9. Figure 8.9 shows three situations involving a plane that is not frictionless and a block sliding along the plane. The block begins with the same speed in all three situations and slides until the kinetic frictional force has stopped it. Rank the situations according to the increase in thermal energy due to the sliding, greatest first.

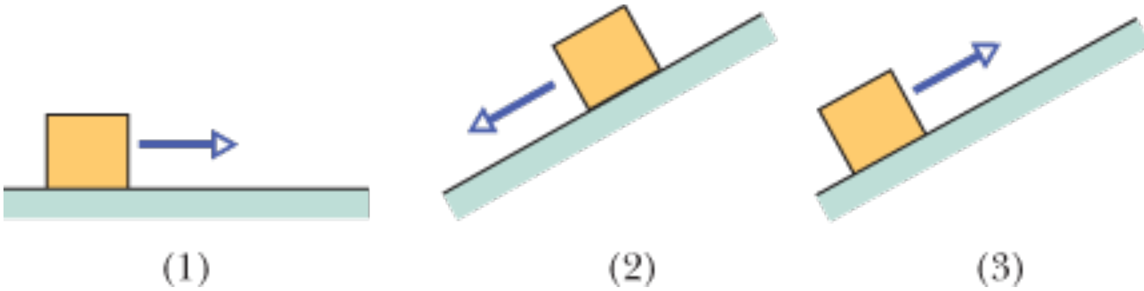


Figure 8.9 Question 9.

I confirm that the **answer is 2, 1, 3**. We use the conservation of energy equation 8.35:

$$W = \Delta E_{mec} + \Delta E_{th} + \Delta E_{int}$$

Clearly W = 0 (no external force) and $\Delta E_{int} = 0$ (assume no change in internal energy), and the change in Thermal Energy $\Delta E_{th} = f_k L$. Note that f_k and L will be different for each case. Hence using W = 0 and $\Delta E_{int} = 0$ and $\Delta E_{mec} = \Delta K + \Delta \text{U}$:

$$0 = \Delta E_{mec} + \Delta E_{th} \rightarrow \Delta E_{th} = -\Delta K - \Delta \text{U}$$

It is assumed that the friction will be enough to stop the motion ($v_f = 0$) of the box in all cases, so in all cases $\Delta K = K_f - K_i = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = -\frac{1}{2} m v_i^2$. In all cases (1,2, and 3) the initial speed, v_i, are the same. The change in gravitational potential energy is $\Delta \text{U} = \text{mg}\Delta \text{y}$, where Δy is the change in height. Combining all of this we have

$$\Delta E_{th} = -\Delta K - \Delta \text{U} = \frac{1}{2} m v_i^2 - \text{mg}\Delta \text{y}$$

We will assume that $\Delta E_{th} > 0$.

Case (1): $\Delta \text{y} = 0$

Case (2): $\Delta \text{y} > 0$ or $\Delta \text{y} < 0$?

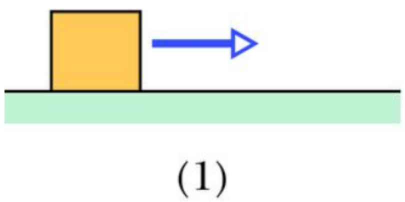
Case (3): $\Delta \text{y} > 0$ or $\Delta \text{y} < 0$?

The challenge is to use the above information to convince yourself that the order is:
answer is 2, 1, 3

Go to next page for explanation

Solution:

Case (1):



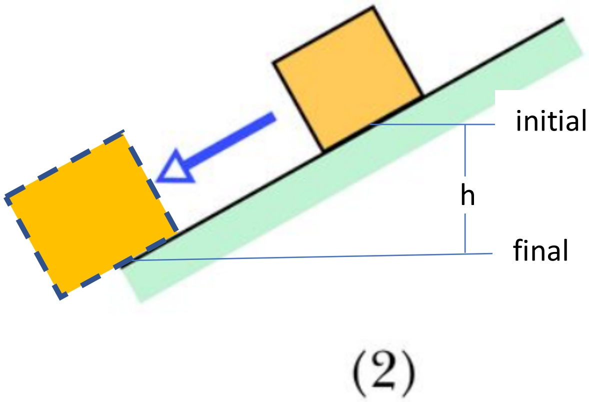
$$\Delta E_{th} = -\Delta K - \Delta \mathrm{U} = \frac{1}{2} m v_i^2 - \mathrm{mg}\Delta \mathrm{y}$$

There is no change in height, $\Delta \mathrm{y} = 0$:

$$\Delta E_{th,case\ 1} = \frac{1}{2} m v_i^2$$

Note that despite the fact that the thermal energy $\Delta E_{th,case\ 1} = f_k L$, we don't need to know the actual force of friction, f_k or the distance traveled by the box L, since $\Delta E_{th,case\ 1}$ must equal the initial kinetic energy of the box.

Case (2):



The box is initially moving down with speed v_i, as it moves down it will be opposed by the friction force f_k, but there will also be a gravitational force $mgsin\theta$ pushing the box down the incline. It is assumed (see question above) that $f_k > mgsin\theta$, so that eventually the kinetic friction will stop the motion of the box.

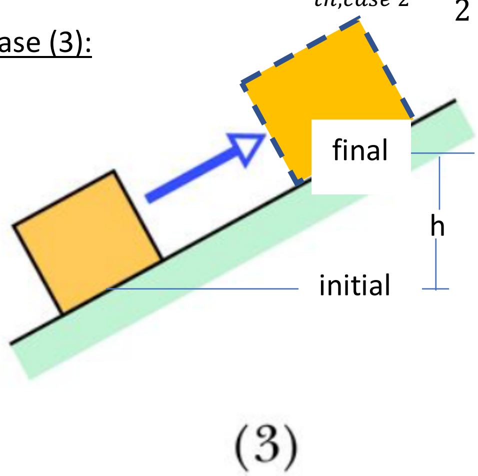
As shown to the left, the box **moves down** a **height** of h, $\Delta \mathrm{y} = -\mathrm{h}$, so that we have:

$$\Delta E_{th,case\ 2} = -\Delta K - \Delta \mathrm{U} = \frac{1}{2} m v_i^2 + \mathrm{mgh}$$

Clearly

$$\Delta E_{th,case\ 2} = \frac{1}{2} m v_i^2 + \mathrm{mgh} > \Delta E_{th,case\ 1} = \frac{1}{2} m v_i^2$$

Case (3):



The box is initially moving up with speed v_i, as it moves down it will be opposed by the friction force f_k, and also by a gravitational force $mgsin\theta$, with both forces pushing the box down the incline, against the velocity of the box. This will slow the box down till it stop moving. It is assumed (see question above) that $f_{s,max} > f_k > mgsin\theta$, so that the box will not start moving again.

As shown to the left, the box **moves up** a **height** of h, $\Delta \mathrm{y} = \mathrm{h}$, so that we have:

$$\Delta E_{th,case\ 2} = -\Delta K - \Delta \mathrm{U} = \frac{1}{2} m v_i^2 - \mathrm{mgh}$$

Clearly

$$\Delta E_{th,case\ 2} = \frac{1}{2} m v_i^2 - \mathrm{mgh} < \Delta E_{th,case\ 1} = \frac{1}{2} m v_i^2$$

Putting this all together $\Delta E_{th,case\ 2} > \Delta E_{th,case\ 1} > \Delta E_{th,case\ 3}$. Hence the order is 2, 1 , 3.