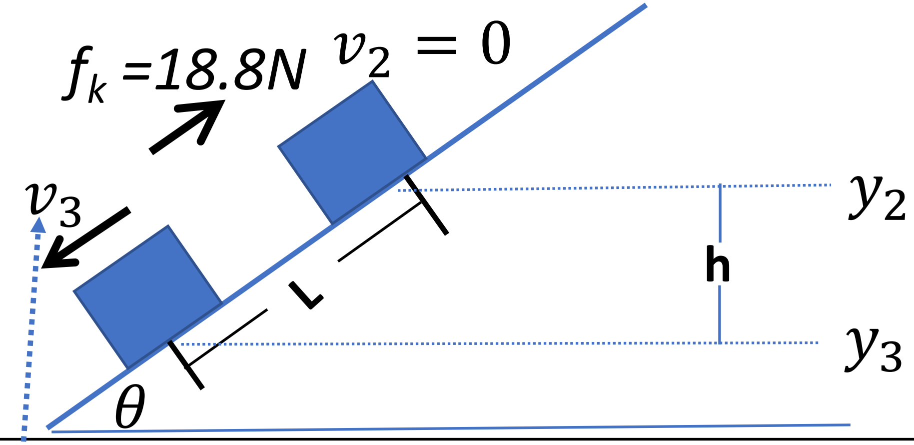
When box is moving **up** incline friction is **down**

When box is moving **down** incline friction is **up**

Work by Gravity on incline with friction, Part II

- A **box** of **mass m = 8 kg** is **moving up** $v_1 = 10m\cdot s^{-1}$ on a $\theta = 53.1^{\circ}$ frictionless incline, with $\mu_k = 0.4$ and $\mu_s = 0.5$.
- **A)** Find max length, L, it travels up incline. **B)** Find speed when it returns to its initial height.





On way up:

Work done: $W = W_g + W_{f_k} + W_{F_N}$

$$W = -mgh - f_kL + 0$$

Work-Energy:

$$W = -mgL\sin\theta - f_kL = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = -\frac{1}{2}mv_1^2$$

$$L = -\frac{1}{2}\frac{v_1^2}{9.8m\cdot s^{-1}\sin 53.1^{\circ} + \frac{18.8N}{8kg}} = 4.9m$$

On way down:

Work done: $W = W_g + W_{f_k} + W_{F_N}$

$$W = +mgh - f_kL + 0$$

Work-Energy:

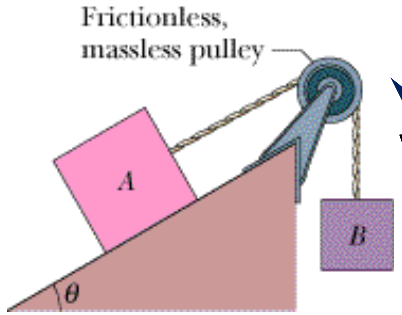
$$W = mgL\sin\theta - f_kL = \frac{1}{2}mv_3^2 - \frac{1}{2}mv_2^2 = \frac{1}{2}mv_3^2$$

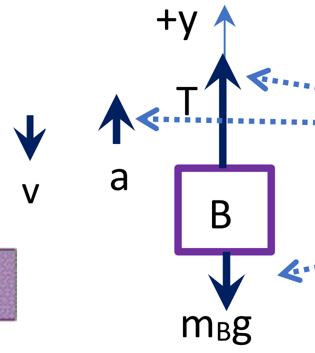
$$v_3 = \sqrt{2gL\sin 53.1^{\circ} - 2Lf_k/m} = 7.33m\cdot s^{-1}$$

$v_3 < v_1$, Kinetic Energy is lost

2-Body incline problem (10 Points)

In the diagram below, box A (mass m_A = 5 kg) is on a $\theta = 53.1°$ incline with friction coefficients: $\mu_s = 0.5$ and $\mu_k = 0.15$. It is connected to a hanging Box B by an ideal rope passed through a **frictionless pulley**. Box B has mass m_B = 4 kg. If Box B is falling find the **acceleration** of Box B and the **tension** in the rope.





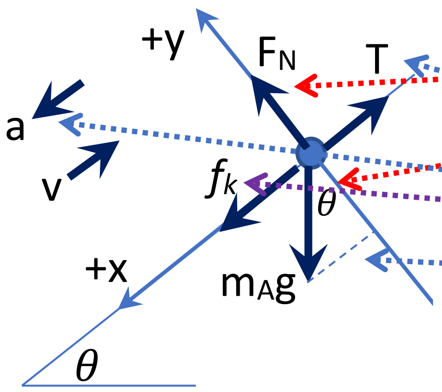
FBD of **Box B** that assumes **B accelerates up**, but B is **moving down** (**velocity** is **down**): FBD is **1 point**

Use 2[nd] law, $F_y^{net} = T - m_B g = m_B a$, or in numerical form

$$T - (4kg)g = (4kg)a$$

$$T - 39.2N = (4kg)a, \qquad [1]$$

Equation [1] has two unknowns tension T and acceleration a. (**2 points**)



FBD of **Box A** that assumes **A accelerates down incline**, but **A** is **moving up incline**: FBD is **2 points**

Y-com – $F_y^{net} = F_N - m_A g cos\theta = 0 \rightarrow$

$$F_N = (5kg)9.8\frac{m}{s^2}cos53.1 = 29.4N$$

Kinetic Friction: $f_k = F_N \mu_k = 29.4N \times 0.15 = 4.41N$, (**1 point**)

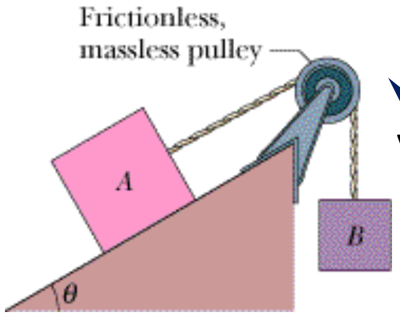
X-com 2[nd] law, $F_x^{net} = m_A g sin\theta + f_k - T = m_A a$,

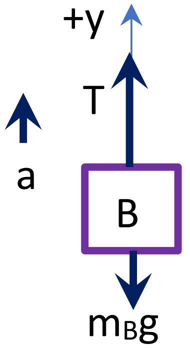
$$43.61N - T = (5kg)a. \qquad [2]$$

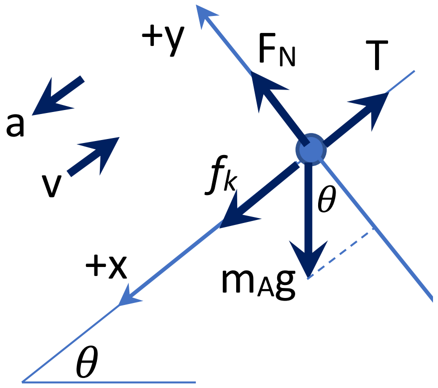
Equation [2] has two unknowns tension T and a. (**2 points**)

2-Body incline problem (10 Points), Part 2

In the diagram below, box A (mass m_A = 5 kg) is on a $\theta = 53.1°$ incline with friction coefficients: $\mu_s = 0.5$ and $\mu_k = 0.15$. It is connected to a hanging Box B by an ideal rope passed through a **frictionless pulley**. Box B has mass m_B = 4 kg. If Box B is falling find the **acceleration** of Box B and the **tension** in the rope.







$$T - 39.2N = (4kg)a, \qquad [1]$$

$$43.61N - T = (5kg)a. \qquad [2]$$

The solution is found by adding equation [1] and [2] to obtain

$$T - 39.2N + 43.61 - T = (4kg)a + (5kg)a$$

$$4.41N = (9kg)a \rightarrow a = 0.49\frac{m}{s^2}, (\mathbf{1\ point})$$

Substituting $a = 0.49\frac{m}{s^2}$ into equation [1] gives $T - 39.2N = (4kg)\left(0.49\frac{m}{s^2}\right) = 41.16N$. (**1 point**)

Students should verify the value of the tension with equation [2]. Since a > 0, our assumption is correct A accelerates down, and B is accelerating up.

Another multiple choice

- A 100-kg piano rolls down a 20° incline. A man tries to keep it from accelerating, and manages to keep its acceleration to 1.2 m/s^2. If the piano rolls 5 m, what is the net work done on it by all the forces acting on it?
- A) 60J B) 100J C) 600J D) 100J E) 490 J