Conservation of Momentum in a Nutshell

In section 9.1 we define the center of mass (com) position (9.1.7) and (9.1.8) of a system of N particle:

$$\vec{r}_{com} = \frac{1}{M}\sum_{i=1}^{N} m_i\vec{r}_i = x_{com}\hat{\imath} + y_{com}\hat{\jmath} + z_{com}\hat{k},$$

where the index i = 1, 2, 3, …N, where m_i is the mass of the i^th particle, $M = \sum_{i=1}^{N} m_i$ is the total mass, and $\vec{r}_i$ is its position. The relation for the x, y and z component are (9.1.5):

$$x_{com} = \frac{1}{M}\sum_{i=1}^{N} m_i x_i\,;\, y_{com} = \frac{1}{M}\sum_{i=1}^{N} m_i y_i\,;\, z_{com} = \frac{1}{M}\sum_{i=1}^{N} m_i z_i.$$

We can also define the **com velocity**:

$$\vec{v}_{com} = \frac{1}{M}\sum_{i=1}^{N} m_i\vec{v}_i = v_{com,x}\hat{\imath} + v_{com,y}\hat{\jmath} + v_{com,z}\hat{k},$$

where $\vec{v}_i$ is the velocity of the i^th particle. We can also define the **com acceleration**:

$$\vec{a}_{com} = \frac{1}{M}\sum_{i=1}^{N} m_i\vec{a}_i = a_{com,x}\hat{\imath} + a_{com,y}\hat{\jmath} + a_{com,z}\hat{k},$$

where $\vec{a}_i$ is the acceleration of the i^th particle.

Newton's second law for system of particles (section 9.2)

For a system of N particles, Equation 9.2.1 states Newton's second law for a system of N particles

$$\vec{F}_{net} = M\vec{a}_{com},$$

and equation 9.2.2 gives the components

$$F_{net,x} = Ma_{com,x}; F_{net,y} = Ma_{com,y}; F_{net,z} = Ma_{com,z}.$$

The **net force** $\vec{F}_{net} = \sum_{i=1}^{N} \vec{F}_{net,i}$ is the sum of all the individual net force acting on all particles, with $\vec{F}_{net,i}$ being the **net force** on the i^th **particle**. I will state without proof that the net force on the system $\vec{F}_{net}$must be from an external agent. For example, consider a **rock** made up of many atoms, the net force on it, $\vec{F}_{net}$, includes **gravity** on the **rock**, the **normal force** of the **ground** on the **rock**, and perhaps a **man pushing** the **rock**. The man, the ground and gravity are external agent (i.e. not part of the rock).

Conclusion: for a system of particle the effect of a net force is the same as that on a point at the **com position**, $\vec{r}_{com}$, with total mass M.

Isolated System and Conservation of Momentum

We define the **linear momentum**, $\vec{p} = m\vec{v}$, with unit $kg \cdot m \cdot s^{-1}$, see equation 9.3.1.

Now we define an **isolated system** of N particles to one with no external force acting on it, $\vec{F}_{net} = 0$, which gives $\vec{a}_{com} = 0$. Since $\vec{a}_{com} = \frac{d\vec{v}_{com}}{dt} = 0$, which means that

$$\vec{v}_{com} = constant$$

Hence for an isolated system, the com moves in a straight line at constant speed. But from earlier:

$$\vec{v}_{com} = \frac{1}{M}\sum_{i=1}^{N} m_i\vec{v}_i \rightarrow M\vec{v}_{com} = \sum_{i=1}^{N} m_i\vec{v}_i = constant$$

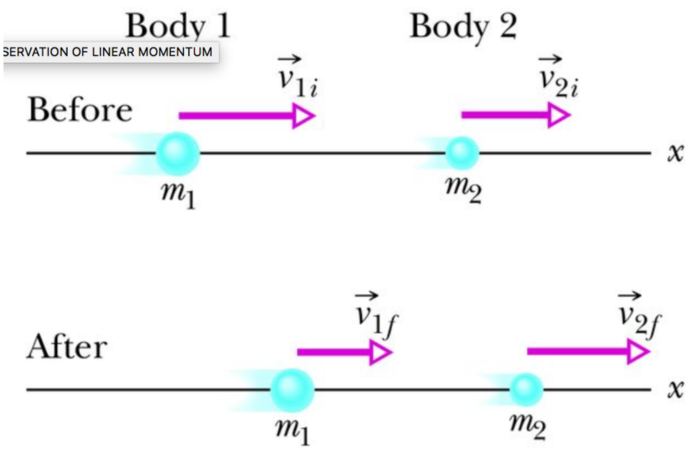
But $m_i\vec{v}_i = \vec{p}_i$ is the momentum of the i[th] particle:

$$M\vec{v}_{com} = \sum_{i=1}^{N} \vec{p}_i = \vec{P} = constant,$$

where $\vec{P} = \sum_{i=1}^{N} \vec{p}_i$ is the **total momentum** of the system. This means that the Total momentum of the **system** does not **change** with **time**.

<u>Simple conservation of momentum example in 1d</u>

The diagram below shows a 1D collision problem:



m_1 = 6 kg
m_2 = 2 kg
initial:
$v_{1i} = 4m/s;\, v_{2i} = 2m/s$
Final:
$v_{1f} = 1m/s;\, v_{2f} = ?$

Do this exercise to prepare for quiz 4

1) Use Conservation of Momentum to calculate the final velocity of body 2, v_{2f}.
   ANSWER: $v_{2f} = 11m/s$, to the right.
2) Calculate the change in Kinetic Energy:

$$\Delta K = K_f - K_i = \left(\frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2\right) - \left(\frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2\right)$$