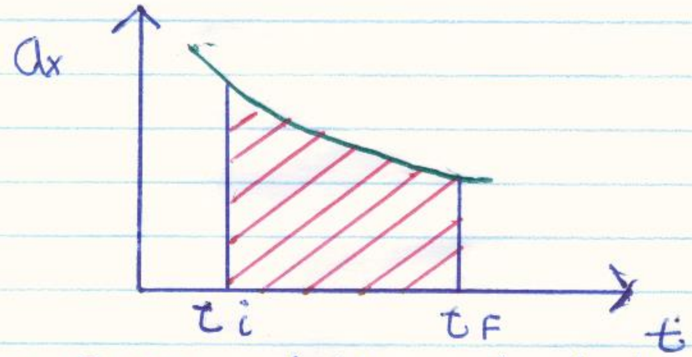
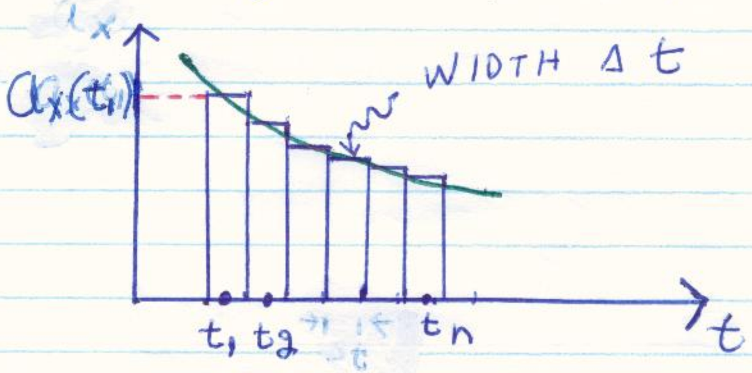
QUICK Introduction to Integrals

GRAPHICAL DEFINITION

Consider the a_x vs. t plot below



The **area** of the shaded portion UNDER the curve CAN be approXIMATED BY DRAWING thin strips of WIDTH Δt



- The height of the strips are $a_x(t_i)$, where $i = 1, 2, 3 \ldots n$
- The area of a strip is $a_x(t_i)\Delta t$

SHADED AREA $\approx a_x(t_1)\Delta t + a_x(t_2)\Delta t + \ldots + a_x(t_n)\Delta t$

The symbol $\approx$ approximately EQUAL

APPROXIMATION is better for SMALL Δt

In the limit $\Delta t \to 0$

$$\text{SHADED AREA} = \lim_{\Delta t \to 0}\left(a_x(t_1)\Delta t + a_x(t_2)\Delta t + \ldots a_x(t_n)\Delta t\right)$$

$$= \int_{t_i}^{t_F} a_x(t)\, dt$$

$\int_{t_i}^{t_F} a_x(t)\, dt$ – is the integral of a_x with respect to t

THE LIMITS OF INTEGRATION are FROM $t = t_i$ to $t = t_F$

Indefinite Integral

Consider two functions of the VARIABLE X: f(X), g(X)
An INDEFINITE INTEGRAL is defined as

$$\int f(x)\,dx = g(x)$$

g(x) is the anti-derivative of f(x)

$$\frac{\partial g(x)}{\partial x} = f(x)$$

$\boxed{\text{EXAMPLE}}$

$$\int (2.9)\,dx = 2.9x$$

$$\int x\,dx = \frac{x^2}{2}$$

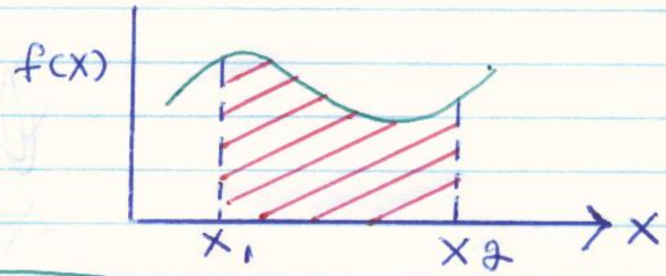
$$\int 5x^2\,dx = \frac{5}{3}x^3$$

DEFINITE INTEGRAL

For the DEFINITE Integral, the anti-derivative $g(x)$ is EVALUATED AT the LIMITS OF INTEGRATION x_1 & x_2

$$\int_{x_1}^{x_2} f(x)\,dx = g(x)\Big|_{x=x_1}^{x=x_2} = g(x_2) - g(x_1)$$

The DEFINITE integral is the area UNDER THE CURVE in the INTERVAL $x_1 < x < x_2$



EXAMPLE

$$\int_{-1}^{2} 3.6\,t^2\,dt = 1.2\,t^3\Big|_{-1}^{2} = 1.2(2)^3 - 1.2(-1)^3$$

$$= 9.6 + 1.2 = 10.8$$

$$\int_{0}^{t} 2t\,dt = t^2\Big|_{0}^{t} = t^2 - (0)^2$$

$$= t^2$$